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Hedging Global Stock Markets with Bitcoin, Precious Metals, Copper, Crude Oil, and Agricultural Commodities: Evidence from Bivariate Threshold GARCH Approach

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Abstract

Purpose: This study examines the effectiveness of hedging global stock markets with hedge assets, including bitcoin, precious metals, copper, crude oil, and agricultural commodities. To achieve this, we selected eleven global stock market indices, including ASX 200, MSCI US, MSCI Europe, MSCI Japan, HSI, IBOVESPA, BSESN, SSECI, STI, TAIEX, and TSXCI to evaluate the effectiveness of hedging with the aforementioned hedge assets. Consequently, our current study offers a more up-to-date and comprehensive comparison of hedging effectiveness among multiple classes of hedge assets than earlier academic work.

Study design/methodology/approach: We collected weekly price data, denominated in USD, from Eikon (https://eikon.refinitiv.com/), covering the period from May 1, 2018, to March 2, 2023. For the analysis, this study utilized the bivariate diagonal BEKK-TGARCH and OLS models to estimate time-varying and static hedge ratios, respectively, with the goal of measuring the effectiveness of hedging.

Findings: In the empirical analysis of the BEKK-TGARCH model, we find that the return spillover effects are weak, and past information shocks influence the current variance-covariances of returns on most hedge assets but not on stock market returns. Moreover, the stock markets exhibit a stronger asymmetric leverage effect than the hedge assets. Furthermore, the BEKK-TGARCH model demonstrates greater hedging effectiveness than the OLS. Silver, copper, and crude oil emerge as highly effective hedge assets, whereas agricultural commodities are the least effective. Finally, ASX 200 and TSXCI are the most effectively hedged stock markets.

Practical Implications: This study evaluates the effectiveness of various hedge assets for hedging global stock markets and identifies the most effective hedge assets. Thus, our research is connected to the field of decision sciences, providing insights into hedging processes and optimal strategies for portfolio managers and hedgers.

Keywords: hedge ratio, hedging effectiveness, BEKK-TGARCH

JEL: C58, G11

1. Introduction

Hedging is a risk management strategy that helps reduce potential losses from fluctuations in the prices of financial securities, currencies, and commodities by trading in hedge assets that are negatively correlated or uncorrelated with the original investment. Hedging is most effective when it minimizes the variance of the hedged portfolio. Moreover, hedging aids in formulating investment strategies that enhance expected returns per unit of risk. Investors may also adopt more aggressive strategies when their investment risk is controlled within a limited range through hedging.

Since futures hedging is cost-effective (Lien & Tse, 2002; Arenas-Falótico & Scudiero, 2023), hedge assets are commonly traded in futures markets in the literature. For example, crude oil futures are among the most widely utilized hedge assets (Yu, et al., 2023). Additionally, earlier studies have shown that metal futures exhibit a positive correlation with stock markets and are suitable for hedging purposes. For instance, copper futures are often employed to hedge against investment risks (Chen, 2023). Therefore, it is crucial to enhance the effectiveness of both crude oil futures and copper futures in hedging against stocks. Importantly, Bitcoin is a new product in financial markets, making it worthwhile to examine its hedging effectiveness (Haliplii, 2020). Additionally, futures contracts for precious metals and agricultural commodities are traded on various commodity exchanges around the world, and their hedging ability is also worth studying (Hanif, et al., 2023).

Previous academic work shows that the results of hedging performance are mixed so far, depending upon the stock markets being hedged, the selection of hedge assets, the methods adopted, and the sample periods covered. Furthermore, typically only one or two hedge assets were employed in each study (e.g., Batten, et al., 2021; Bunditsakulporn, 2022; Chen, 2023; Kangalli Uyar, et al., 2022; Okorie, 2020). making it difficult to compare hedging performance among different hedge assets. Our study aims to examine the effectiveness of hedging global stock markets with multiple classes of hedge assets, including futures of bitcoin (cryptocurrency) gold, silver, palladium (precious metals), copper (industrial metal), crude oil (energy commodity), corn, orange juice, and wheat (agricultural commodities). Eleven stock market indices were selected to represent the global stock markets: ASX 200 Index in Australia, MSCI USA Index, MSCI Europe Index, MSCI Japan Index, Bombay Stock Exchange Sensitive Index (BSESN) in India, Bovespa Index (IBOVESPA) in Brazil, Hang Seng Index (HSI) in Hong Kong, Shanghai Stock Exchange Composite Index (SSECI) in China, Strait Times Index (STI) in Singapore, Taiwan Stock Exchange Capitalization Weighted Stock Index (TAIEX) in Taiwan and TSX Composite Index (TXCX) in Canada. The selected stock markets cover the areas in Asia, Europe, and America, and are located in both developed and emerging regions. We employ the threshold GARCH (TGARCH) and OLS methods for model estimation during the recent period. Hence, our findings are expected to provide a more up-to-date and comprehensive comparison of the hedging performance of different hedge assets in global stock markets than those in previous studies.

The rest of this paper is organized as follows. Section 2 reviews the literature; Section 3 introduces research methods, followed by data description in Section 4. Section 5 documents empirical results, and Section 6 concludes.

2. Literature Review

A large body of extant literature has focused on estimating hedge ratios between pairs of returns on stocks and hedge assets to estimate hedging effectiveness. Hedge assets vary across different asset classes, such as cryptocurrency, precious and industrial metals, and energy and agricultural commodities.

