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Research Article

Input and Output Passivity of Complex Dynamical Networks with General Topology

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The input passivity and output passivity are investigated for a generalized complex dynamical network, in which the coupling may be nonlinear, time-varying, and nonsymmetric. By constructing some suitable Lyapunov functionals, some input and output passivity criteria are derived in form of linear matrix inequalities (LMIs) for complex dynamical network. Finally, a numerical example and its simulation are given to illustrate the efficiency of the derived results.

1. Introduction

In the past few decades, the passivity theory provides a nice tool for analyzing the stability of systems and has found applications in diverse areas such as stability, complexity, signal processing, chaos control and synchronization, and fuzzy control. Many interesting results in linear and nonlinear systems have been derived (see [1–21]). Especially, the passivity of neural networks [5, 7, 11–13, 21] and fuzzy systems [6, 10, 16, 19] have been extensively investigated because of their great significance for both practical and theoretical purposes.

Recently, the complex networks have been gaining increasing recognition as a fundamental tool in understanding dynamical behavior and the response of real systems across many fields of science and engineering. In particular, synchronization is one of the most significant and interesting dynamical properties of the complex networks which has been carefully studied [22–46]. However, few authors have considered the problem on the passivity of complex dynamical networks. Therefore, it is interesting to study the passivity of complex dynamical networks.

It is well known that the power supply is an important role for stability analysis and controller synthesis of linear or nonlinear systems. Based on the concept of power supply, the input passivity is introduced. For instance, Chang et al. [19] dealt with developing the relaxed stability conditions for continuous-time affine Takagi-Sugeno (T-S) fuzzy models by applying the input passivity and Lyapunov theory. However, to the best of our knowledge, the intput passivity of complex dynamical networks with time-varying delays has not yet been established in the literature. Therefore, it is interesting to study input passivity of complex dynamical networks possessing general topology. As a natural extension of input passivity, we also introduce the output passivity in order to study dynamical behavior of complex networks much better.

It should be pointed out that many literatures on the dynamical behavior of complex dynamical networks are considered under some simplified assumptions. For instance, the coupling among the nodes of the complex networks is linear, time invariant, symmetric, and so on. In fact, such simplification does not match the peculiarities of real networks in many circumstances. Firstly, the interplay of two different nodes in a network cannot be described accurately by linear functions of their states because they are naturally nonlinear functions of states. Secondly, many real-world networks are more likely to have different coupling strengths for different connections, and coupling strength are frequently varied with time. Moreover, the coupling topology is likely directed and weighted in many real-world networks such as the food web, metabolic networks, World-Wide Web, epidemic networks, document citation networks, and so on. Hence, it is interesting to remove the assumptions mentioned above in order to investigate the passivity of complex networks much better. Additionally, it is well known that the phenomena of time delays are common in complex networks, in which delays even are frequently varied with time. Therefore, It is interesting to study such a complex dynamical network model with nonlinear, time-varying, nonsymmetric, and delayed coupling.

Motived by the above discussion, we formulate a delayed dynamical network model with general topology. The objective of this paper is to study the input and output passivity of complex networks. Some sufficient conditions on input and output passivity are obtained by LMI and Lyapunov functional method for complex dynamical networks.

The rest of this paper is organized as follows. In Section 2, a complex dynamical network model is introduced and some useful preliminaries are presented. Some input and output passivity conditions are presented in Section 3. In Section 4, a numerical example and its simulation are given to illustrate the theoretical results. Finally, conclusions are drawn in Section 5.

2. Network Model and Preliminaries

In this paper, we consider a dynamical network consisting of N identical nodes with diffusive and delay coupling. The mathematical model of the coupled network can be described as follows:

$$\begin{gathered}\dot{x}_i(t) = f(x_i(t)) + \sum_{j=1}^{N} G_{ij}(t) h(x_j(t)) + \sum_{j=1}^{N} \widehat{G}_{ij}(t) \widehat{h}\left(\overline{x_j(t-\tau_j(t))}\right) + B_i(t) u_i(t), \\ y_i(t) = C_i(t) x_i(t) + D_i(t) u_i(t),\end{gathered} \tag{2.1}$$

where $i = 1, 2, \ldots, N$, $f(\cdot)$ is continuously differentiable function, $x_i(t) = (x_{i1}(t), x_{i2}(t), \ldots, x_{in}(t))^T \in R^n$ is the state variable of node i, $\tau_{il}(t)$ is the time-varying delays with

$0 \leqslant \tau_{il}(t) \leqslant \tau_{il} \leqslant \tau$, $l = 1, 2, \ldots, n$, $y_i(t) \in R^n$ is the output of node i, and $u_i(t) \in R^n$ is the input vector of node i, $B_i(t)$, $C_i(t)$ and $D_i(t)$ are known matrices with appropriate dimensions, $h(x_i(t))=[h_1(x_i(t)),h_2(x_i(t)),\ldots,h_n(x_i(t))]^T$ and $\widehat{h}(\overline{x_i(t-\tau_i(t))}) = [\widehat{h}_1(x_i(t-\tau_{i1}(t))),\widehat{h}_2(x_i(t-\tau_{i2}(t))),\ldots,\widehat{h}_n(x_i(t-\tau_{in}(t)))]^T$ are continuously functions that describe the coupling relations between two nodes for nondelayed configuration and delayed one at time t, respectively, $G(t) = (G_{ij}(t))_{N\times N}$ and $\widehat{G}(t) = (\widehat{G}_{ij}(t))_{N\times N}$ represent the topological structure of the complex network and coupling strength between nodes for nondelayed configuration and delayed one at time t, respectively, and the diagonal elements of $G(t)$ and $\widehat{G}(t)$ are defined as follows:

$$G_{ii}(t) = -\sum_{\substack{j=1\\ j\neq 1}}^{N} G_{ij}(t), \quad \widehat{G}_{ii}(t) = -\sum_{\substack{j=1\\ j\neq 1}}^{N} \widehat{G}_{ij}(t), \quad i = 1, 2, \ldots, N. \tag{2.2}$$

One should note that, in this model, the individual couplings between two connected nodes may be nonlinear, and the coupling configurations are not restricted to the symmetric and irreducible connections or the nonnegative off-diagonal links. In [47], Yu and his colleagues investigated a linearly hybrid coupled network with time-varying delay, in which the coupling is linear, time invariant. In addition, the coupling relation and the coupling configuration are not related to the current states and the delayed states. Hence, the complex dynamical network discussed in this paper can describe the real-world networks much better.

In the following, we give several useful denotations, definitions, and lemmas.

Let R^n be the n-dimensional Euclidean space and $R^{n\times m}$ the space of $n \times m$ real matrices. $P \geqslant 0$ ($P \leqslant 0$) means matrix P is symmetrical and semipositive (seminegative) definite. $P > 0$ ($P < 0$) means that matrix P is symmetrical and positive (negative) definite. $\|\cdot\|$ is the Euclidean norm. I_n denotes the $n \times n$ real identity matrix.

