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Research Article

Possibility Fuzzy Soft Set

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We introduce the concept of possibility fuzzy soft set and its operation and study some of its properties. We give applications of this theory in solving a decision-making problem. We also introduce a similarity measure of two possibility fuzzy soft sets and discuss their application in a medical diagnosis problem.

1. Introduction

Fuzzy set was introduced by Zadeh in [1] as a mathematical way to represent and deal with vagueness in everyday life. After that many authors have studied the applications of fuzzy sets in different areas (see Klir and Yuan [2]). Molodtsov [3] initiated the theory of soft sets as a new mathematical tool for dealing with uncertainties which traditional mathematical tools cannot handle. He has shown several applications of this theory in solving many practical problems in economics, engineering, social science, medical science, and so forth. Maji et al. [4, 5] have further studied the theory of soft sets and used this theory to solve some decision-making problems. They have also introduced the concept of fuzzy soft set, a more general concept, which is a combination of fuzzy set and soft set and studied its properties [6], and also Roy and Maji used this theory to solve some decision-making problems [7]. Alkhazaleh et al. [8] introduced soft multiset as a generalization of Molodtsov's soft set. They also introduced in [9] the concept of fuzzy parameterized interval-valued fuzzy soft set and gave its application in decision making. Zhu and Wen in [10] incorporated Molodtsov's soft set theory with the probability theory and proposed the notion of probabilistic soft sets. In [11] Chaudhuri et al. defined the concepts of soft relation and fuzzy soft relation and then applied them to solve a number of decision-making problems. Majumdar and Samanta [12] defined and studied the generalised fuzzy soft sets where the degree is attached with the parameterization of fuzzy sets while defining a fuzzy soft set. In this paper, we generalise the concept of fuzzy soft sets as introduced by Maji et al. [6] to the possibility fuzzy soft set. In our generalisation of fuzzy soft set, a possibility of each element in the universe is attached with the parameterization of fuzzy sets while defining a fuzzy soft set. Also we give some applications of the possibility fuzzy soft set in decision-making problem and medical diagnosis.

2. Preliminaries

In this section, we recall some definitions and properties regarding fuzzy soft set and generalised fuzzy soft set required in this paper.

Let U be a universe set, and let E be a set of parameters. Let $P(U)$ denote the power set of U and $A \subseteq E$.

Definition 2.1 (see [3]). *A* pair (F, E) is called a *soft set* over U, where F is a mapping given by $F : E \to P(U)$. In other words, a soft set over U is a parameterized family of subsets of the universe U.

Definition 2.2 (see [6]). Let U be an initial universal set, and let E be a set of parameters. Let I^U denote the power set of all fuzzy subsets of U. Let $A \subseteq E$. *A* pair (F, E) is called a *fuzzy soft set* over U where F is a mapping given by $F : A \to I^U$.

The following definitions and propositions are due to Majumdar and Samanta [12].

Definition 2.3. Let $U = \{x_1, x_2, \ldots, x_n\}$ be the universal set of elements, and let $E = \{e_1, e_2, \ldots, e_m\}$ be the universal set of parameters. The pair (U, E) will be called a soft universe. Let $F : E \to I^U$ and μ be a fuzzy subset of E, that is, $\mu : E \to I = [0, 1]$, where I^U is the collection of all fuzzy subsets of U. Let $F_\mu : E \to I^U \times I$ be a function defined as follows:

$$F_\mu(e) = \big(F(e), \mu(e)\big). \tag{2.1}$$

Then F_μ is called a *generalized fuzzy soft set* (GFSS in short) over the soft universe (U, E). Here for each parameter e_i, $F_\mu(e_i) = (F(e_i), \mu(e_i))$ indicates not only the degree of belongingness of the elements of U in $F(e_i)$ but also the degree of possibility of such belongingness which is represented by $\mu(e_i)$. So we can write $F_\mu(e_i)$ as follows:

$$F_\mu(e_i) = \left(\left\{\frac{x_1}{F(e_i)(x_1)}, \frac{x_2}{F(e_i)(x_2)}, \ldots, \frac{x_n}{F(e_i)(x_n)}\right\}, \mu(e_i)\right), \tag{2.2}$$

where $F(e_i)(x_1), F(e_i)(x_2), \ldots, F(e_i)(x_n)$ are the degrees of belongingness and $\mu(e_i)$ is the degree of possibility of such belongingness.