Bitcoin is widely recognized as the most actively traded cryptocurrency, dominating the cryptocurrency markets in terms of trading volume and market capitalization (Mikhaylov, 2020; Fang, et al., 2022). It is the most popular cryptocurrency compared with others such as Ethereum, Litecoin, Tether, and Solana. Bitcoin is not only considered an investment vehicle but also has the potential to provide hedging against stock market fluctuations (Xu & Kinkyo, 2023). Most importantly, negative or zero correlations with other asset classes make bitcoin attractive for hedging purposes (Wong, et al., 2018).

Precious metals, particularly gold and silver, have also been popular as hedge assets in investment portfolios. Gold is widely regarded as a safe-haven asset, and investors often hold gold during times of economic uncertainty or financial crises (Rizvi, et al., 2022). Furthermore, gold tends to retain its value during periods of inflation. As fiat currencies lose purchasing power, gold value often increases, making it an effective hedge against inflation (Valadkhani, et al., 2022). Gold has shown a negative correlation with stock market movements, providing diversification and serving as a hedging asset in investment portfolios (Ali, et al., 2020). Besides gold, silver also has a low correlation with other asset classes, so investors can diversify their portfolios with silver for risk management (Chiang, 2022; Paul, et al., 2023). Silver also serves as a hedge against inflation risk (Adekoya, et al., 2021). Furthermore, like gold, silver is considered a safe haven during periods of economic uncertainty and financial distress (Dibooglu, et al., 2022). Furthermore, like gold and silver, palladium also helps in hedging against inflation, market fluctuations, and financial crises (Salisu, et al., 2019; Naeem, et al., 2022; Mensi, et al., 2023).

Copper is a key industrial metal used in many sectors, such as construction, renewable energy, and electronics. Its unique characteristics make it attractive for hedging and diversification purposes (Chen, 2023). Copper is an important raw material, and therefore, its price often moves together with inflation. As inflation rises, demand for copper in manufacturing and other industrial applications also increases, thereby supporting the price of copper. Hence, copper can be an effective hedge against inflation (Adekoya, et al., 2023).

Crude oil is a prominent commodity, and its prices affect the international economy and trade. Furthermore, oil prices and exchange rates influence each other (Beckmann, et al., 2020). Hedging against fluctuations in oil prices and exchange rates can be beneficial to investments in currencies and oil (Olstad, et al., 2021). Hedging and diversification functions of crude oil are important for oil exporters as well as importers (Ji, et al., 2020; Liu & Lee, 2022; Ming, et al., 2023). Due to low or negative correlations, especially during crisis periods, oil performs better in hedging and diversification functions (Liu, et al., 2020). Moreover, Batten, et al. (2021) argue that hedging stocks with crude oil is possible when crude oil and the stock markets are relatively independent of each other in terms of returns. Because of this, crude oil is used to hedge against the risk of changes in stock markets (Foroni, et al., 2017; Xu, et al., 2020; Shahzad, et al., 2022), especially when the stock markets are extremely volatile (Boubaker & Larbi, 2022).

Agricultural commodities and their prices move differently from the prices of non-agricultural financial assets. Prices of agricultural commodities are usually determined by their own demand and supply dynamics in their respective physical markets. Under these circumstances, agricultural commodities can be used to hedge against inflation and protect investors against a decrease in the purchasing power of money during inflationary periods (Spencer, et al., 2018). Further, compared with financial products, agricultural commodities have specific risk factors, which include unpredictable weather conditions, supply chain disruptions, and changing demand. The unique characteristics of agricultural markets often result in a low correlation with stock markets, enabling agricultural commodities to act as excellent diversification instruments for investors and providing the opportunity to hedge stock portfolio risk (Hernandez, et al., 2021; Gunera, 2023).

3. Methodology

This paper employs the bivariate asymmetric diagonal Baba-Engle-Kraft-Kroner-Threshold GARCH (BEKK-TGARCH) model of Engle and Kroner (1995) to estimate the time-varying hedge ratios for the construction of hedged portfolios, which is useful to measure hedging effectiveness. We also estimate static hedge ratios using the ordinary least squares (OLS) model for comparison.[1]

3.1 Bivariate Diagonal BEKK-TGARCH Model

To begin with, we define $R_{f,t}$ and $R_{s,t}$ as returns of hedge assets and stock market indices, respectively, from time t_{-1} to time t. Then, we use the Vector Autoregressive Model of order 1, VAR (1), to model the mean equations of $R_{f,t}$ and $R_{s,t}$.

The mean equations of $R_{f,t}$ and $R_{s,t}$ are modelled in bivariate VAR (1), as follows:

$$R_{f,t} = \mu_{f0} + \mu_{f1}R_{f,t-1} + \mu_{f2}R_{s,t-1} + \varepsilon_{f,t}, \tag{1}$$

$$R_{s,t} = \mu_{s0} + \mu_{s1}R_{f,t-1} + \mu_{s2}R_{s,t-1} + \varepsilon_{s,t}, \tag{2}$$

where $\begin{bmatrix}\varepsilon_{f,t}\\ \varepsilon_{s,t}\end{bmatrix} \mid \Omega_{t-1} \sim N\,(0, H_t)$.