Definition 2.1 (see [3, 4, 19]). Network (2.1) is called input passive if there exist two constants β and $\gamma > 0$ such that

$$2\int_0^{t_p} y^T(s)u(s)ds \geqslant -\beta^2 + \gamma\int_0^{t_p} u^T(s)u(s)ds, \tag{2.3}$$

for all $t_p \geqslant 0$.

Definition 2.2. Network (2.1) is called output passive if there exist two constants β and $\gamma > 0$ such that

$$2\int_0^{t_p} y^T(s)u(s)ds \geqslant -\beta^2 + \gamma\int_0^{t_p} y^T(s)y(s)ds, \tag{2.4}$$

for all $t_p \geqslant 0$.

Lemma 2.3 (see [32]). *For any vectors $x, y \in R^n$ and $n \times n$ square matrix $P > 0$, the following LMI holds:*

$$x^T y + y^T x \leqslant x^T P x + y^T P^{-1} y. \tag{2.5}$$

3. Main Results

In order to obtain our main results, two useful assumptions are introduced.

(A1) There exist two constants $L_1 < 0,\ L_2 > 0$ such that

$$x^T(t)f(x(t)) \leqslant L_1 x^T(t)x(t),$$
$$f^T(y(t))f(y(t)) \leqslant L_2 y^T(t)y(t) \tag{3.1}$$

hold for any t. Here $x(t), y(t) \in R^n$ are time-varying vectors.

(A2) There exist two positive constants $L_3, L_4 > 0$ such that

$$\|h(x(t))\| \leqslant L_3\|x(t)\|,$$
$$\left\|\widehat{h}(y(t))\right\| \leqslant L_4\|y(t)\| \tag{3.2}$$

hold for any t. Here $x(t), y(t) \in R^n$ are time-varying vectors.

In the following, we first give some input passivity criteria.

Theorem 3.1. *Let (A1) and (A2) hold, and $\dot{\tau}_{il}(t) \leqslant \sigma < 1$. Suppose that there exist matrices $P = \mathrm{diag}(P_1, \ldots, P_N)$, $P_i = \mathrm{diag}(p_{i1}, p_{i2}, \ldots, p_{in})$, $p_{il} > 0$, and two positive constants $\xi, \gamma > 0$ such that*

$$\begin{gathered}\left(2L_1 + aL_4^2 + L_3^2\right)I + \frac{\left(\widehat{G}(t)\otimes I_n\right)P^{-1}\left(\widehat{G}(t)\otimes I_n\right)^T}{1-\sigma} + \frac{\left(B(t)-C^T(t)\right)\left(B^T(t)-C(t)\right)}{\xi} \\ + (G(t)\otimes I_n)(G(t)\otimes I_n)^T \leqslant 0, \\ D(t) + D^T(t) - (\xi+\gamma)I \geqslant 0,\end{gathered} \tag{3.3}$$

where

$$a = \max\{p_{il}, i = 1,\ 2, \ldots, N,\ l = 1, 2, \ldots, n\}, \tag{3.4}$$

$i = 1, 2, \ldots, N$, $l = 1, 2, \ldots, n$. *I denotes the $Nn \times Nn$ real identity matrix. Then the network* (2.1) *is input passive.*

Proof. Firstly, we can rewrite network (2.1) in a compact form as follows:

$$\begin{gathered}\dot{x}(t) = F(x(t)) + (G(t)\otimes I_n)H(x(t)) + \left(\widehat{G}(t)\otimes I_n\right)\widehat{H}\left(\overline{x(t-\tau(t))}\right) + B(t)u(t), \\ y(t) = C(t)x(t) + D(t)u(t),\end{gathered} \tag{3.5}$$

where

$$
\begin{gathered}
x(t)=\left[x_1^T(t),x_2^T(t),\ldots,x_N^T(t)\right]^T,\qquad y(t)=\left[y_1^T(t),y_2^T(t),\ldots,y_N^T(t)\right]^T,\\
F(x(t))=\left[f^T(x_1(t)),f^T(x_2(t)),\ldots,f^T(x_N(t))\right]^T,\\
B(t)=\operatorname{diag}[B_1(t),B_2(t),\ldots,B_N(t)],\\
\widehat{H}\left(\overline{x(t-\tau(t))}\right)=\left[\hat{h}^T\left(\overline{x_1(t-\tau_1(t))}\right),\hat{h}^T\left(\overline{x_2(t-\tau_2(t))}\right),\ldots,\hat{h}^T\left(\overline{x_N(t-\tau_N(t))}\right)\right]^T,\\
C(t)=\operatorname{diag}[C_1(t),C_2(t),\ldots,C_N(t)],\qquad H(x(t))=\left[h^T(x_1(t)),h^T(x_2(t)),\ldots,h^T(x_N(t))\right]^T,\\
D(t)=\operatorname{diag}[D_1(t),D_2(t),\ldots,D_N(t)],\qquad u(t)=\left[u_1^T(t),u_2^T(t),\ldots,u_N^T(t)\right]^T.
\end{gathered}
\tag{3.6}
$$

In the following, construct Lyapunov functional for model (3.5) as follows:

$$
V(x(t))=x^T(t)x(t)+\sum_{i=1}^{N}\sum_{l=1}^{n}\int_{t-\tau_{il}(t)}^{t}p_{il}\hat{h}_l^2(x_i(\alpha))d\alpha. \tag{3.7}
$$

The derivative of $V(x(t))$ satisfies

$$
\begin{aligned}
&\dot{V}(x(t))\\
&\quad\leqslant 2x^T(t)\dot{x}(t)+\widehat{H}^T(x(t))P\widehat{H}(x(t))-(1-\sigma)\widehat{H}^T\left(\overline{x(t-\tau(t))}\right)P\widehat{H}\left(\overline{x(t-\tau(t))}\right)\\
&\quad=2x^T(t)F(x(t))+2x^T(t)(G(t)\otimes I_n)H(x(t))+2x^T(t)\left(\widehat{G}(t)\otimes I_n\right)\widehat{H}\left(\overline{x(t-\tau(t))}\right)\\
&\qquad+2x^T(t)B(t)u(t)+\widehat{H}^T(x(t))P\widehat{H}(x(t))-(1-\sigma)\widehat{H}^T\left(\overline{x(t-\tau(t))}\right)P\widehat{H}\left(\overline{x(t-\tau(t))}\right),
\end{aligned}
\tag{3.8}
$$

where $\widehat{H}(x(t))=(\hat{h}^T(x_1(t)),\hat{h}^T(x_2(t)),\ldots,\hat{h}^T(x_N(t)))^T$. Then we have

$$
\begin{aligned}
&\dot{V}(x(t))-2y^T(t)u(t)+\gamma u^T(t)u(t)\\
&\quad\leqslant 2x^T(t)F(x(t))+\widehat{H}^T(x(t))P\widehat{H}(x(t))+2x^T(t)(G(t)\otimes I_n)H(x(t))+2x^T(t)B(t)u(t)\\
&\qquad+2x^T(t)\left(\widehat{G}(t)\otimes I_n\right)\widehat{H}\left(\overline{x(t-\tau(t))}\right)-(1-\sigma)\widehat{H}^T\left(\overline{x(t-\tau(t))}\right)P\widehat{H}\left(\overline{x(t-\tau(t))}\right)\\
&\qquad-2x^T(t)C^T(t)u(t)-u^T(t)\left[D(t)+D^T(t)\right]u(t)+\gamma u^T(t)u(t).
\end{aligned}
\tag{3.9}
$$