Definition 2.4. Let F_μ and G_δ be two GFSSs over (U, E). F_μ is said to be a generalised fuzzy soft subset of G_δ if

(i) μ is a fuzzy subset of δ;

(ii) $F(e)$ is also a fuzzy subset of $G(e)$, forall $e \in E$.

In this case, we write $F_\mu \subseteq G_\delta$.

Definition 2.5. Union of two GFSSs F_μ and G_δ, denoted by $F_\mu \tilde{\cup} G_\delta$, is a GFSS H_ν, defined as $H_\nu : E \longrightarrow I^U \times I$ such that

$$H_\nu(e) = (H(e), \nu(e)), \tag{2.3}$$

where $H(e) = s(F(e), G(e))$, $\nu(e) = s(\mu(e), \delta(e))$, and s is any s-norm.

Definition 2.6. Intersection of two GFSSs F_μ and G_δ, denoted by $F_\mu \tilde{\cap} G_\delta$, is a GFSS H_ν, defined as $H_\nu : E \longrightarrow I^U \times I$ such that

$$H_\nu(e) = (H(e), \nu(e)), \tag{2.4}$$

where $H(e) = t(F(e), G(e))$, $\nu(e) = t(\mu(e), \delta(e))$, and t is any t-norm.

Definition 2.7. A GFSS is said to be a *generalised null fuzzy soft set*, denoted by ϕ_θ, if $\phi_\theta : E \longrightarrow I^U \times I$ such that $\phi_\theta(e) = (F(e), \theta(e))$, where $F(e) = \overline{0}$, forall $e \in E$ and $\theta(e) = \overline{0}$ forall $e \in E$.

Definition 2.8. A GFSS is said to be a *generalised absolute fuzzy soft set*, denoted by $\tilde{A}_\alpha$, if $\tilde{A}_\alpha$: $E \longrightarrow I^U \times I$ such that $\tilde{A}_\alpha(e) = (A(e), \alpha(e))$, where $A(e) = \overline{1}$, forall $e \in E$ and $\alpha(e) = \overline{1}$ forall $e \in E$.

Proposition 2.9. *Let F_μ be a GFSS over (U, E). Then the following holds:*

(i) $F_\mu \subseteq F_\mu \tilde{\cup} F_\mu$,

(ii) $F_\mu \tilde{\cap} F_\mu \subseteq F_\mu$,

(iii) $F_\mu \tilde{\cup} \phi_\theta = F_\mu$,

(iv) $F_\mu \tilde{\cap} \phi_\theta = \phi_\theta$,

(v) $F_\mu \tilde{\cup} \tilde{A}_\alpha = \tilde{A}_\alpha$,

(vi) $F_\mu \tilde{\cup} \tilde{A}_\alpha = F_\mu$.

Proposition 2.10. *Let F_μ, G_δ, and H_λ be any three GFSSs over (U, E). Then the following holds:*

(i) $F_\mu \tilde{\cup} G_\delta = G_\delta \tilde{\cup} F_\mu$,

(ii) $F_\mu \tilde{\cap} G_\delta = G_\delta \tilde{\cap} F_\mu$,

(iii) $F_\mu \tilde{\cup} (G_\delta \tilde{\cup} H_\lambda) = (F_\mu \tilde{\cap} G_\delta) \tilde{\cup} H_\lambda$,

(iv) $F_\mu \tilde{\cap} (G_\delta \tilde{\cap} H_\lambda) = (F_\mu \tilde{\cap} G_\delta) \tilde{\cap} H_\lambda$.