$\varepsilon_{f,t}$and $\varepsilon_{s,t}$ represent the returns of the selected hedge assets and stock indices, at time t, while Ω_{t-1} represents the information available at time t-1. Besides, H_t represents the conditional variance-covariance matrix of the error terms at time t. Also, $\mu_{f\,i}$ and $\mu_{s\,i}$, i = 0,1,2, denote the coefficients of mean Equations (1) and (2). μ_{f2} and μ_{s1} measure the return spillovers from stock market indices to

[1] DCC proposed by Engle (2002) is another popular GARCH model. However, Caporin and McAleer (2013) discuss caveats and limits about the use of DCC and consider that BEKK is more general as it allows for direct spillovers and feedback effects across conditional variance and covariances, as well as indirect spillovers and feedback effects across conditional correlations.

hedge assets and from hedge assets to stock market indices, respectively.[2] Moreover, μ_{f1} (μ_{s2}) measures how past returns on a hedge asset (a stock market index) influence its current returns.

Further, we adopt the diagonal BEKK-TGARCH model to estimate the variance-covariance matrix H_t. The method helps ensure H_t to be positive definite so that it addresses the problem with the diagonal VECH model, and it allows for the threshold asymmetric leverage effect in GARCH specification. A general form of the bivariate BEKK-TGARCH (1,1,1) is:

$$H_t = C'C + A'\varepsilon_{t-1}\varepsilon'_{t-1}A + B'H_{t-1}B + D\varepsilon_{t-1}I_{t-1}D'\varepsilon'_{t-1}I_{t-1}, \tag{3}$$

where A, B, and D are 2×2 diagonal matrices of parameters and C is an upper triangular matrix of parameters. H_t is a 2×2 conditional variance-covariance matrix, $I_{t-1} = 1$ if $\varepsilon_{t-1} < 0$ and $= 0$ otherwise. I_{t-1} is a dummy and is used to measure the leverage effect. Alternatively, Equation (3) can be shown as:

$$\begin{bmatrix} h_{11,t} & h_{12,t} \\ h_{21,t} & h_{22,t} \end{bmatrix} = CC' + \begin{bmatrix} a_{11} & 0 \\ 0 & a_{22} \end{bmatrix} \begin{bmatrix} \varepsilon^2_{1,t-1} & \varepsilon_{1,t-1}\varepsilon_{2,t-1} \\ \varepsilon_{2,t-1}\varepsilon_{1,t-1} & \varepsilon^2_{2,t-1} \end{bmatrix} \begin{bmatrix} a_{11} & 0 \\ 0 & a_{22} \end{bmatrix} + \begin{bmatrix} b_{11} & 0 \\ 0 & b_{22} \end{bmatrix} \begin{bmatrix} h_{11,t-1} & h_{12,t-1} \\ h_{21,t-1} & h_{22,t-1} \end{bmatrix} \begin{bmatrix} b_{11} & 0 \\ 0 & b_{22} \end{bmatrix} + \begin{bmatrix} d_{11} & 0 \\ 0 & d_{22} \end{bmatrix} \begin{bmatrix} \varepsilon^2_{1,t-1} & \varepsilon_{1,t-1}\varepsilon_{2,t-1} \\ \varepsilon_{2,t-1}\varepsilon_{1,t-1} & \varepsilon^2_{2,t-1} \end{bmatrix} \begin{bmatrix} d_{11} & 0 \\ 0 & d_{22} \end{bmatrix} * I_{t-1},$$

where $C = \begin{bmatrix} C_{11} & C_{12} \\ 0 & C_{22} \end{bmatrix}$ is the matrix of intercept coefficients C_{11}, C_{12} and C_{22}, and a_{ij} denotes the coefficient of the ARCH term $\varepsilon^2_{i,t-1}$ while b_{ij} is the coefficient of the GARCH term $h_{ij,t-1}$. In addition, d_{ij} is the coefficient to measure the threshold asymmetric effects of negative and positive news. Equation (3) can be alternatively written as:

$$h_{f,t} \equiv h_{11,t} = C_{11} + a^2_{11}\varepsilon^2_{1,t-1} + b^2_{11}h_{11,t-1} + d^2_{11}\varepsilon^2_{1,t-1}I1, \tag{4}$$

$$h_{s,t} \equiv h_{22,t} = C_{22} + a^2_{22}\varepsilon^2_{2,t-1} + b^2_{22}h_{22,t-1} + d^2_{22}\varepsilon^2_{2,t-1}I2, \tag{5}$$

$$h_{f.s,t} \equiv h_{12,t} = C_{12} + a_{11}a_{22}\varepsilon_{1,t-1}\varepsilon_{2,t-1} + b_{11}b_{22}h_{12,t-1} + d_{11}d_{22}\varepsilon_{1,t-1}I_{1,t-1}\varepsilon_{2,t-1}I_{2,t-1}, \tag{6}$$

where a_{11} and a_{22} are used to measure the impacts of past information shocks on the current conditional variance-covariances of hedge asset returns and stock index returns, respectively. b_{11} and b_{22} measure the effects of past conditional variance-covariances of hedge asset returns and stock index returns on their current conditional variance-covariances. d_{11} and d_{22} indicate the asymmetric leverage effect of hedge assets and stock indices on the current conditional variance-covariances.

3.2 Optimal Hedge Ratio

[2] Coronado, et al. (2020) and Feng, et al. (2023) document recent methods to measure time-varying spillovers.