Applying Lemma 2.3, then we can easily obtain

$$
\begin{aligned}
2x^T(t)(G(t)\otimes I_n)H(x(t)) &\leqslant H^T(x(t))H(x(t)) + x^T(t)(G(t)\otimes I_n)(G(t)\otimes I_n)^T x(t),\\
2x^T(t)\left[B(t)-C^T(t)\right]u(t) &\leqslant \frac{x^T(t)\left[B(t)-C^T(t)\right]\left[B^T(t)-C(t)\right]x(t)}{\xi} + \xi u^T(t)u(t),\\
2x^T(t)\left(\widehat{G}(t)\otimes I_n\right)\widehat{H}\left(\overline{x(t-\tau(t))}\right) &\leqslant (1-\sigma)\widehat{H}^T\left(\overline{x(t-\tau(t))}\right)P\widehat{H}\left(\overline{x(t-\tau(t))}\right)\\
&\quad + \frac{x^T(t)\left(\widehat{G}(t)\otimes I_n\right)P^{-1}\left(\widehat{G}(t)\otimes I_n\right)^T x(t)}{1-\sigma}.
\end{aligned}
\tag{3.10}
$$

Hence, we have

$$
\begin{aligned}
&\dot{V}(x(t)) - 2y^T(t)u(t) + \gamma u^T(t)u(t)\\
&\quad\leqslant 2x^T(t)F(x(t)) + \widehat{H}^T(x(t))P\widehat{H}(x(t)) + x^T(t)(G(t)\otimes I_n)(G(t)\otimes I_n)^T x(t)\\
&\qquad + H^T(x(t))H(x(t)) + \frac{x^T(t)\left[B(t)-C^T(t)\right]\left[B^T(t)-C(t)\right]x(t)}{\xi}\\
&\qquad + \frac{x^T(t)\left(\widehat{G}(t)\otimes I_n\right)P^{-1}\left(\widehat{G}(t)\otimes I_n\right)^T x(t)}{1-\sigma}.
\end{aligned}
\tag{3.11}
$$

According to (A1) and (A2), we have

$$
\begin{aligned}
x^T(t)F(x(t)) &= \sum_{i=1}^{N} x_i^T(t)f(x_i(t)) \leqslant \sum_{i=1}^{N} L_1 x_i^T(t)x_i(t) = L_1 x^T(t)x(t),\\
H^T(x(t))H(x(t)) &= \sum_{i=1}^{N} h^T(x_i(t))h(x_i(t)) \leqslant \sum_{i=1}^{N} L_3^2 x_i^T(t)x_i(t) = L_3^2 x^T(t)x(t),\\
\widehat{H}^T(x(t))\widehat{H}(x(t)) &= \sum_{i=1}^{N} \widehat{h}^T(x_i(t))\widehat{h}(x_i(t)) \leqslant \sum_{i=1}^{N} L_4^2 x_i^T(t)x_i(t) = L_4^2 x^T(t)x(t).
\end{aligned}
\tag{3.12}
$$

It follows from inequalities (3.12) that

$$
\begin{aligned}
&\dot{V}(x(t)) - 2y^T(t)u(t) + \gamma u^T(t)u(t)\\
&\quad\leqslant x^T(t)\left[\left(2L_1 + aL_4^2 + L_3^2\right)I + \frac{\left(\widehat{G}(t)\otimes I_n\right)P^{-1}\left(\widehat{G}(t)\otimes I_n\right)^T}{1-\sigma}\right.\\
&\qquad\left. + \frac{\left(B(t)-C^T(t)\right)\left(B^T(t)-C(t)\right)}{\xi} + (G(t)\otimes I_n)(G(t)\otimes I_n)^T\right]x(t)\\
&\quad\leqslant 0.
\end{aligned}
\tag{3.13}
$$

By integrating (3.13) with respect to t over the time period 0 to t_p, we get

$$2\int_0^{t_p} y^T(s)u(s)ds \geqslant V(x(t_p)) - V(x(0)) + \gamma\int_0^{t_p} u^T(s)u(s)ds. \tag{3.14}$$

From the definition of $V(x(t))$, we have $V(x(t_p) \geqslant 0$. Thus,

$$2\int_0^{t_p} y^T(s)u(s)ds \geqslant -V(x(0)) + \gamma\int_0^{t_p} u^T(s)u(s)ds, \tag{3.15}$$

for all $t_p \geqslant 0$. The proof is completed. □

Theorem 3.2. *Let (A1) and (A2) hold, and $\dot{\tau}_{il}(t) \leqslant \sigma < 1$. Assume that there exist two matrices $Z = \operatorname{diag}(Z_1,\dots,Z_N)$, $Z_i = \operatorname{diag}(z_{i1}, z_{i2},\dots,z_{in})$, $P = \operatorname{diag}(P_1, P_2,\dots,P_N)$, $P_i = \operatorname{diag}(p_{i1}, p_{i2},\dots,p_{in})$, $z_{il}, p_{il} > 0$, and two positive constants $\xi, \gamma > 0$ such that*

$$2L_1 + aL_4^2 + \xi\left(1 + L_2 + L_3^2\right) \leqslant 0, \tag{3.16}$$

$$\begin{bmatrix} 0 & 0 & G(t)\otimes I_n & \widehat{G}(t)\otimes I_n & B(t) - C^T(t) \\ 0 & M & M(G(t)\otimes I_n) & M\left(\widehat{G}(t)\otimes I_n\right) & MB(t) \\ \mathfrak{A} & \mathfrak{A}M & \mathfrak{A}M(G(t)\otimes I_n) & \mathfrak{A}M\left(\widehat{G}(t)\otimes I_n\right) & \mathfrak{A}MB(t) \\ \mathfrak{B} & \mathfrak{B}M & \mathfrak{B}M(G(t)\otimes I_n) & \mathfrak{B}M\left(\widehat{G}(t)\otimes I_n\right) & \mathfrak{B}MB(t) \\ B^T(t) - C(t) & B^T(t)M & B^T(t)M(G(t)\otimes I_n) & B^T(t)M\left(\widehat{G}(t)\otimes I_n\right) & \gamma I + B^T(t)MB(t) \end{bmatrix}$$
$$-\xi\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & I \end{bmatrix} \leqslant 0, \tag{3.17}$$

