Hindawi Publishing Corporation
Journal of Applied Mathematics and Decision Sciences
Volume 2009, Article ID 215163, 11 pages
doi:10.1155/2009/215163

Research Article

Valuation for an American Continuous-Installment Put Option on Bond under Vasicek Interest Rate Model

Guoan Huang,[1] Guohe Deng,[2] and Lihong Huang[3]

[1] *Department of Computer Science, Guilin College of Aerospace Technology, Guilin 541004, China*
[2] *School of Mathematics Science, Guangxi Normal University, Guilin 541004, China*
[3] *College of Mathematics and Econometrics, Hunan University, Changsha 410082, China*

Correspondence should be addressed to Guohe Deng, dengguohe@mailbox.gxnu.edu.cn

Received 2 December 2008; Accepted 6 March 2009

Recommended by Lean Yu

The valuation for an American continuous-installment put option on zero-coupon bond is considered by Kim's equations under a single factor model of the short-term interest rate, which follows the famous Vasicek model. In term of the price of this option, integral representations of both the optimal stopping and exercise boundaries are derived. A numerical method is used to approximate the optimal stopping and exercise boundaries by quadrature formulas. Numerical results and discussions are provided.

1. Introduction

Although there has been a large literature dealing with numerical methods for American options on stocks [1] and references cited therein, [2], there are not many papers for American options on default-free bonds, see, for example, [3–7], and so on. Numerical methods such as finite differences, binomial tree methods and Least-Square Monte Carlo simulations are still widely used. However, these methods have several shortcomings including time consuming, unbounded domain and discontinuous derivative with respect to the variate of payoff function. The most recent papers, like [8–11] provide different types of methods.

In this paper we consider an alternative form of American option in which the buyer pays a smaller up-front premium and then a constant stream of installments at a certain rate per unit time. So the buyer can choose at any time to stop making installment payments by either exercising the option or stopping the option contract. This option is called American continuous-installment (CI) option. Installment options are a recent financial innovation that helps the buyer to reduce the cost of entering into a hedging strategy and the liquidity risk. Nowadays, the installment options are the most actively traded warrant throughout the financial world, such as the installment warrants on Australian stock and a 10-year warrant with 9 annual payments offered by Deutsche bank, and so on. There is very little literature on pricing the installment option, in particular, for pricing the American CI options. Ciurlia and Roko [12], and Ben-Ameur et al. [13] provide numerical procedures for valuing American CI options on stock under the geometric Brownian motion framework. However, in practice the option on bond is more useful than option on stock, and pricing the former is more complicated, because it is dependent on interest rates variable which is modelled by many economical models.

The aim of this paper is to present an approximation method for pricing American CI put option written on default-free, zero-coupon bond under Vasicek interest rate model. This method is based on Kim integral equations using quadrature formula approximations, such as the trapezoidal rule and the Simpson rule. The layout of this paper is as follows. Section 2 introduces the model and provides some preliminary results. In Section 3 we formulate the valuation problem for the American CI put option on bond describe as a free boundary problem and describe the Kim integral equations. Numerical method and results are presented in Section 4. Section 5 concludes.

2. The Model and Preliminary Results

In the one-factor Vasicek model [14], the short-term interest rate r_t is modeled as a mean-reverting Gaussian stochastic process on a probability space $(\Omega, \mathcal{F}, P)$ equipped with a filtration $(\mathcal{F}_t)_{t\geq 0}$. Under the the risk-neutral probability measure Q, it satisfies the linear stochastic differential equation (SDE)

$$dr_t = \kappa(r_\infty - r_t)dt + \sigma dW_t, \tag{2.1}$$

where $(W_t)_{t\geq 0}$ is a standard Q-Brownian motion, $\kappa > 0$ is the speed of mean reversion, $r_\infty > 0$ is the long-term value of interest rate, and σ is a constant volatility.