The optimal hedge ratio is then estimated to construct the minimum-variance hedged portfolio:

$$R_{p,t} = R_{s,t} - \beta_t R_{f,t}, \tag{7}$$

where $R_{p,t}$, $R_{s,t}$ and $R_{f,t}$ are returns of hedged portfolios, stock market indices, and hedge asset futures, respectively, between time t_{-1} and time t. β_t represents the optimal hedge ratio between time t_{-1} and time t. The optimal hedge ratio is obtained from the minimization of the conditional variance of the return on the hedged portfolio ($h_{p,t}$) (Baillie and Myers, 1991):

$$\beta_t = \frac{h_{f,s,t}}{h_{f,t}} \tag{8}$$

where $h_{f.s,t}$ and $h_{f,t}$ are time-varying conditional covariances between hedge asset futures returns and global stock market index returns, and the conditional variance of hedge asset returns, respectively, at time t, estimated from Equations (4) and (6) using the diagonal BEKK-TGARCH approach. When $h_{f,s,t}$ is positive (negative), β_t would be positive (negative).

Furthermore, we use OLS to calculate static optimal hedge ratios for comparison:

$$R_{s,t} = \alpha + \beta R_{f,t} + \mu_t, \tag{9}$$

where the OLS estimated hedge ratio is $\beta = \frac{h_{f,s}}{h_f}$. (10)

3.3 Hedging Effectiveness

Hedging effectiveness (HE) is a measure of the capability of a hedge asset to mitigate the risk level of a hedged portfolio (Ku, et al., 2007), and it is written as:

$$\text{Hedging effectiveness (HE)} = \frac{\text{Variance}_{\text{unhedged}} - \text{Variance}_{\text{hedged}}}{\text{Variance}_{\text{unhedged}}}, \tag{11}$$

where $Variance_{unhedged}$ denotes the variance of return on the unhedged portfolio, while $Variance_{hedged}$ denotes the variance of return on the hedged portfolio.

4. Data

This study collected weekly indices of eleven global stock markets, which include ASX 200 (Australia), MSCI USA (United States), MSCI Europe (Europe), MSCI Japan (Japan), IBOVESPA (Brazil), BSESN (India), HSI (Hong Kong), SSECI (China), STI (Singapore), TAIEX (Taiwan), and TSXCI (Canada). We also collected futures price data on nine hedge assets, including bitcoin (cryptocurrency), gold, silver, palladium (precious metals), copper (industrial metal), crude oil (energy commodity), as well as corn, orange juice, and wheat (agricultural commodities). The data are all denominated in USD and are sourced from Eikon (https://www.lseg.com/en/data-analytics/refinitiv). The sample period spans May 1, 2018, to March 2, 2023, with a total of 266 observations. We converted the data into a natural logarithmic form for analysis. Tables 1 and 2 provide the descriptive statistics of all return series generated by the difference of the natural logarithm of the series at the level.[3]

From Table 1, IBOVESPA has the highest returns and the highest standard deviation, indicating that the Brazilian stock market fluctuates the most compared with other stock markets. Likewise, as shown in Table 2, bitcoin's return fluctuates the most among hedge assets. Furthermore, not all return series are normally distributed. It justifies the use of the t-distribution instead of the normal distribution for estimation of the parameters in Equations (1)–(3).

Table 1. Descriptive Statistics of Stock Index Return Series

	ASX200	MSCI US	MSCI Europe	MSCI Japan	IBOV-ESPA	BSESN	HSI	SSECI	STI	TAIEX	TSXCI
Mean	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Median	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00
Maximum	0.12	0.12	0.09	0.16	0.16	0.13	0.08	0.08	0.11	0.10	0.11
Minimum	-0.20	-0.16	-0.23	-0.17	-0.25	-0.15	-0.10	-0.10	-0.14	-0.12	-0.19
Std. Dev.	0.03	0.03	0.03	0.03	0.05	0.03	0.03	0.03	0.03	0.03	0.03
Skewness	-1.73	-0.83	-1.73	-0.20	-0.50	-0.45	-0.22	-0.37	-0.51	-0.68	-1.94
Kurtosis	14.05	8.66	15.60	12.86	5.88	7.31	3.33	3.67	9.21	5.35	15.22
Jarque-Bera	1481.23*	383.67*	1886.48*	1075.79*	102.30*	213.74*	3.34	10.87*	436.96*	81.16*	1813.85*
Obs.	266.00	266.00	266.00	266.00	266.00	266.00	266.00	266.00	266.00	266.00	266.00

Notes: The Jarque-Bera statistic is used to test the null hypothesis that the data have a normal distribution and follow an asymptotically chi-squared distribution with two degrees of freedom. *, **, *** denote statistical significance at the 1%, 5% and 10% level, respectively.

Table 2. Descriptive Statistics of Hedge Asset Futures Return Series

	BITCOIN	GOLD	SILVER	PALLA-DIUM	COPPER	CRUDE OIL	CORN	ORANGE JUICE	WHEAT
Mean	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Median	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Maximum	0.30	0.09	0.16	0.35	0.10	0.15	0.14	0.16	0.47
Minimum	0.54	-0.10	-0.17	-0.47	-0.12	-0.18	-0.25	-0.15	-0.21
Std. Dev.	0.10	0.02	0.04	0.06	0.03	0.00	0.04	0.04	0.05
Skewness	0.80	-0.15	-0.36	-1.00	-0.19	-0.39	-0.85	0.11	2.50
Kurtosis	6.39	6.45	6.64	18.43	4.28	6.88	9.60	4.04	28.71
Jarque-Bera	154.93*	132.61*	152.24*	2674.08*	19.67*	173.19*	512.51*	12.40*	7573.69*
Obs.	266.00	266.00	266.00	266.00	266.00	266.00	266.00	266.00	266.00

See notes to Table 1.