where $\mathfrak{A}$ denotes $(G(t)\otimes I_n)^T$ and $\mathfrak{B}$ denotes $(\widehat{G}(t)\otimes I_n)^T$

$$\begin{gathered} D(t) + D^T(t) - \xi I \geqslant 0, \\ b(1-\sigma) - \xi \geqslant 0, \end{gathered} \tag{3.18}$$

where

$$\begin{aligned} a &= \max\{P_{il},\ i = 1,2,\dots,N,\ l = 1,2,,\dots,n\}, \qquad M_i = \operatorname{diag}(\tau_{i1}z_{i1}, \tau_{i2}z_{i2},\dots,\tau_{in}z_{in}), \\ b &= \mathit{min}\{P_{il},\ i = 1,2,\dots,N,\ l = 1,2,\dots,n\}, \qquad M = \operatorname{diag}(M_1, M_2,\dots,M_N), \end{aligned} \tag{3.19}$$

$i = 1, 2, 3, \ldots, N$, $l = 1, 2, \ldots, n$. *I denotes the $Nn \times Nn$ real identity matrix. Then the network* (2.1) *is input passive.*

Proof. Firstly, construct the following Lyapunov functional for system (3.5):

$$V(x(t)) = x^T(t)x(t) + \sum_{i=1}^{N}\sum_{l=1}^{n}\int_{-\tau_{il}(t)}^{0}\int_{t+\beta}^{t} z_{il}\dot{x}_{il}^2(\alpha)d\alpha d\beta + \sum_{i=1}^{N}\sum_{l=1}^{n}\int_{t-\tau_{il}(t)}^{t} p_{il}\widehat{h}_l^2(x_i(\alpha))d\alpha. \quad (3.20)$$

The derivative of $V(x(t))$ satisfies

$$\begin{aligned}
&\dot{V}(x(t)) \\
&\leqslant 2x^T(t)\dot{x}(t) + \sum_{i=1}^{N}\sum_{l=1}^{n}\dot{\tau}_{il}(t)\int_{t-\tau_{il}(t)}^{t} z_{il}\dot{x}_{il}^2(\alpha)d\alpha + \sum_{i=1}^{N}\sum_{l=1}^{n}\tau_{il}(t)z_{il}\dot{x}_{il}^2(t) - \sum_{i=1}^{N}\sum_{l=1}^{n}\int_{t-\tau_{il}(t)}^{t} z_{il}\dot{x}_{il}^2(\alpha)d\alpha \\
&\quad + \widehat{H}^T(x(t))P\widehat{H}(x(t)) - (1-\sigma)\widehat{H}^T\left(\overline{x(t-\tau(t))}\right)P\widehat{H}\left(\overline{x(t-\tau(t))}\right) \\
&\leqslant 2x^T(t)F(x(t)) + 2x^T(t)(G(t)\otimes I_n)H(x(t)) + 2x^T(t)\left(\widehat{G}(t)\otimes I_n\right)\widehat{H}\left(\overline{x(t-\tau(t))}\right) \\
&\quad + 2x^T(t)B(t)u(t) + \left[F(x(t)) + (G(t)\otimes I_n)H(x(t)) + \left(\widehat{G}(t)\otimes I_n\right)\widehat{H}\left(\overline{x(t-\tau(t))}\right) + B(t)u(t)\right]^T \\
&\quad \times M\left[F(x(t)) + (G(t)\otimes I_n)H(x(t)) + \left(\widehat{G}(t)\otimes I_n\right)\widehat{H}\left(\overline{x(t-\tau(t))}\right) + B(t)u(t)\right] \\
&\quad + \widehat{H}^T(x(t))P\widehat{H}(x(t)) - (1-\sigma)\widehat{H}^T\left(\overline{x(t-\tau(t))}\right)P\widehat{H}\left(\overline{x(t-\tau(t))}\right).
\end{aligned} \quad (3.21)$$

Then we can easily obtain

$$\begin{aligned}
&\dot{V}(x(t)) - 2y^T(t)u(t) + \gamma u^T(t)u(t) \\
&\quad \leqslant 2x^T(t)F(x(t)) + 2x^T(t)(G(t)\otimes I_n)H(x(t)) \\
&\qquad + 2x^T(t)\left(\widehat{G}(t)\otimes I_n\right)\widehat{H}\left(\overline{x(t-\tau(t))}\right) + 2x^T(t)B(t)u(t) \\
&\qquad + \left[F(x(t)) + (G(t)\otimes I_n)H(x(t)) + \left(\widehat{G}(t)\otimes I_n\right)\widehat{H}\left(\overline{x(t-\tau(t))}\right) + B(t)u(t)\right]^T \\
&\qquad \times M\left[F(x(t)) + (G(t)\otimes I_n)H(x(t)) + \left(\widehat{G}(t)\otimes I_n\right)\widehat{H}\left(\overline{x(t-\tau(t))}\right) + B(t)u(t)\right] \\
&\qquad + \widehat{H}^T(x(t))P\widehat{H}(x(t)) - (1-\sigma)\widehat{H}^T\left(\overline{x(t-\tau(t))}\right)P\widehat{H}\left(\overline{x(t-\tau(t))}\right) \\
&\qquad - 2x^T(t)C^T(t)u(t) - u^T(t)\left[D(t) + D^T(t)\right]u(t) + \gamma u^T(t)u(t).
\end{aligned} \quad (3.22)$$

Set

$$W(t) = \left[x^T(t), F^T(x(t)), H^T(x(t)), \widehat{H}^T\left(\overline{x(t-\tau(t))}\right), u^T(t)\right]^T. \quad (3.23)$$

We have

$$
\begin{aligned}
&\dot{V}(x(t)) - 2y^T(t)u(t) + \gamma u^T(t)u(t) \\
&\quad \leqslant 2x^T(t)F(x(t)) + \widehat{H}^T(x(t))P\widehat{H}(x(t)) - (1-\sigma)\widehat{H}^T\left(\overline{x(t-\tau(t))}\right)P\widehat{H}\left(\overline{x(t-\tau(t))}\right) + W^T(t) \\
&\qquad \times \begin{bmatrix}
0 & 0 & G(t)\otimes I_n & \widehat{G}(t)\otimes I_n & B(t)-C^T(t) \\
0 & M & M(G(t)\otimes I_n) & M\left(\widehat{G}(t)\otimes I_n\right) & MB(t) \\
\mathfrak{A} & \mathfrak{A}M & \mathfrak{A}M(G(t)\otimes I_n) & \mathfrak{A}M\left(\widehat{G}(t)\otimes I_n\right) & \mathfrak{A}MB(t) \\
\mathfrak{B} & \mathfrak{B}M & \mathfrak{B}M(G(t)\otimes I_n) & \mathfrak{B}M\left(\widehat{G}(t)\otimes I_n\right) & \mathfrak{B}MB(t) \\
B^T(t)-C(t) & B^T(t)M & B^T(t)M(G(t)\otimes I_n) & B^T(t)M\left(\widehat{G}(t)\otimes I_n\right) & \gamma I + B^T(t)MB(t)
\end{bmatrix} \\
&\qquad \times W(t) - u^T(t)\left[D(t)+D^T(t)\right]u(t).
\end{aligned}
\tag{3.24}
$$