Consider a frictionless and no-arbitrage financial market which consists of a bank account A_t with its price process given by $dA_t = r_t A_t dt$ and a T_1-maturity default-free, zero-coupon bond $B(t, r, T_1) = B_t$ with its no-arbitrage price at time t given by

$$B(t, r, T_1) = E_Q\left\{e^{-\int_t^{T_1} r_u du}\mathcal{F}_t\right\} \widehat{=} E_Q^t\left[e^{-\int_t^{T_1} r_s ds}\right], \tag{2.2}$$

where E_Q is the expectation under the risk-neutral probability measure Q. Vasicek [14] provides the explicit form of the zero-bond as follows:

$$B(t, r, T_1) = a(T_1 - t)e^{-b(T_1 - t)r_t} \tag{2.3}$$

with

$$a(u) = \exp\left\{-\left[R_\infty u - R_\infty b(u) + \frac{\sigma^2}{4\kappa} b^2(u)\right]\right\},$$
$$b(u) = \frac{1 - e^{-\kappa u}}{\kappa}, \qquad R_\infty = r_\infty - \frac{\sigma^2}{2\kappa^2}. \tag{2.4}$$

From (2.3), we are easy to obtain the following partial differential equation (P.D.E.):

$$\frac{\partial B_t}{\partial t} + \kappa(r_\infty - r)\frac{\partial B_t}{\partial r} + \frac{1}{2}\sigma^2 \frac{\partial^2 B_t}{\partial r^2} - rB_t = 0 \tag{2.5}$$

with terminal condition $B(T_1, r, T_1) = 1$.

The payoff of a European-style put option without paying any dividends written on the zero-coupon bond $B(t, r, T_1)$ with maturity T $(T < T_1)$, and strike price K is $h(T, r) = \max\{K - B(T, r, T_1), 0\}$. The no-arbitrage price at time t $(0 \le t \le T)$ of this option is denoted by $p_e(t, r, K; T)$. Following Jamshidian [15], the price of this option can generally be expressed as follows:

$$p_e(t, r, K; T) = E_Q^t\left[e^{-\int_t^T r_s ds} h(T, r)\right] = KB(t, r, T)N(-d_2) - B(t, r, T_1)N(-d_1), \tag{2.6}$$

where $N(\cdot)$ is the 1-dimensional standard cumulative normal distribution, and

$$d_{1,2} = \frac{1}{\sigma_0} \ln \frac{B(t, r, T_1)}{KB(t, r, T)} \pm \frac{1}{2}\sigma_0,$$
$$\sigma_0 = \sigma b(T_1 - T)\sqrt{\frac{1 - e^{-2\kappa(T-t)}}{2\kappa}}. \tag{2.7}$$

Now we consider a CI option written on the zero-coupon bond $B(t, r, T_1)$. Denote the initial premium of this option to be $V_t = V(t, r; q)$, which depends on the interest rate, time t, and the continuous-installment rate q. Applying Ito's Lemma to V_t, the dynamics for the initial value of this option is obtained as follows:

$$dV_t = \left[\frac{\partial V_t}{\partial t} + \frac{1}{2}\sigma^2 \frac{\partial^2 V_t}{\partial r^2} + \kappa(r_\infty - r_t)\frac{\partial V_t}{\partial r} - q\right]dt + \sigma \frac{\partial V_t}{\partial r} dW_t. \tag{2.8}$$

Theorem 2.1. *In the Vasicek interest rates term structure model* (2.1)*. The contingent claim* $V(t, r; q)$ *satisfies the inhomogeneous partial differential equation*

$$\frac{\partial V_t}{\partial t} + \frac{1}{2}\sigma^2 \frac{\partial^2 V_t}{\partial r^2} + \kappa(r_\infty - r_t)\frac{\partial V_t}{\partial r} - rV_t = q. \tag{2.9}$$