5. Empirical Results

5.1 Unit Root Test

[3] The price trends of all stock indices and hedge asset futures are depicted in Figures A1 and A2 in Woo and Zheng (2025). Available at: http://hdl.handle.net/20.500.11861/10707.

The unit root test is conducted using the Augmented Dickey-Fuller (ADF) test, with the results shown in Table 3. This test shows that the price series are all non-stationary in level and their return series are stationary. The subsequent results from the diagonal BEKK-TGARCH model are not spurious.

Table 3. Results of Unit Root Tests

Stock indexes	Level	First Difference
ASX 200	-2.98	-10.00*
MSCI US	-2.36	-9.01*
MSCI Europe	-2.53	-9.00*
MSCI Japan	-2.13	-9.12*
IBOVESPA	-2.85	-10.06*
BSESN	-2.19	-6.35*
his	-2.52	-7.77*
SSECI	-2.33	-5.94*
STI	-2.04	-10.94*
TAIEX	-1.62	-3.14***
TSXCI	-2.64	-10.47*
Hedge Asset Futures		
Bitcoin	-2.16	-3.85**
Gold	-1.66	-5.43*
Silver	-2.27	-5.59*
Palladium	-0.48	-17.08*
Copper	-2.06	-3.60**
Crude oil	-1.52	-7.01*
Corn	-2.18	-4.66*
Orange Juice	-0.90	-16.17*
Wheat	-2.85	-9.49*

Notes: An intercept and a linear trend are included in the ADF regression. The critical value is -3.99 at the 1% level, -3.43 at the 5% level, and -3.14 at the 10% level. *, **, *** denote statistical significance at the 1%, 5%, 10% level, respectively

5.2 Estimation of Bivariate Diagonal BEKK-TGARCH Model

5.2.1 Results of Mean Equations

We estimate the mean Equations (1) and (2) in a bivariate VAR (1). The return spillovers from stock market indices to hedge assets and from hedge assets to stock market indices can be measured from the coefficients μ_{f2} and μ_{s1} respectively. Suppose that μ_{f2} is positive (negative) and statistically significant; in this case, an increase in lagged stock market returns leads to a rise (fall) in current hedge asset returns. On the other hand, if μ_{s1} is positive (negative) and statistically significant, a rise in lagged hedge asset returns has positive (negative) impacts on current stock market returns.

The results (which are unreported) demonstrate that the evidence of return spillovers from lagged stock indices to hedge assets is negligible, except in some cases of hedging with gold (copper and crude oil) where the parameters μ_{f2} are significant and negative (positive).[4] In other words, the hedged asset returns cannot be predicted by the past stock market returns in most cases. However, a rise in stock

[4] To save space, the empirical results of mean equations are all reported in Tables 4-12 in Woo and Zheng (2025).

markets (ASX 200, MSCI US, MSCI Japan, HSI, SSECI, and STI) would lead to a fall in the next period's return of gold. Also, a rise in returns from stock markets (except HSI and SSECI) would lead to a rise in the next period's return on copper. Similarly, a rise in returns on oil is predicted by a rise in past returns of stock markets (ASX 200, MSCI Europe, BSESN, TAIEX, and TSXCI).

Moreover, evidence of spillovers from the lagged returns of hedge assets to current stock index returns is also negligible when the coefficient μ_{s1} is insignificant. For example, lagged returns on gold and wheat futures cannot influence the current returns of all stock markets. Moreover, a rise in past returns on bitcoin, oil, and corn helps predict a fall in current returns of stock markets, as two cases have significant and negative values of μ_{s1}. Therefore, it is concluded that current stock index returns are not likely to have been affected by lagged returns in hedge asset markets. We conclude that evidence of return spillovers between stock markets and hedge assets is weak.

In addition, we can measure how past returns on one asset (hedge asset or stock index) influence its current returns from the coefficients μ_{f1} and μ_{s2}. However, such evidence is also scant since μ_{f1} and μ_{s2} are insignificant in most cases. Furthermore, μ_{f1} is negative and significant in some cases where copper, oil, corn, and wheat act as hedge assets. This indicates a reversal pattern of returns, implying that current positive (negative) returns are predicated by their past negative (positive) returns.

5.2.2 Results of Variance-Covariance Equations

Transmission of conditional variance-covariances between hedge assets and stock market indices is measured from the estimated coefficients in Equations (4)-(6).[5] Empirical results (unreported) indicate that most of the a_{11} coefficients are statistically significant (except for bitcoin and palladium), and almost all a_{22} coefficients are insignificant. Hence, past information shocks mostly increase the current conditional variance of the returns on hedge assets (except for bitcoin and palladium) but not stock market returns.[6]

Moreover, almost all d_{22} coefficients are significant (except in cases where bitcoin is used as a hedge asset), but almost all d_{11} coefficients are insignificant. Therefore, our study finds strong evidence of an asymmetric leverage effect on the conditional variance of stock market returns.[7] However, there is limited evidence of threshold asymmetry in the conditional variance of returns on hedge assets.