Definition 2.11. Similarity between the two GFSSs F_μ and G_δ, denoted by $S(F_\mu, G_\delta)$, is defined as follows:

$$S(F_\mu, G_\delta) = M(F(e), G(e)) \cdot m(\mu(e), \delta(e)) \tag{2.5}$$

such that $M(F(e), G(e)) = \max_i M_i(F(e), G(e))$, where

$$M_i(F(e), G(e)) = 1 - \frac{\sum_{j=1}^{n} |F_{ij}(e) - G_{ij}(e)|}{\sum_{j=1}^{n} |F_{ij}(e) + G_{ij}(e)|}, \qquad m(\mu(e), \delta(e)) = 1 - \frac{\sum |\mu(e) - \delta(e)|}{\sum |\mu(e) + \delta(e)|}. \tag{2.6}$$

Definition 2.12. Let F_μ and G_δ be two GFSSs over the same universe (U, E). We call the two GFSSs to be *significantly similar* if $S(F_\mu, G_\delta) \geq 1/2$.

Proposition 2.13. *Let F_μ and G_δ be any two GFSSs over (U, E). Then the following holds:*

(i) $S(F_\mu, G_\delta) = S(G_\delta, F_\mu)$,

(ii) $0 \leq S(F_\mu, G_\delta) \leq 1$,

(iii) $F_\mu = G_\delta \Rightarrow S(F_\mu, G_\delta) = 1$,

(iv) $F_\mu \subseteq G_\delta \subseteq H_\lambda \Rightarrow S(F_\mu, H_\lambda) \leq S(G_\delta, H_\lambda)$,

(v) $F_\mu \tilde{\cap} G_\delta = \varphi \Leftrightarrow S(F_\mu, G_\delta) = 0$.

3. Possibility Fuzzy Soft Sets

In this section, we generalise the concept of fuzzy soft sets as introduced by Maji et al. [6]. In our generalisation of fuzzy soft set, a possibility of each element in the universe is attached with the parameterization of fuzzy sets while defining a fuzzy soft set.

Definition 3.1. Let $U = \{x_1, x_2, \ldots, x_n\}$ be the universal set of elements and let $E = \{e_1, e_2, \ldots, e_m\}$ be the universal set of parameters. The pair (U, E) will be called a soft universe. Let $F : E \to I^U$ and μ be a fuzzy subset of E, that is, $\mu : E \to I^U$, where I^U is the collection of all fuzzy subsets of U. Let $F_\mu : E \to I^U \times I^U$ be a function defined as follows:

$$F_\mu(e) = \big(F(e)(x), \mu(e)(x)\big), \quad \forall x \in U. \tag{3.1}$$

Then F_μ is called a *possibility fuzzy soft set* (PFSS in short) over the soft universe (U, E). For each parameter e_i, $F_\mu(e_i) = (F(e_i)(x), \mu(e_i)(x))$ indicates not only the degree of belongingness of the elements of U in $F(e_i)$ but also the degree of possibility of belongingness of the elements of U in $F(e_i)$, which is represented by $\mu(e_i)$. So we can write $F_\mu(e_i)$ as follows:

$$F_\mu(e_i) = \left\{\left(\frac{x_1}{F(e_i)(x_1)}, \mu(e_i)(x_1)\right), \left(\frac{x_2}{F(e_i)(x_2)}, \mu(e_i)(x_2)\right), \ldots, \left(\frac{x_n}{F(e_i)(x_n)}, \mu(e_i)(x_n)\right)\right\}. \tag{3.2}$$

Sometime we write F_μ as (F_μ, E). If $A \subseteq E$, we can also have a PFSS (F_μ, A).

Example 3.2. Let $U = \{x_1, x_2, x_3\}$ be a set of three blouses. Let $E = \{e_1, e_2, e_3\}$ be a set of qualities where e_1 = bright, e_2 = cheap, and e_3 = colourful, and let $\mu : E \to I^U$. We define a function $F_\mu : E \to I^U \times I^U$ as follows:

$$\begin{aligned}
F_\mu(e_1) &= \left\{\left(\frac{x_1}{0.3}, 0.7\right), \left(\frac{x_2}{0.7}, 0.5\right), \left(\frac{x_3}{0.5}, 0.6\right)\right\}, \\
F_\mu(e_2) &= \left\{\left(\frac{x_1}{0.5}, 0.6\right), \left(\frac{x_2}{0.6}, 0.5\right), \left(\frac{x_3}{0.6}, 0.5\right)\right\}, \\
F_\mu(e_3) &= \left\{\left(\frac{x_1}{0.7}, 0.5\right), \left(\frac{x_2}{0.6}, 0.5\right), \left(\frac{x_3}{0.5}, 0.7\right)\right\}.
\end{aligned} \tag{3.3}$$