According to LMI (3.17)-(3.18), we have

$$
\begin{aligned}
&\dot{V}(x(t)) - 2y^T(t)u(t) + \gamma u^T(t)u(t) \\
&\quad \leqslant 2x^T(t)F(x(t)) + \widehat{H}^T(x(t))P\widehat{H}(x(t)) - (1-\sigma)\widehat{H}^T\left(\overline{x(t-\tau(t))}\right)P\widehat{H}\left(\overline{x(t-\tau(t))}\right) \\
&\qquad - u^T(t)\left[D(t)+D^T(t)-\xi I\right]u(t) \\
&\qquad + \xi x^T(t)x(t) + \xi F^T(x(t))F(x(t)) + \xi H^T(x(t))H(x(t)) + \xi\widehat{H}^T\left(\overline{x(t-\tau(t))}\right)\widehat{H}\left(\overline{x(t-\tau(t))}\right) \\
&\quad \leqslant 2x^T(t)F(x(t)) + \widehat{H}^T(x(t))P\widehat{H}(x(t)) + \xi x^T(t)x(t) + \xi F^T(x(t))F(x(t)) + \xi H^T(x(t))H(x(t)).
\end{aligned}
\tag{3.25}
$$

Since (A1), we can easily obtain

$$
F^T(x(t))F(x(t)) = \sum_{i=1}^{N} f^T(x_i(t))f(x_i(t)) \leqslant \sum_{i=1}^{N} L_2 x_i^T(t)x_i(t) = L_2 x^T(t)x(t). \tag{3.26}
$$

It follows from inequalities (3.12) and (3.26) that

$$
\dot{V}(x(t)) - 2y^T(t)u(t) + \gamma u^T(t)u(t) \leqslant x^T(t)\left[2L_1 + aL_4^2 + \xi\left(1 + L_2 + L_3^2\right)\right]x(t) \leqslant 0. \tag{3.27}
$$

By integrating (3.27) with respect to t over the time period 0 to t_p, we get

$$2\int_0^{t_p} y^T(s)u(s)ds \geqslant V(x(t_p)) - V(x(0)) + \gamma\int_0^{t_p} u^T(s)u(s)ds. \tag{3.28}$$

From the definition of $V(x(t))$, we have $V(x(t_p) \geqslant 0$. Thus,

$$2\int_0^{t_p} y^T(s)u(s)ds \geqslant -V(x(0)) + \gamma\int_0^{t_p} u^T(s)u(s)ds, \tag{3.29}$$

for all $t_p \geqslant 0$. The proof is completed. □

In the above, two sufficient conditions are given to ensure the input passivity of network (2.1). In the following, we discuss the output passivity of network (2.1).

Theorem 3.3. *Let (A1) and (A2) hold, and* $\dot{\tau}_{il}(t) \leqslant \sigma < 1$. *Suppose that there exist matrices* $P = \mathrm{diag}(P_1, \ldots, P_N)$, $P_i = \mathrm{diag}(P_{i1}, P_{i2}, \ldots, P_{in})$, $P_{il} > 0$, *and two positive constants* $\xi, \gamma > 0$ *such that*

$$\begin{gathered}
\left(2L_1 + aL_4^2 + L_3^2\right)I + \gamma C^T(t)C(t) + \frac{\left(\widehat{G}(t)\otimes I_n\right)P^{-1}\left(\widehat{G}(t)\otimes I_n\right)^T}{1-\sigma} + \frac{W(t)W^T(t)}{\xi} \\
+ (G(t)\otimes I_n)(G(t)\otimes I_n)^T \leqslant 0, \\
D(t) + D^T(t) - \gamma D^T(t)D(t) - \xi I \geqslant 0,
\end{gathered} \tag{3.30}$$

where

$$\begin{gathered}
a = \max\{P_{il},\ i = 1,2,\ldots,N,\ l = 1,2,\ldots,n\}, \\
W(t) = B(t) - C^T(t) + \gamma C^T(t)D(t),
\end{gathered} \tag{3.31}$$

$i = 1,2,\ldots,N,\ l = 1,2,\ldots,n. I$ *denotes the* $Nn \times Nn$ *real identity matrix. Then the network* (2.1) *is output passive.*

Proof. Firstly, we construct the same Lyapunov functional as (3.7) for system (3.5), then we can obtain

$$\begin{aligned}
\dot{V}(x(t)) \leqslant{}& 2x^T(t)F(x(t)) + 2x^T(t)(G(t)\otimes I_n)H(x(t)) + 2x^T(t)\left(\widehat{G}(t)\otimes I_n\right)\widehat{H}\left(\overline{x(t-\tau(t))}\right) \\
&+ 2x^T(t)B(t)u(t) + \widehat{H}^T(x(t))P\widehat{H}(x(t)) - (1-\sigma)\widehat{H}^T\left(\overline{x(t-\tau(t))}\right)P\widehat{H}\left(\overline{x(t-\tau(t))}\right).
\end{aligned} \tag{3.32}$$

Then we have

$$
\begin{aligned}
&\dot{V}(x(t)) - 2y^T(t)u(t) + \gamma y^T(t)y(t) \\
&\quad\leqslant 2x^T(t)F(x(t)) + 2x^T(t)(G(t)\otimes I_n)H(x(t)) + 2x^T(t)\left(\widehat{G}(t)\otimes I_n\right)\widehat{H}\left(\overline{x(t-\tau(t))}\right) \\
&\qquad + 2x^T(t)B(t)u(t) + \widehat{H}^T(x(t))P\widehat{H}(x(t)) - (1-\sigma)\widehat{H}^T\left(\overline{x(t-\tau(t))}\right)P\widehat{H}\left(\overline{x(t-\tau(t))}\right) \\
&\qquad - 2x^T(t)C^T(t)u(t) - u^T(t)\left[D(t) + D^T(t)\right]u(t) + \gamma[C(t)x(t) + D(t)u(t)]^T \\
&\qquad \times [C(t)x(t) + D(t)u(t)] \\
&\quad= 2x^T(t)F(x(t)) + 2x^T(t)(G(t)\otimes I_n)H(x(t)) + 2x^T(t)\left(\widehat{G}(t)\otimes I_n\right)\widehat{H}\left(\overline{x(t-\tau(t))}\right) \\
&\qquad + \widehat{H}^T(x(t))P\widehat{H}(x(t)) - (1-\sigma)\widehat{H}^T\left(\overline{x(t-\tau(t))}\right)P\widehat{H}\left(\overline{x(t-\tau(t))}\right) \\
&\qquad + 2x^T(t)\left[B(t) - C^T(t) + \gamma C^T(t)D(t)\right]u(t) + \gamma x^T(t)C^T(t)C(t)x(t) \\
&\qquad - u^T(t)\left[D(t) + D^T(t) - \gamma D^T(t)D(t)\right]u(t).
\end{aligned}
\tag{3.33}
$$