Proof. We now consider a self-financing trading strategy $\psi = (\psi_1, \psi_2)$, where ψ_1 and ψ_2 represent positions in bank account and T_1-maturity zero-coupon bonds, respectively. It is apparent that the wealth process π_t satisfies

$$\pi_t = \psi_1 A_t + \psi_2 B_t = V_t, \tag{2.10}$$

where the second equality is a consequence of the assumption that the trading strategy ψ replicate the option. Furthermore, since ψ is self-financing, its wealth process π_t also satisfies

$$d\pi_t = \psi_1 dA_t + \psi_2 dB_t, \tag{2.11}$$

so that

$$d\pi_t = \psi_1 r_t \; A_t dt + \psi_2 \left[\frac{\partial B_t}{\partial t} + \kappa(r_\infty - r_t)\frac{\partial B_t}{\partial r} + \frac{1}{2}\sigma^2 \frac{\partial^2 B_t}{\partial r^2} \right] dt + \sigma \psi_2 \frac{\partial B_t}{\partial r} dW_t. \tag{2.12}$$

From (2.8) and (2.10), we get

$$\begin{aligned} &\left[\frac{\partial V_t}{\partial t} + \frac{1}{2}\sigma^2 \frac{\partial^2 V_t}{\partial r^2} + \kappa(r_\infty - r_t)\frac{\partial V_t}{\partial r} - q - rV_t \right] dt + \sigma \frac{\partial V_t}{\partial r} dW_t \\ &\quad = \psi_2 \left[\frac{\partial B_t}{\partial t} + \kappa(r_\infty - r_t)\frac{\partial B_t}{\partial r} + \frac{1}{2}\sigma^2 \frac{\partial^2 B_t}{\partial r^2} - rB_t \right] dt + \psi_2 \sigma \frac{\partial B_t}{\partial r} dW_t. \end{aligned} \tag{2.13}$$

Setting $\psi_2 = (\partial B_t/\partial r)/(\partial V_t/\partial r)$ the coefficient of dW_t vanishes. It follows from (2.5) that, V_t satisfies (2.9). □

3. Kim Equations for the Price of American CI Put Option

Consider an American CI put option written on the zero-coupon bond B_t with the same strike price K and maturity time T ($T < T_1$). Although the underlying asset is the bond, the independent variable is the interest rate. Similar to American continuous-installment option on stock [12], there is an upper critical interest rate r_t^u above which it is optimal to stop the installment payments by exercising the option early, as well as a lower critical interest rate r_t^l below which it is advantageous to terminate payments by stopping the option contract. We may call r_t^u to be exercising boundary and r_t^l to be stopping boundary. Denote the initial premium of this put option at time t by $P(t, r; q) = P_t$, defined on the domain

$\mathfrak{D} = \{(r_t, t) \in [0, +\infty) \times [0, T]\}$. It is known that $P(t, r; q)$, r_t^u and r_t^l are the solution of the following free boundary problem [4]:

$$\begin{gathered}
\frac{\partial P_t}{\partial t} + \frac{1}{2}\sigma^2 \frac{\partial^2 P_t}{\partial r^2} + \kappa(r_\infty - r_t)\frac{\partial P_t}{\partial r} - rP_t = q, \quad \forall (r, t) \in \mathcal{C}, \\
P(t, r; q) = 0, \quad (r, t) \in \mathcal{S}, \\
P(t, r; q) = K - B(t, r, T_1), \quad (r, t) \in \mathcal{E}, \\
P(T, r; q) = h(T, r), \quad r \geq 0, \\
P(t, r_t^u; q) = K - B(t, r_t^u, T_1), \\
P\left(t, r_t^l; q\right) = 0, \quad t \in [0, T],
\end{gathered} \tag{3.1}$$

where $\mathcal{C} = \{(r_t, t) \in (r_t^l, r_t^u) \times [0, T)\}$ is a continuation region, $\mathcal{S} = \{((r_t, t) \in [0, r_t^l] \times [0, T]\}$ is a stopping region, and $\mathcal{E} = \{((r_t, t) \in [r_t^u, +\infty) \times [0, T]\}$ is a exercise region.

Remark 3.1. Due to the decreasing property of the price $B(t, r, T_1)$ on the state variable r, the strike price K should be strictly less than $B(T, 0, T_1)$. Otherwise, exercise would never be optimal.