Then F_μ is a PFSS over (U,E). In matrix notation, we write

$$F_\mu = \begin{pmatrix} 0.3, 0.7 & 0.7, 0.5 & 0.5, 0.6 \\ 0.5, 0.6 & 0.6, 0.5 & 0.6, 0.5 \\ 0.7, 0.5 & 0.6, 0.5 & 0.5, 0.7 \end{pmatrix}. \tag{3.4}$$

Definition 3.3. Let F_μ and G_δ be two PFSSs over (U,E). F_μ is said to be a *possibility fuzzy soft subset* (PFS subset) of G_δ, and one writes $F_\mu \subseteq G_\delta$ if

(i) $\mu(e)$ is a fuzzy subset of $\delta(e)$, forall $e \in E$,

(ii) $F(e)$ is a fuzzy subset of $G(e)$, forall $e \in E$.

Example 3.4. Let $U = \{x_1, x_2, x_3\}$ be a set of three cars, and let $E = \{e_1, e_2, e_3\}$ be a set of parameters where e_1 = cheap, e_2 = expensive, and e_3 = red. Let F_μ be a PFSS over (U,E) defined as follows:

$$\begin{aligned} F_\mu(e_1) &= \left\{ \left(\frac{x_1}{0.2}, 0.4 \right), \left(\frac{x_2}{0.6}, 0.5 \right), \left(\frac{x_3}{0.5}, 0.6 \right) \right\}, \\ F_\mu(e_2) &= \left\{ \left(\frac{x_1}{0.7}, 0.5 \right), \left(\frac{x_2}{0.6}, 0.6 \right), \left(\frac{x_3}{0.8}, 0.6 \right) \right\}, \\ F_\mu(e_3) &= \left\{ \left(\frac{x_1}{0}, 0.1 \right), \left(\frac{x_2}{0.5}, 0.3 \right), \left(\frac{x_3}{0.3}, 0.1 \right) \right\}. \end{aligned} \tag{3.5}$$

Let $G_\delta : E \to I^U \times I^U$ be another PFSS over (U,E) defined as follows:

$$\begin{aligned} G_\delta(e_1) &= \left\{ \left(\frac{x_1}{0.3}, 0.6 \right), \left(\frac{x_2}{0.7}, 0.6 \right), \left(\frac{x_3}{0.6}, 0.7 \right) \right\}, \\ G_\delta(e_2) &= \left\{ \left(\frac{x_1}{0.8}, 0.6 \right), \left(\frac{x_2}{0.7}, 0.7 \right), \left(\frac{x_3}{0.9}, 0.8 \right) \right\}, \\ G_\delta(e_3) &= \left\{ \left(\frac{x_1}{0.1}, 0.2 \right), \left(\frac{x_2}{0.6}, 0.5 \right), \left(\frac{x_3}{0.5}, 0.2 \right) \right\}. \end{aligned} \tag{3.6}$$

It is clear that F_μ is a PFS subset of G_δ.

Definition 3.5. Let F_μ and G_δ be two PFSSs over (U,E). Then F_μ and G_δ are said to be *equal*, and one writes $F_\mu = G_\delta$ if F_μ is a PFS subset of G_δ and G_δ is a PFS subset of F_μ.

In other words, $F_\mu = G_\delta$ if the following conditions are satisfied:

(i) $\mu(e)$ is equal to $\delta(e)$, forall $e \in E$,

(ii) $F(e)$is equal to $G(e)$, forall $e \in E$.

Definition 3.6. A PFSS is said to be a *possibility null fuzzy soft set*, denoted by φ_0, if $\varphi_0 : E \to I^U \times I^U$ such that

$$\varphi_0(e) = \left(F(e)(x), \mu(e)(x)\right), \quad \forall e \in E, \tag{3.7}$$

where $F(e) = 0$, and $\mu(e) = 0$, forall $e \in E$.

Definition 3.7. A PFSS is said to be a *possibility absolute fuzzy soft set*, denoted by A_1, if $A_1 : E \longrightarrow I^U \times I^U$ such that

$$A_1(e) = \left(F(e)(x), \mu(e)(x)\right), \quad \forall e \in E, \tag{3.8}$$

where $F(e) = 1$ and $\mu(e) = 1$, forall $e \in E$.