Since a_{22} and d_{11} coefficients are mostly insignificant, evidence for the overall impacts of past shocks and asymmetric leverage on the conditional covariance in Equation (6) is weak. In other words, past information shocks from hedge assets or stock market indices do not significantly and asymmetrically influence the current conditional covariance.

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[5] Ibid, Tables 13-21 for all the results of the variance-covariance equations.
[6] a_{11}^2 and $\varepsilon_{1,t-1}^2$ must be positive (even though a_{11} and $\varepsilon_{1,t-1}$ may be negative) so that the associated effects of shocks on conditional variance in Equation (4) must be positive. The same logic can be applied to other coefficients in Equations (4) and (5).
[7] There is no evidence of asymmetry in the conditional variance of return on SSECI except when oil is a hedge asset.

On the other hand, most of the b_{11} and b_{22} coefficients are significant, and therefore, there is strong evidence that the current conditional variance-covariances can be predicted by their past data.

5.3 Optimal Hedge Ratio

5.3.1 Time-Varying Optimal Hedge Ratio

We estimate the optimal hedge ratios β_t to construct hedged portfolios using $h_{f.s,t}$ and $h_{f,t}$ (Equation 8) obtained from the diagonal BEKK-TGARCH model. The minimum-variance hedged portfolio (Equation 7) is created by shorting (buying) β_t dollars of hedge asset for every dollar of stock index in a long position if β_t is positive (negative).

Since the estimates of β_t vary over time, we use descriptive statistics (mean, median, standard deviation, minimum, and maximum) to illustrate their characteristics throughout the sample period, with the results for all hedged portfolios reported by Woo and Zheng (2025).[8] Specifically, Table 4 presents the mean optimal hedge ratios of all stock-copper pairs. We calculate the average of these mean values (0.38), which is the highest among all hedged portfolios, followed by the stock-gold and stock-oil pairs (0.31).[9] Table 5 indicates that the average of the mean optimal hedge ratios of all stock-wheat pairs (0.02) is the lowest among all hedged portfolios, slightly below the stock-corn and stock-orange juice pairs (0.053).[10] Thus, the values of β_t influence the construction of hedged portfolios.

Particularly, the IBOVESPA-copper pair in Table 4 exhibits the highest mean optimal hedge ratio of 0.57 among all stock-copper pairs. This implies that an investor, on average, can short 0.57 dollars of copper futures for each dollar of IBOVESPA in a long position. Its maximum hedge ratio is up to 2.06, the largest among all stock-copper pairs, meaning that an investor needs to short 2.06 dollars of copper futures at the maximum for hedging his one dollar of IBOVESPA. For the ASX 200-copper pair, its minimum hedge ratio is -0.40, the lowest among all stock-copper pairs. A negative hedge ratio implies that the prices of ASX 200 and copper futures move in opposite directions with negative conditional covariance as shown in Equation (8), thus warranting the hedgers to take long positions in both assets. Hence, a hedger can long 0.4 dollars of copper futures for each dollar of ASX 200 purchased in the hedged portfolio. Moreover, the TAIEX-wheat pair and the SSECI-wheat pair (Table 5) have the lowest mean optimal hedge ratio of -0.02 among all stock-wheat pairs, suggesting that, on average, for every dollar invested in TAIEX and SSECI, 0.02 dollars should be invested in wheat futures for hedging.

Table 4. Descriptive Statistics for Time-Varying Optimal Hedge Ratio of Stock-Copper Pairs

	Mean	Median	St. Dev	Minimum	Maximum
ASX 200/copper	0.39	0.42	0.13	-0.40	1.33
MSCI US/copper	0.28	0.25	0.15	-0.09	1.11
MSCI Europe/copper	0.37	0.40	0.10	0.10	1.24
MSCI Japan/copper	0.27	0.28	0.07	0.03	0.98

[8] Ibid, Tables 22-30.

[9] Ibid, Tables 23 and 27.

[10] bid, Tables 28-30.

HSI/copper	0.50	0.54	0.09	0.17	0.60
IBOVESPA/copper	0.57	0.51	0.22	0.15	2.06
BSESN/copper	0.27	0.23	0.13	0.09	1.21
SSECI/copper	0.44	0.47	0.10	0.09	0.58
STI/copper	0.33	0.35	0.08	0.13	0.68
TAIEX/copper	0.41	0.43	0.09	0.18	0.76
TSXCI/copper	0.37	0.34	0.17	0.15	1.71

Note: The average of the mean hedge ratios of all stock-copper pairs is around 0.38.

Table 5. Descriptive Statistics for Time-Varying Optimal Hedge Ratio of Stock-Wheat Pairs

	Mean	Median	St. Dev	Minimum	Maximum
ASX 200/ wheat	0.03	0.04	0.09	-0.57	0.46
MSCI US/ wheat	0.01	0.02	0.12	-0.86	0.59
MSCI Europe/ wheat	0.01	0.01	0.05	-0.11	0.24
MSCI Japan/wheat	0.04	0.03	0.02	0.00	0.18
HSI/ wheat	0.01	0.02	0.03	-0.11	0.12
IBOVESPA/ wheat	0.18	0.21	0.11	-0.15	0.73
BSESN/ wheat	0.00	0.00	0.04	-0.10	0.12
SSECI/ wheat	-0.02	-0.02	0.04	-0.18	0.24
STI/ wheat	0.01	0.00	0.03	-0.02	0.15
TAIEX/ wheat	-0.02	-0.01	0.03	-0.19	0.02
TSXCI/ wheat	0.02	0.03	0.02	-0.06	0.07

Note: The average of the mean hedge ratios of all stock-wheat pairs is around 0.02.