It should be noted that although the value of the American CI put option has been expressed through the use of PDEs and their boundary conditions, there is still no explicit solution for the P.D.E. in (3.1). Numerical methods must be applied to value the price of the American CI option on bond. In the following we will solve this problem (3.1) with the integral equation method discussed in [8–12]. This method expresses the price of the American option as the sum of the price of the corresponding European option and the early exercise gains depending on the optimal exercise boundary. Jamshidian [3] uses this method to value the American bond option in Vasicek model.

Theorem 3.2. *Let the short interest rate r_t satisfy model* (2.1). *Then the initial premium of the American CI put option , $P(t, r; q)$, can be written as*

$$\begin{aligned}
P(t, r; q) = p_e(t, r, K; T) &+ q\int_t^T B(t, r, s) N\left(e\left(r, r_s^l\right)\right) ds \\
&+ \int_t^T B(t, r, s)\Bigg\{-qN(e(r, r_s^u)) + K[r_s^u - \sigma_1 e(r, r_s^u)]N(-e(r, r_s^u)) \\
&\qquad + \frac{K\sigma_1}{\sqrt{2\pi}} \exp\left\{-\frac{e^2(r, r_s^u)}{2}\right\}\Bigg\} ds.
\end{aligned} \tag{3.2}$$

Moreover, the optimal stopping and exercise boundaries, r^u and r^l, are solutions to the following system of recursive integral equations:

$$
\begin{aligned}
K - B(t,r^u,T_1) = p_e(t,r^u,K;T) + q\int_t^T B(t,r^u,s)N\left(c\left(r^u,r_s^l\right)\right)ds \\
+\int_t^T B(t,r^u,s)\Bigg\{-qN(e(r^u,r_s^u)) + K[r_s^u - \sigma_1 e(r^u,r_s^u)]N(-e(r^u,r_s^u)) \\
+\frac{K\sigma_1}{\sqrt{2\pi}}\exp\left\{-\frac{e^2(r^u,r_s^u)}{2}\right\}\Bigg\}ds, \\
0 = p_e\left(t,r^l,K;T\right) + q\int_t^T B\left(t,r^l,s\right)N\left(c\left(r^l,r_s^l\right)\right)ds \\
+\int_t^T B\left(t,r^l,s\right)\Bigg\{-qN\left(e\left(r^l,r_s^u\right)\right) + K\left[r_s^u - \sigma_1 e\left(r^l,r_s^u\right)\right]N\left(-e\left(r^l,r_s^u\right)\right) \\
+\frac{K\sigma_1}{\sqrt{2\pi}}\exp\left\{-\frac{e^2(r^l,r_s^u)}{2}\right\}\Bigg\}ds,
\end{aligned}
\tag{3.3}
$$

subject to the boundary conditions

$$
B(T,r^u,T_1) = K, \qquad B\left(T,r^l,T_1\right) = K, \tag{3.4}
$$

where $e(r,r_s^*) = ((r_s^* - r_t) - \kappa(r_t - r_\infty)b(s-t) + (1/2)\sigma^2 b^2(s-t))/\sigma_1)$ *and* $\sigma_1^2 = (\sigma^2/2\kappa)(1-e^{-2\kappa(s-t)})$.

Proof. Let $Z(s,r) = e^{-\int_0^s r_u\,du}P(s,r;q)$ be the discounted initial premium function of the American CI put option in the domain $\mathfrak{D}$. It is known that the function $Z(s,r) \in C^{1,2}(\mathfrak{D})$. We can apply Ito Lemma to $Z(s,r)$ and write

$$
Z(T,r) = Z(t,r) + \int_t^T\left[\frac{\partial Z(s,r)}{\partial s}ds + \frac{\partial Z(s,r)}{\partial r}dr + \frac{1}{2}\sigma^2\frac{\partial^2 Z(s,r)}{\partial r^2}ds\right]. \tag{3.5}
$$