Example 3.8. Let $U = \{x_1, x_2, x_3\}$ be a set of three blouses. Let $E = \{e_1, e_2, e_3\}$ be a set of qualities where e_1 = bright, e_2 = cheap, and e_3 = colorful, and let $\mu : E \longrightarrow I^U$. We define a function $F_\mu : E \longrightarrow I^U \times I^U$ which is a PFSS over (U, E) defined as follows:

$$\begin{aligned}
F_\mu(e_1) &= \left\{\left(\frac{x_1}{0}, 0\right), \left(\frac{x_2}{0}, 0\right), \left(\frac{x_3}{0}, 0\right)\right\}, \\
F_\mu(e_2) &= \left\{\left(\frac{x_1}{0}, 0\right), \left(\frac{x_2}{0}, 0\right), \left(\frac{x_3}{0}, 0\right)\right\}, \\
F_\mu(e_3) &= \left\{\left(\frac{x_1}{0}, 0\right), \left(\frac{x_2}{0}, 0\right), \left(\frac{x_3}{0}, 0\right)\right\}.
\end{aligned} \tag{3.9}$$

Then F_μ is a possibility null fuzzy soft set.

Let $\mu : E \longrightarrow I^U$, and we define the function $F_\mu : E \longrightarrow I^U \times I^U$ which is a PFSS over (U, E) as follows:

$$\begin{aligned}
F_\mu(e_1) &= \left\{\left(\frac{x_1}{1}, 1\right), \left(\frac{x_2}{1}, 1\right), \left(\frac{x_3}{1}, 1\right)\right\}, \\
F_\mu(e_2) &= \left\{\left(\frac{x_1}{1}, 1\right), \left(\frac{x_2}{1}, 1\right), \left(\frac{x_3}{1}, 1\right)\right\}, \\
F_\mu(e_3) &= \left\{\left(\frac{x_1}{1}, 1\right), \left(\frac{x_2}{1}, 1\right), \left(\frac{x_3}{1}, 1\right)\right\}.
\end{aligned} \tag{3.10}$$

Then F_μ is a possibility absolute fuzzy soft set.

Definition 3.9. Let F_μ be a PFSS over (U, E). Then the *complement* of F_μ, denoted by F_μ^c, is defined by $F_\mu^c = G_\delta$ such that $\delta(e) = c(\mu(e))$ and $G(e) = c(F(e))$, forall $e \in E$, where c is a fuzzy complement.

Example 3.10. Consider the matrix notation in Example 3.2:

$$F_\mu = \begin{pmatrix} 0.3, 0.7 & 0.7, 0.5 & 0.5, 0.6 \\ 0.5, 0.6 & 0.6, 0.5 & 0.6, 0.5 \\ 0.7, 0.5 & 0.6, 0.5 & 0.5, 0.7 \end{pmatrix}. \tag{3.11}$$

By using the basic fuzzy complement, we have $F_\mu^c = G_\delta$ where G_δ

$$G_\delta = \begin{pmatrix} 0.7, 0.3 & 0.3, 0.5 & 0.5, 0.4 \\ 0.5, 0.4 & 0.4, 0.5 & 0.4, 0.5 \\ 0.3, 0.5 & 0.4, 0.5 & 0.5, 0.3 \end{pmatrix}. \tag{3.12}$$

4. Union and Intersection of PFSS

In this section, we introduce the definitions of union and intersection of PFSS, derive some properties, and give some examples.

Definition 4.1. Union of two PFSSs F_μ and G_δ, denoted by $F_\mu \tilde{\cup} G_\delta$, is a PFSS $H_\nu : E \to I^U \times I^U$ defined by

$$H_\nu(e) = (H(e)(x), \nu(e)(x)), \quad \forall e \in E, \tag{4.1}$$

such that $H(e) = s(F(e), G(e))$ and $\nu(e) = s(\mu(e), \delta(e))$ where s is an s-norm.