Applying Lemma 2.3, we have

$$
\begin{aligned}
2x^T(t)(G(t)\otimes I_n)H(x(t)) &\leqslant H^T(x(t))H(x(t)) + x^T(t)(G(t)\otimes I_n)(G(t)\otimes I_n)^T x(t), \\
2x^T(t)W(t)u(t) &\leqslant \frac{x^T(t)W(t)W^T(t)x(t)}{\xi} + \xi u^T(t)u(t), \\
2x^T(t)\left(\widehat{G}(t)\otimes I_n\right)\widehat{H}\left(\overline{x(t-\tau(t))}\right) &\leqslant (1-\sigma)\widehat{H}^T\left(\overline{x(t-\tau(t))}\right)P\widehat{H}\left(\overline{x(t-\tau(t))}\right) \\
&\quad + \frac{x^T(t)\left(\widehat{G}(t)\otimes I_n\right)P^{-1}\left(\widehat{G}(t)\otimes I_n\right)^T x(t)}{1-\sigma}.
\end{aligned}
\tag{3.34}
$$

Hence, we can easily obtain

$$
\begin{aligned}
&\dot{V}(x(t)) - 2y^T(t)u(t) + \gamma y^T(t)y(t) \\
&\quad\leqslant 2x^T(t)F(x(t)) + x^T(t)(G(t)\otimes I_n)(G(t)\otimes I_n)^T x(t) \\
&\qquad + \frac{x^T(t)\left(\widehat{G}(t)\otimes I_n\right)P^{-1}\left(\widehat{G}(t)\otimes I_n\right)^T x(t)}{1-\sigma} + \frac{x^T(t)W(t)W^T(t)x(t)}{\xi} \\
&\qquad + H^T(x(t))H(x(t)) + \widehat{H}^T(x(t))P\widehat{H}(x(t)) + \gamma x^T(t)C^T(t)C(t)x(t).
\end{aligned}
\tag{3.35}
$$

It follows from inequalities (3.12) that

$$
\begin{aligned}
&\dot{V}(x(t)) - 2y^T(t)u(t) + \gamma y^T(t)y(t) \\
&\quad \leqslant x^T(t)\left[\left(2L_1 + aL_4^2 + L_3^2\right)I + \gamma C^T(t)C(t) + \frac{\left(\widehat{G}(t)\otimes I_n\right)P^{-1}\left(\widehat{G}(t)\otimes I_n\right)^T}{1-\sigma}\right. \\
&\qquad \left. + \frac{W(t)W^T(t)}{\xi} + (G(t)\otimes I_n)(G(t)\otimes I_n)^T\right]x(t) \leqslant 0.
\end{aligned}
\tag{3.36}
$$

By integrating (3.36) with respect to t over the time period 0 to t_p, we get

$$
2\int_0^{t_p} y^T(s)u(s)ds \geqslant V\left(x\left(t_p\right)\right) - V(x(0)) + \gamma\int_0^{t_p} y^T(s)y(s)ds. \tag{3.37}
$$

From the definition of $V(x(t))$, we have $V(x(t_p)) \geqslant 0$. Thus,

$$
2\int_0^{t_p} y^T(s)u(s)ds \geqslant -V(x(0)) + \gamma\int_0^{t_p} y^T(s)y(s)ds, \tag{3.38}
$$

for all $t_p \geqslant 0$. The proof is completed. □

Theorem 3.4. *Let (A1) and (A2) hold, and $\dot{\tau}_{il}(t) \leqslant \sigma < 1$. Assume that there exist two matrices $Z = \mathrm{diag}(Z_1, \ldots, Z_N)$, $Z_i = \mathrm{diag}(z_{i1}, z_{i2}, \ldots, z_{in})$, $P = \mathrm{diag}(P_1, P_2, \ldots, P_N)$, $P_i = \mathrm{diag}(p_{i1}, p_{i2}, \ldots, p_{in})$, $z_{il}, p_{il} > 0$, and two positive constants $\xi, \gamma > 0$ such that*

$$
2L_1 + aL_4^2 + \xi\left(1 + L_2 + L_3^2\right) \leqslant 0, \tag{3.39}
$$

$$
\begin{aligned}
&\begin{bmatrix}
\gamma C^T(t)C(t) & 0 & G(t)\otimes I_n & \widehat{G}(t)\otimes I_n & W_1(t) \\
0 & M & M(G(t)\otimes I_n) & M\mathfrak{D} & MB(t) \\
\mathfrak{A} & \mathfrak{A}M & \mathfrak{A}M(G(t)\otimes I_n) & \mathfrak{A}M\left(\widehat{G}(t)\otimes I_n\right) & \mathfrak{A}MB(t) \\
\mathfrak{B} & \mathfrak{B}M & \mathfrak{B}M(G(t)\otimes I_n) & \mathfrak{B}M\left(\widehat{G}(t)\otimes I_n\right) & \mathfrak{B}MB(t) \\
W_1^T(t) & B^T(t)M & B^T(t)M(G(t)\otimes I_n) & B^T(t)M\left(\widehat{G}(t)\otimes I_n\right) & W_2(t)
\end{bmatrix} \\
&\quad -\xi\begin{bmatrix}
I & 0 & 0 & 0 & 0 \\
0 & I & 0 & 0 & 0 \\
0 & 0 & I & 0 & 0 \\
0 & 0 & 0 & I & 0 \\
0 & 0 & 0 & 0 & I
\end{bmatrix} \leqslant 0,
\end{aligned}
\tag{3.40}
$$

$$D(t) + D^T(t) - \xi I \geqslant 0, \tag{3.41}$$

$$b(1-\sigma) - \xi \geqslant 0, \tag{3.42}$$

where

$$\begin{gathered}
a = \max\{p_{il},\ i = 1,2,\ldots,N,\quad l = 1,2,\ldots,n\}, \qquad W_1(t) = B(t) - C^T(t) + \gamma C^T(t)D(t), \\
b = min\{p_{il}, i = 1,2,\cdots,N,\quad l = 1,2,\ldots,n\}, \qquad W_2(t) = \gamma D^T(t)D(t) + B^T(t)MB(t), \\
M_i = \operatorname{diag}(\tau_{i1}z_{i1}, \tau_{i2}z_{i2}, \ldots, \tau_{in}z_{in}), \qquad M = \operatorname{diag}(M_1, M_2, \ldots, M_N),
\end{gathered} \tag{3.43}$$