In terms of $P(t,r;q)$ this means

$$
\begin{aligned}
e^{-\int_t^T r_s ds}P(T,r;q) = P(t,r;q) + \int_t^T e^{-\int_t^s r_u du}\left[\frac{\partial P_s}{\partial s} + \frac{1}{2}\sigma^2\frac{\partial^2 P_s}{\partial r^2} + \kappa(r_\infty - r)\frac{\partial P_s}{\partial r} - rP_s\right]ds \\
+\int_t^T e^{-\int_t^s r_u du}\sigma\frac{\partial P_s}{\partial r}dW_s.
\end{aligned}
\tag{3.6}
$$

From (3.1) we know that $P(T,r;q) = h(T,r)$ and $P(s,r;q) = P(s,r;q)\mathbf{1}_{(r,s)\in\mathcal{C}} + P(s,r;q)\mathbf{1}_{(r,s)\in\mathcal{S}} + P(s,r;q)\cdot\mathbf{1}_{(r,s)\in\mathcal{E}} = P(s,r;q)\mathbf{1}_{(r,s)\in\mathcal{C}} + [K - B(s,r,T_1)]\mathbf{1}_{(r,s)\in\mathcal{E}}$. Substituting and taking expectation under Q on both sides of (3.6) give

$$\begin{aligned}
p_e(t,r,K;T) &= E_Q^t\left[e^{-\int_t^T r_s ds} g(T,r)\right] \\
&= P(t,r;q) + \int_t^T E_Q^t\left\{e^{-\int_t^s r_u du}\left[\frac{\partial P_s}{\partial s} + \frac{1}{2}\sigma^2\frac{\partial^2 P_s}{\partial r^2} + \kappa(r_\infty - r)\frac{\partial P_s}{\partial r} - rP_s\right]\right\}ds \\
&= P(t,r;q) + q\int_t^T E_Q^t\left[e^{-\int_t^s r_u du}1_{(r_s^l<r_s<r_s^u)}\right]ds - K\int_t^T E_Q^t\left\{e^{-\int_t^s r_u du} r_s 1_{(r_s\geq r_s^u)}\right\}ds.
\end{aligned} \tag{3.7}$$

From (2.1), it is easy to obtain that the state variable r_s follows

$$r_s = r_t e^{-\kappa(s-t)} + r_\infty\left(1 - e^{-\kappa(s-t)}\right) + \sigma\int_t^s e^{-\kappa(s-u)} dW_u \tag{3.8}$$

for every $s > t$. Then the state variable r_s follows the normal distribution. Furthermore, using s-forward measures discussed in [16] and the normal distribution produces the representation (3.2). The recursive equations (3.3) for the optimal stopping and exercise boundaries are obtained by imposing the boundary conditions $P(t,r_t^u;q) = K - B(t,r_t^u,T_1)$ and $P(t,r_t^l;q) = 0$. The boundary conditions (3.4) hold since the limitation for (3.3) as $t \uparrow T$. □

Remark 3.3. From (3.2), when r_t^l and r_t^u are obtained by (3.3), the value of American CI put option is also derived. However, (3.3) are Volterra integral equations and can be solved numerically. Notice that the stopping and exercise boundary functions, r_t^l and r_t^u, cannot be proved to be monotone function of time t. So we use trapezoidal rule method to deal with them.

4. Numerical Method and Results

In this section we provide our method for pricing American CI put option by solving the Kim equations and present numerical results. This method consists of the following three steps. The first is to approximate the quadrature representations in (3.3) by using the trapezoidal rule. The second step is needed to find the numerical values of both the stopping and exercise boundaries, r_t^l and r_t^u from the equations approximated above with the Newton-Raphson (NR) iteration approach. When the values of r_t^l and r_t^u are obtained, the third step, numerical integration of (3.2), yields the value of a given American CI put option. This method is widely used to value American option by several authors, for example, [8, 11].