All time-varying optimal hedge ratios are depicted in Figure A3 as in Woo and Zheng (2025), where the hedge ratios, in most cases, increase in the first quarter of 2020 at the outbreak of the COVID-19 epidemic and then drop in the second quarter. The reason is that the COVID-19 epidemic initially led to an instantaneous and sharp collapse in both stock markets and hedge assets, together with an upsurge in their covariances and hedge ratios. After one quarter, the panic in the markets gradually faded, and then prices rebounded, albeit at varying paces for different stock markets and hedge assets. Their covariances and the hedge ratios accordingly plunged back to levels prevailing before the outbreak of COVID-19.

5.3.2. Static Optimal Hedge Ratio

Table 6 reports the static optimal hedge ratios estimated using OLS as shown in Equation (10). The average static optimal hedge ratio of all stock-gold portfolios is 0.47, which is the highest, followed by the stock-copper pairs (0.39) and stock-oil pairs (0.36). At the same time, the wheat futures still provide the lowest static optimal hedge ratios on average (0.01), with several zero and even negative hedge ratios. Particularly, the IBOVESPA-oil pair has the highest static optimal hedge ratio of 0.75 among all hedged portfolios. The MSCI Europe-wheat pair has the lowest (and negative) static optimal hedge ratio of -0.06.

Table 6. Static Optimal Hedge Ratios

	Bitcoin	Gold	Silver	Palladium	Copper	Oil
ASX 200	0.54	0.60	0.39	0.21	0.09	0.49
MSCI US	0.07	0.41	0.28	0.17	0.38	0.39
MSCI Europe	0.11	0.55	0.32	0.21	0.39	0.29
MSCI Japan	0.08	0.48	0.26	0.19	0.32	0.32

HSI	0.05	0.46	0.21	0.14	0.46	0.23
IBOVESPA	0.13	0.65	0.51	0.29	0.68	0.75
BSESN	0.07	0.31	0.22	0.10	0.30	0.23
SSECI	0.04	0.37	0.19	0.11	0.39	0.20
STI	0.07	0.37	0.22	0.15	0.37	0.28
TAIEX	0.06	0.40	0.24	0.15	0.40	0.24
TSXCI	0.11	0.61	0.38	0.22	0.50	0.50
Average	0.12	0.47	0.29	0.18	0.39	0.36
	Corn	Orange Juice	Wheat			
ASX 200	0.10	0.04	0.05			
MSCI US	0.04	0.04	0.00			
MSCI Europe	0.01	0.06	-0.06			
MSCI Japan	0.03	0.07	0.03			
HSI	0.04	0.05	-0.01			
IBOVESPA	0.29	0.16	0.16			
BSESN	0.03	0.05	-0.05			
SSECI	0.04	0.07	0.00			
STI	0.07	0.02	-0.01			
TAIEX	0.04	0.03	0.00			
TSXCI	0.10	0.07	0.03			
Average	0.07	0.06	0.01			

Note: The static optimal hedge ratios are estimated using OLS.

5.4 Hedging Effectiveness (HE)

Hedging effectiveness (HE), as written in Equation (11), is the measure of the proportional reduction in the variance of a hedged portfolio to the variance of an unhedged portfolio. The HE is positive (negative) when the variance of the return of a hedged portfolio is smaller (larger) than that of an unhedged portfolio. Table 7 summarizes the results of HEs estimated using the diagonal BEKK-TGARCH and OLS models.

First, the results of HEs estimated using the TGARCH model are, on average, greater than those of OLS. For example, the average HE for bitcoin (palladium) is 0.09 (0.15) estimated by employing the TGARCH model, whereas it is 0.07 (0.13) estimated by employing the OLS model. Similarly, for the ASX 200-orange juice (MSCI US-copper) pair, the HE achieved with the TGARCH model is 0.10 (0.25), but the HE decreases to 0.00 (0.17) when employing the OLS model. This indicates that the time-varying hedge ratios (Equation 8) capture the nonlinear fluctuations of conditional variance-covariances between stock market returns and asset futures over time, compared with the static hedge ratio (Equation 10). The subsequent analysis of HE is therefore based on the results of the TGARCH Model.

Moreover, silver, copper, and crude oil are the preferred hedge assets that generate average HEs of 23%, 21%, and 18%, respectively. The HEs for the hedged portfolios of ASX-200 (TSXCI) with silver are up to 36% (39%); with copper, 35% (38%); and with crude oil, 38% (41%). Hedging with silver, copper, and crude oil attains HEs of over 20% in 7, 8, and 5 out of 11 global stock markets, respectively.

However, hedging stock markets with agricultural commodities (corn, orange juice, and wheat) provides an average HE of 0.04 or less, and a HE of 0.02 or less in 22 out of 33 cases. For example, the HE for the TAIEX-corn pair is 0.01, the BSESN-orange juice pair is -0.01, and the HSI-wheat pair is 0.00. Hence, agricultural commodities perform the worst for hedging global stock markets in our study.

Also, bitcoin is not a preferred hedge asset in our study with an average HE of 0.09, and it is consistent with the findings of, for example, Baur, et al. (2022) and Corbet, et al. (2020), though this is in contrast to those of Dyhrberg (2016) and Kliber, et al. (2019). Likewise, gold is a popular hedge asset in extant literature (e.g., Ourir, et al. 2023). However, its average HE is only 0.12, the lowest among all precious metals.