$i = 1,2,3,\ldots,N$, $l = 1,2,\ldots,n$. *I denotes the $Nn \times Nn$ real identity matrix. Then the network* (2.1) *is output passive.*

Proof. Similarly, we construct the same Lyapunov functional as (3.20) for system (3.5), then we can obtain

$$\begin{aligned}
&\dot{V}(x(t)) \\
&\leqslant 2x^T(t)F(x(t)) + 2x^T(t)(G(t)\otimes I_n)H(x(t)) + 2x^T(t)\left(\widehat{G}(t)\otimes I_n\right)\widehat{H}\left(\overline{x(t-\tau(t))}\right) \\
&\quad + 2x^T(t)B(t)u(t) + \left[F(x(t)) + (G(t)\otimes I_n)H(x(t)) + \left(\widehat{G}(t)\otimes I_n\right)\widehat{H}\left(\overline{x(t-\tau(t))}\right) + B(t)u(t)\right]^T \\
&\quad \times M\left[F(x(t)) + (G(t)\otimes I_n)H(x(t)) + \left(\widehat{G}(t)\otimes I_n\right)\widehat{H}\left(\overline{x(t-\tau(t))}\right) + B(t)u(t)\right] \\
&\quad + \widehat{H}^T(x(t))P\widehat{H}(x(t)) - (1-\sigma)\widehat{H}^T\left(\overline{x(t-\tau(t))}\right)P\widehat{H}\left(\overline{x(t-\tau(t))}\right).
\end{aligned} \tag{3.44}$$

Then, we can easily obtain

$$\begin{aligned}
&\dot{V}(x(t)) - 2y^T(t)u(t) + \gamma y^T(t)y(t) \\
&\quad \leqslant 2x^T(t)F(x(t)) + 2x^T(t)(G(t)\otimes I_n)H(x(t)) \\
&\qquad + 2x^T(t)\left(\widehat{G}(t)\otimes I_n\right)\widehat{H}\left(\overline{x(t-\tau(t))}\right) + 2x^T(t)B(t)u(t) \\
&\qquad + \left[F(x(t)) + (G(t)\otimes I_n)H(x(t)) + \left(\widehat{G}(t)\otimes I_n\right)\widehat{H}\left(\overline{x(t-\tau(t))}\right) + B(t)u(t)\right]^T \\
&\qquad \times M\left[F(x(t)) + (G(t)\otimes I_n)H(x(t)) + \left(\widehat{G}(t)\otimes I_n\right)\widehat{H}\left(\overline{x(t-\tau(t))}\right) + B(t)u(t)\right] \\
&\qquad + \widehat{H}^T(x(t))P\widehat{H}(x(t)) - (1-\sigma)\widehat{H}^T\left(\overline{x(t-\tau(t))}\right)P\widehat{H}\left(\overline{x(t-\tau(t))}\right) \\
&\qquad - 2x^T(t)C^T(t)u(t) - u^T(t)\left[D(t) + D^T(t)\right]u(t) \\
&\qquad + \gamma[C(t)x(t) + D(t)u(t)]^T[C(t)x(t) + D(t)u(t)].
\end{aligned} \tag{3.45}$$

Set

$$W(t) = \left[x^T(t), F^T(x(t)), H^T(x(t)), \widehat{H}^T\left(\overline{x(t-\tau(t))}\right), u^T(t)\right]^T. \tag{3.46}$$

So, we have

$$\begin{aligned}
&\dot{V}(x(t)) - 2y^T(t)u(t) + \gamma y^T(t)y(t) \\
&\quad \leqslant 2x^T(t)F(x(t)) + \widehat{H}^T(x(t))P\widehat{H}(x(t)) - (1-\sigma)\widehat{H}^T\left(\overline{x(t-\tau(t))}\right)P\widehat{H}\left(\overline{x(t-\tau(t))}\right) + W^T(t) \\
&\qquad \times \begin{bmatrix} \gamma C^T(t)C(t) & 0 & G(t)\otimes I_n & \widehat{G}(t)\otimes I_n & W_1(t) \\ 0 & M & M(G(t)\otimes I_n) & M\left(\widehat{G}(t)\otimes I_n\right) & MB(t) \\ \mathfrak{A} & \mathfrak{A}M & \mathfrak{A}M(G(t)\otimes I_n) & \mathfrak{A}M\left(\widehat{G}(t)\otimes I_n\right) & \mathfrak{A}MB(t) \\ \mathfrak{B} & \mathfrak{B}M & \mathfrak{B}M(G(t)\otimes I_n) & \mathfrak{B}M\left(\widehat{G}(t)\otimes I_n\right) & \mathfrak{B}MB(t) \\ W_1^T(t) & B^T(t)M & B^T(t)M(G(t)\otimes I_n) & B^T(t)M\left(\widehat{G}(t)\otimes I_n\right) & W_2(t) \end{bmatrix} \\
&\qquad \times W(t) - u^T(t)\left[D(t) + D^T(t)\right]u(t).
\end{aligned} \tag{3.47}$$

According to LMI (3.40), we have

$$\begin{aligned}
&\dot{V}(x(t)) - 2y^T(t)u(t) + \gamma y^T(t)y(t) \\
&\quad \leqslant 2x^T(t)F(x(t)) + \widehat{H}^T(x(t))P\widehat{H}(x(t)) + \xi x^T(t)x(t) \\
&\qquad + \xi F^T(x(t))F(x(t)) + \xi H^T(x(t))H(x(t)).
\end{aligned} \tag{3.48}$$

It follows from inequalities (3.12) and (3.26) that

$$\dot{V}(x(t)) - 2y^T(t)u(t) + \gamma y^T(t)y(t) \leqslant x^T(t)\left[2L_1 + aL_4^2 + \xi\left(1 + L_2 + L_3^2\right)\right]x(t) \leqslant 0. \tag{3.49}$$

By integrating (3.49) with respect to t over the time period 0 to t_p, we get

$$2\int_0^{t_p} y^T(s)u(s)ds \geqslant V\left(x\left(t_p\right)\right) - V(x(0)) + \gamma\int_0^{t_p} y^T(s)y(s)ds. \tag{3.50}$$

From the definition of $V(x(t))$, we have $V(x(t_p) \geqslant 0$. Thus,

$$2\int_0^{t_p} y^T(s)u(s)ds \geqslant -V(x(0)) + \int_0^{t_p} y^T(s)y(s)ds, \tag{3.51}$$

for all $t_p \geqslant 0$. The proof is completed. □

Remark 3.5. The conditions in the above theorems do not restrict the coupling among the nodes of the complex networks to be linear, time invariant, symmetric, and so on. Therefore, our criteria are flexible and convenient.

4. Example

In this section, we give an example and its simulation to show the effectiveness of the above obtained theoretical criteria.