We now divide the time interval $[0,T]$ into N subintervals: $t_i = i\Delta t$, $i = 0,1,2,\ldots,N$, $\Delta t = T/N$. Denote $r_{t_i}^l = r_i^l$ and $r_{t_i}^u = r_i^u$ for $i = 0,1,2,\ldots,N$. Since $T_N = T$, we get by (2.3) and(3.4)

$$r_N^l = r_N^u = \frac{1}{b(T_1 - T)}\ln\frac{a(T_1 - T)}{K}. \tag{4.1}$$

We define the integrand of (3.3) as the following functions:

$$\begin{aligned} f(t,r,s,r_s^*) &= B(t,r,s)\left\{-qN(e(r,r_s^*)) + K[r_s^* - \sigma_1 e(r,r_s^*)]N(-e(r,r_s^*)) \right.\\ &\qquad\left. +\frac{K\sigma_1}{\sqrt{2\pi}}\exp\left\{-\frac{e^2(r,r_s^*)}{2}\right\}\right\}, \\ g(t,r,s,r_s^*) &= qB(t,r,s)N(c(r,r_s^*)). \end{aligned} \tag{4.2}$$

We use the trapezoidal rule to represent the system of recursive integral equations (3.3) as follows:

$$\begin{aligned} &p(t_i,r_i^u,K;T) + \Delta t\left[\frac{1}{2}g\left(t_i,r_i^u,t_i,r_i^l\right) + \sum_{j=i+1}^{N-1} g\left(t_i,r_i^u,t_j,r_j^l\right) + \frac{1}{2}g\left(t_i,r_i^u,t_N,r_N^l\right)\right] \\ &\qquad + \Delta t\left[\frac{1}{2}f(t_i,r_i^u,t_i,r_i^u) + \sum_{j=i+1}^{N-1} f\left(t_i,r_i^u,t_j,r_j^u\right) + \frac{1}{2}f(t_i,r_i^u,t_N,r_N^u)\right] \\ &\qquad + B(t_i,r_i^u,T_1) - K = 0, \\ &p\left(t_i,r_i^l,K;T\right) + \Delta t\left[\frac{1}{2}g\left(t_i,r_i^l,t_i,r_i^l\right) + \sum_{j=i+1}^{N-1} g\left(t_i,r_i^l,t_j,r_j^l\right) + \frac{1}{2}g\left(t_i,r_i^l,t_N,r_N^l\right)\right] \\ &\qquad + \Delta t\left[\frac{1}{2}f\left(t_i,r_i^l,t_i,r_i^u\right) + \sum_{j=i+1}^{N-1} f\left(t_i,r_i^l,t_j,r_j^u\right) + \frac{1}{2}f\left(t_i,r_i^l,t_N,r_N^u\right)\right] \\ &\qquad = 0, \quad i = 0,\ldots,N-1. \end{aligned} \tag{4.3}$$

Since there are nonlinear system equations, one can solve it using the NR iteration. In a similar way, numerical values of both r_i^l and r_i^u, $i = N-1, N-2, \ldots, 0$ can be obtained recursively from (4.3). We denote the representation of left side in (4.3) by $F_1(r_i^l, r_i^u)$ and $F_2(r_i^l, r_i^u)$, respectively. Then, by the NR iteration the values (r_i^l, r_i^u) have approximations $(r_i^l(k), r_i^u(k))$ of order k, where $k = 0, 1, 2, \ldots$

$$\begin{pmatrix} r_i^l(k+1) \\ r_i^u(k+1) \end{pmatrix} = \begin{pmatrix} r_i^l(k) \\ r_i^u(k) \end{pmatrix} - \begin{pmatrix} \dfrac{\partial F_1}{\partial x} & \dfrac{\partial F_1}{\partial y} \\ \dfrac{\partial F_2}{\partial x} & \dfrac{\partial F_2}{\partial y} \end{pmatrix}^{-1} \cdot \begin{pmatrix} F_1(x,y) \\ F_2(x,y) \end{pmatrix}\Big|_{(x,y)=(r_i^l(k),r_i^u(k))}, \tag{4.4}$$

where $\partial F_j/\partial x$ and $\partial F_j/\partial y$, $j = 1,2$ are, respectively, partial derivatives of functions $F_j(x,y)$ with respect to x and y. When the values of all (r_i^l, r_i^u) for $i = N,\ldots,0$ are obtained, using

Table 1: Value of parameters.