Finally, the most effectively hedged global stock markets include Australia and Canada, with an average HE of 0.21, whereas Hong Kong, India, and Mainland China are not effectively hedged, as per the tests conducted, with average HEs of less than 0.10.

6. Conclusion

Investors often use hedging strategies to reduce investment risk. The results of hedging effectiveness in extant literature are, however, mixed, depending upon the stock markets being hedged, the selection of hedge assets, the methods adopted, and the sample periods covered. Moreover, typically one or two hedge assets were employed in each study, making it difficult to compare the hedging performance of different hedge assets together. This paper aims to measure and compare the effectiveness of bitcoin, gold, silver, palladium, copper, crude oil, corn, orange juice, and wheat in hedging against stock market risks using BEKK-TGARCH and OLS methods adopted during the recent period. The selected hedge assets comprise classes of cryptocurrency, precious and industrial metals, as well as energy and agriculture commodities. Our findings indicate that silver, copper, and crude oil perform the best. Surprisingly, bitcoin and gold are not well-performed in our study, which is contrary to a large part of the extant literature. The agricultural commodities perform the worst and are then not recommended for hedging against stock market risk. Further, the stock market risks are effectively hedged (except with agricultural commodities) in Australia and Canada but are poorly hedged in Hong Kong, India, and Mainland China. Additionally, the HEs of time-varying hedge ratios, as estimated by TGARCH, are on average higher than those of static hedge ratios as estimated by OLS. Hence, our results provide a more complete picture of the hedging performance of multiple classes of hedge assets in global stock markets than those in prior studies.

Our study is connected to the field of decision sciences (Chang, et al., 2018; Hasan-Zadeh, 2019; Tuan, et al., 2022). The empirical findings provide investors and portfolio managers with an understanding of the hedging performance of various classes of hedge assets in global stock markets, which are useful for making optimal hedging decisions, i.e., selecting hedging strategies and tools.

Table 7. Estimates of Hedging Effectiveness (HE)

TGARCH	ASX 200	MSCI US	MSCI Europe	MSCI Japan	HSI	IBOVESPA	BSESN	SSECI	STI	TAIEX	TSXCI	Average [a]
Bitcoin	0.09	0.06	0.15	0.07	0.04	0.13	0.10	0.03	0.09	0.10	0.11	0.09
Gold	0.14	0.07	0.16	0.21	0.10	0.08	0.03	0.09	0.12	0.10	0.21	0.12
Silver	0.36	0.24	0.23	0.20	0.10	0.20	0.10	0.09	0.24	0.12	0.39	0.21
Palladium	0.15	0.13	0.26	0.27	0.11	0.12	0.07	0.15	0.15	0.12	0.21	0.15
Copper	0.35	0.25	0.13	0.09	0.24	0.23	0.16	0.24	0.25	0.23	0.38	0.23
Crude oil	0.38	0.31	0.08	-0.08	0.01	0.29	0.16	0.07	0.21	0.11	0.41	0.18
Corn	0.06	0.05	0.02	0.02	0.01	0.09	0.09	0.00	0.05	0.01	0.10	0.04
Orange juice	0.10	0.01	0.02	0.03	0.02	0.01	-0.01	-0.01	-0.02	0.01	0.02	0.02
Wheat	0.07	0.07	0.03	0.01	0.00	0.04	0.01	0.01	0.00	0.00	0.00	0.02
Average [b]	0.21	0.15	0.13	0.10	0.08	0.14	0.09	0.08	0.14	0.10	0.21	
OLS	**ASX 200**	**MSCI US**	**MSCI Europe**	**MSCI Japan**	**HSI**	**IBOVESPA**	**BSESN**	**SSECI**	**STI**	**TAIEX**	**TSXCI**	**Average [a]**
Bitcoin	0.11	0.06	0.13	0.09	0.03	0.06	0.05	0.02	0.08	0.05	0.13	0.07
Gold	0.14	0.08	0.14	0.13	0.07	0.06	0.05	0.08	0.09	0.09	0.17	0.10
Silver	0.26	0.16	0.19	0.16	0.09	0.16	0.09	0.08	0.13	0.13	0.28	0.16
Palladium	0.17	0.12	0.19	0.20	0.09	0.12	0.05	0.06	0.13	0.11	0.21	0.13
Copper	0.28	0.17	0.17	0.14	0.23	0.17	0.11	0.21	0.21	0.21	0.28	0.20
Crude oil	0.29	0.21	0.12	0.16	0.07	0.25	0.07	0.07	0.15	0.09	0.33	0.16
Corn	0.01	0.00	0.00	0.00	0.00	0.05	0.00	0.00	0.01	0.00	0.02	0.01
Orange juice	0.00	0.00	0.01	0.01	0.01	0.02	0.01	0.01	0.00	0.00	0.01	0.01
Wheat	0.01	0.00	0.01	0.00	0.00	0.02	0.01	0.00	0.00	0.00	0.00	0.01
Average [b]	0.16	0.10	0.12	0.11	0.07	0.11	0.06	0.07	0.10	0.09	0.18	

Note:

Average[a] denotes the average HE of hedge assets across different stock indices

Average[b] denotes the average HE of stock indices across different hedge assets

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