Example 4.1. Consider the following dynamical network (2.1) with the system parameters:

$$G(t) = \begin{bmatrix} -0.8 & 0.3 & 0.5 \\ 0.3 & 0.1 & -0.4 \\ 0.4 & 0 & -0.4 \end{bmatrix},$$

$$\widehat{G}(t) = \begin{bmatrix} -1 & 0.3 & 0.7 \\ -0.3 & 0.3 & 0 \\ 0.2 & 0.8 & -1 \end{bmatrix},$$

$$B_i(t) = C_i(t) = D_i(t) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \tag{4.1}$$

$$h(x_i(t)) = \left[\frac{\sin(x_{i1}(t))}{5}, \frac{\sin(x_{i2}(t))}{6}, \frac{\sin(x_{i3}(t))}{7}\right]^T,$$

$$\widehat{h}(x_i(t)) = \left[\frac{\sin(x_{i1}(t))}{5}, \frac{\sin(x_{i2}(t))}{7}, \frac{\sin(x_{i3}(t))}{8}\right]^T,$$

$$f(x_i(t)) = [-4x_{i1}(t), -5x_{i2}(t), -6x_{i3}(t)]^T, \qquad x_i(t) \in R^3, \quad i = 1, 2, 3.$$

It is obvious that we can take $L_1 = -4$, $L_2 = 36$, $L_3 = 1/5$, and $L_4 = 1/5$, and the coupling is not restricted to linear, symmetric, and the nonnegative off-diagonal.

Firstly, we analyze the input passivity of network (2.1) with different time-varying delays.

Set $\tau_{il}(t) = 1 - (1/(3+i+l))e^{-t}$, then we have $0 \leqslant \tau_{il}(t) \leq \tau_{il} = \tau = 1$, $\dot{\tau}_{il}(t) = (1/(3+i+l))e^{-t} \leqslant 1/5 < 1$, for $t \geq 0$, $i = 1, 2, 3$, $l = 1, 2, 3$.

Using the MATLAB LMI Toolbox, we can find the following positive-definite matrix P satisfying the LMI (3.3) with $\gamma = 0.5$, $\xi = 1.4$, and $a = 1$,

$$P = \operatorname{diag}(1, 1, \ldots, 1). \tag{4.2}$$

According to Theorem 3.1, we know that network (2.1) with above given parameters is input passive. Then, set $u_i(t) = [\sin(\pi t/5), \sin(\pi t/5), \sin(\pi t/5)]^T$ and the simulation results are shown in Figure 1.

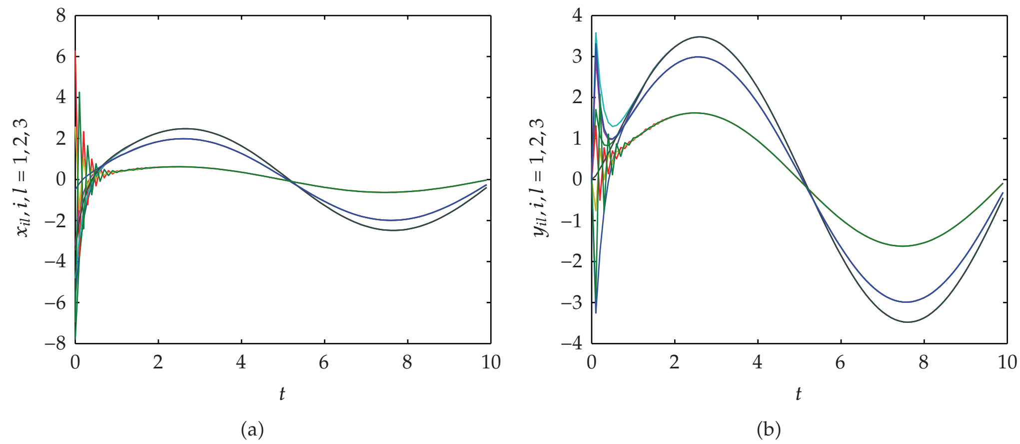

Figure 1: Input passivity of network (2.1) with time-varying delays.

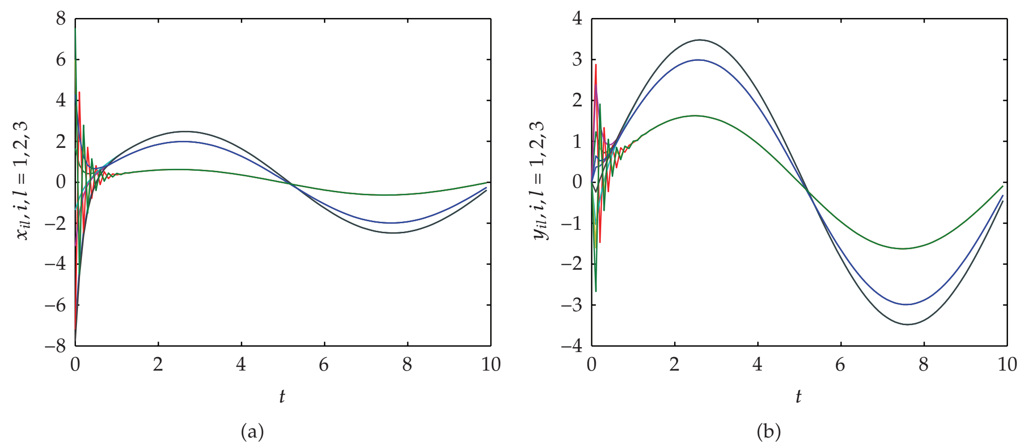

Figure 2: Output passivity of network (2.1) with time-varying delays.

In the following, we analyze the output passivity of the network (2.1) with different time-varying delays.

Set $\tau_{il}(t) = 4/((1 + l + i)t + 4)$, then we have $0 \leqslant \tau_{il}(t) \leq \tau_{il} = \tau = 1$, $\dot{\tau}_{il}(t) = -4(1 + l + i)/[(1 + l + i)t + 4]^2 \leq 0 < 1$, for $t \geq 0$, $i = 1, 2, 3$, $l = 1, 2, 3$.

Using the MATLAB LMI Toolbox, we can find the following positive-definite matrix P satisfying the LMI (3.30) with $\gamma = 1$, $\xi = 1$, and $a = 1$,

$$P = \operatorname{diag}(1, 1, \ldots, 1). \tag{4.3}$$

According to Theorem 3.3, we know that network (2.1) with above given parameters is output passive. Then, set $u_i(t) = [\sin(\pi t/5), \sin(\pi t/5), \sin(\pi t/5)]^T$, and the simulation results are shown in Figure 2.

5. Conclusion

We have studied the input and output passivity of complex dynamical networks. We not only considered the case that the coupling strength and topology structure are frequently varied with time, but also took into account the case that the coupling relation and the coupling configuration are related to the current states and the delayed states. Some input and output passivity criteria have been established in terms of linear matrix inequalities (LMIs) for complex dynamical network by constructing appropriate Lyapunov functionals. An illustrative example was presented to show the efficiency of the derived results.

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