κ	r_∞	σ	K	T	T_1
0.05	0.083	0.015	0.95	1	5

Table 2: Initial premium of option on bond.

r_0	European put option	American CI put option		
		$q = 1$	$q = 10$	$q = 30$
0.04	0.0782	0.4409	0.3766	0.0893
0.10	0.2193	0.5928	0.4432	0.2296
0.15	0.3057	0.6556	0.4953	0.3258

Simpson's rule for (3.2) we get the approximation, $\widehat{P}_0(r,q)$, of the value at time $t = 0$ for the American CI put bond option in the following way: assuming N is an even number we have

$$\begin{aligned}
\widehat{P}_0(r,q) = p(0,r,K;T) &+ \frac{\Delta t}{3}\Big[g\left(0,r,0,r_0^l\right) + 4g\left(0,r,t_1,r_1^l\right) + 2g\left(0,r,t_2,r_2^l\right) \\
&\quad + 4g\left(0,r,t_3,r_3^l\right) + \cdots + 2g\left(0,r,t_{N-2},r_{N-2}^l\right) \\
&\quad + 4g\left(0,r,t_{N-1},r_{N-1}^l\right) + g\left(0,r,T,r_T^l\right)\Big] \\
&+ \frac{\Delta t}{3}\left[f\left(0,r,0,r_0^u\right) + 4f\left(0,r,t_1,r_1^u\right) + 2f\left(0,r,t_2,r_2^u\right) + 4f\left(0,r,t_3,r_3^u\right) + \cdots\right. \\
&\quad \left. + 2f\left(0,r,t_{N-2},r_{N-2}^u\right) + 4f\left(0,r,t_{N-1},r_{N-1}^u\right) + f\left(0,r,T,r_T^u\right)\right].
\end{aligned} \tag{4.5}$$

In Table 1, we describe the parameters in this section. In our example, we take $N = 6$. Table 2 provides the initial premium of this put option on bond for different installment rate $q = 1, 10$, and 30 with different initial interest rate $r_0 = 0.04, 0.10$, and 0.15.

Table 2 shows that the larger the initial interest rate is, the higher the price of American CI put option on bond is. However, the larger the installment rate is, the lower the price of this option is.

Figure 1 displays the curves of both the optimal stopping and exercise boundaries versus different installment rates q. We find out that the two boundaries decrease when the installment rate is arising. That shows that the larger the installment rate is, the higher probability the exercising of the option is.

5. Conclusions

A simple approximated method for pricing the American CI option written on the zero-bond under Vasicek model is proposed. Numerical example is provided to analyze the effects of the installment rate q on the price of this option and the optimal stopping and exercise boundaries. However, the Vasicek model allows for negative values of interest rate. This property is manifestly incompatible with reality. For this reason, work is ongoing to extend them to other models.

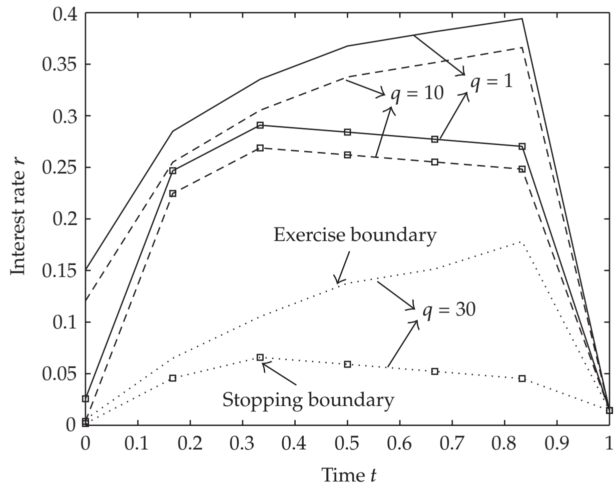

Figure 1: Optimal stopping and exercise boundaries for different installment rates.

Acknowledgment

The work has been partially supported by the NNSF of China with no. 40675023 and the Guangxi Natural Science Foundation with no. 0991091.

References

[1] W. Allegretto, Y. Lin, and H. Yang, "Finite element error estimates for a nonlocal problem in American option valuation," *SIAM Journal on Numerical Analysis*, vol. 39, no. 3, pp. 834–857, 2001.

[2] M. Broadie and J. B. Detemple, "Option pricing: valuation models and applications," *Management Science*, vol. 50, no. 9, pp. 1145–1177, 2004.

[3] F. Jamshidian, "An analysis of American options," *Review of Futures Markets*, vol. 11, no. 1, pp. 72–80, 1992.

[4] M. Chesney, R. Elliott, and R. Gibson, "Analytical solutions for the pricing of American bond and yield options," *Mathematical Finance*, vol. 3, no. 3, pp. 277–294, 1993.

[5] T. S. Ho, R. C. Stapleton, and M. G. Subrahmanyam, "The valuation of American options on bonds," *Journal of Banking & Finance*, vol. 21, no. 11-12, pp. 1487–1513, 1997.

[6] W. Allegretto, Y. Lin, and H. Yang, "Numerical pricing of American put options on zero-coupon bonds," *Applied Numerical Mathematics*, vol. 46, no. 2, pp. 113–134, 2003.

[7] L. ShuJin and L. ShengHong, "Pricing American interest rate option on zero-coupon bond numerically," *Applied Mathematics and Computation*, vol. 175, no. 1, pp. 834–850, 2006.

[8] I. J. Kim, "The analytic valuation of American options," *Review of Financial Studies*, vol. 3, no. 4, pp. 547–572, 1990.

[9] S. D. Jacka, "Optimal stopping and the American put," *Mathematical Finance*, vol. 1, no. 1, pp. 1–14, 1991.

[10] P. Carr, R. Jarrow, and R. Myneni, "Alternative characterizations of American put options," *Mathematical Finance*, vol. 2, no. 1, pp. 87–106, 1992.

[11] S. Kallast and A. Kivinukk, "Pricing and hedging American options using approximations by Kim integral equations," *European Finance Review*, vol. 7, no. 3, pp. 361–383, 2003.

[12] P. Ciurlia and I. Roko, "Valuation of American continuous-installment options," *Computational Economics*, vol. 25, no. 1-2, pp. 143–165, 2005.

[13] H. Ben-Ameur, M. Breton, and P. François, "A dynamic programming approach to price installment options," *European Journal of Operational Research*, vol. 169, no. 2, pp. 667–676, 2006.

[14] O. Vasicek, "An equilibrium characterization of the term structure," *Journal of Financial Economics*, vol. 5, no. 2, pp. 177–188, 1977.

[15] F. Jamshidian, "An exact bond option formula," *The Journal of Finance*, vol. 44, no. 1, pp. 205–209, 1989.

[16] H. Geman, N. El Karoui, and J.-C. Rochet, "Changes of numéraire, changes of probability measure and option pricing," *Journal of Applied Probability*, vol. 32, no. 2, pp. 443–458, 1995.

Advances in Operations Research
Advances in Decision Sciences
Journal of Applied Mathematics
Algebra
Journal of Probability and Statistics
The Scientific World Journal
International Journal of Differential Equations
Hindawi
Submit your manuscripts at
http://www.hindawi.com
International Journal of Combinatorics
Advances in Mathematical Physics
Journal of Complex Analysis
Journal of Mathematics
Mathematical Problems in Engineering
Abstract and Applied Analysis
Discrete Dynamics in Nature and Society
International Journal of Mathematics and Mathematical Sciences
Journal of Discrete Mathematics
Journal of Function Spaces
International Journal of Stochastic Analysis
Journal of Optimization