Example 4.2. Let $U = \{x_1, x_2, x_3\}$ and $E = \{e_1, e_2, e_3\}$. Let F_μ be a PFSS defined as follows:

$$\begin{aligned} F_\mu(e_1) &= \left\{ \left(\frac{x_1}{0.7}, 0.4 \right), \left(\frac{x_2}{0.7}, 0.6 \right), \left(\frac{x_3}{0.6}, 0.6 \right) \right\}, \\ F_\mu(e_2) &= \left\{ \left(\frac{x_1}{0.4}, 0.6 \right), \left(\frac{x_2}{0.8}, 0.5 \right), \left(\frac{x_3}{0.3}, 0.8 \right) \right\}, \\ F_\mu(e_3) &= \left\{ \left(\frac{x_1}{0.2}, 0.9 \right), \left(\frac{x_2}{0.8}, 0.8 \right), \left(\frac{x_3}{0.3}, 0.6 \right) \right\}. \end{aligned} \tag{4.2}$$

Let G_δ be another PFSS over (U, E) defined as follows:

$$\begin{aligned} G_\delta(e_1) &= \left\{ \left(\frac{x_1}{0.6}, 0.4 \right), \left(\frac{x_2}{0.3}, 0.5 \right), \left(\frac{x_3}{0.3}, 0.5 \right) \right\}, \\ G_\delta(e_2) &= \left\{ \left(\frac{x_1}{0.7}, 0.7 \right), \left(\frac{x_2}{0.5}, 0.6 \right), \left(\frac{x_3}{0.4}, 0.7 \right) \right\}, \\ G_\delta(e_3) &= \left\{ \left(\frac{x_1}{0.3}, 0.9 \right), \left(\frac{x_2}{0.4}, 0.4 \right), \left(\frac{x_3}{0.6}, 0.5 \right) \right\}. \end{aligned} \tag{4.3}$$

By using the basic fuzzy union, we have $F_\mu \tilde{\cup} G_\delta = H_v$, where

$$\begin{aligned}
H_\nu(e_1) &= \left\{ \left(\frac{x_1}{\max(0.7, 0.6)}, \max(0.4, 0.4) \right), \left(\frac{x_2}{\max(0.7, 0.3)}, \max(0.6, 0.5) \right), \right. \\
&\quad \left. \left(\frac{x_3}{\max(0.6, 0.3)}, \max(0.6, 0.5) \right) \right\} \\
&= \left\{ \left(\frac{x_1}{0.7}, 0.4 \right), \left(\frac{x_2}{0.7}, 0.6 \right), \left(\frac{x_3}{0.6}, 0.6 \right) \right\}.
\end{aligned} \tag{4.4}$$

Similarly we get

$$\begin{aligned}
H_\nu(e_2) &= \left\{ \left(\frac{x_1}{0.7}, 0.7 \right), \left(\frac{x_2}{0.8}, 0.6 \right), \left(\frac{x_3}{0.4}, 0.8 \right) \right\}, \\
H_\nu(e_3) &= \left\{ \left(\frac{x_1}{0.3}, 0.9 \right), \left(\frac{x_2}{0.8}, 0.8 \right), \left(\frac{x_3}{0.6}, 0.6 \right) \right\}.
\end{aligned} \tag{4.5}$$

In matrix notation, we write

$$H_\nu(e) = \begin{pmatrix} 0.7, 0.4 & 0.7, 0.6 & 0.6, 0.6 \\ 0.7, 0.7 & 0.8, 0.6 & 0.4, 0.8 \\ 0.3, 0.9 & 0.8, 0.8 & 0.6, 0.6 \end{pmatrix}. \tag{4.6}$$

Definition 4.3. Intersection of two PFSSs F_μ and G_δ, denoted by $F_\mu \tilde{\cap} G_\delta$, is a PFSS $H_\nu : E \to I^U \times I^U$ defined by

$$H_\nu(e) = (H(e)(x), \nu(e)(x)), \quad \forall e \in E, \tag{4.7}$$

such that $H(e) = t(F(e), G(e))$ and $\nu(e) = t(\mu(e), \delta(e))$ where t is a fuzzy t-norm.

Example 4.4. Consider the Example 4.2 where F_μ and G_δ are PFSSs defined as follows:

$$\begin{aligned}
F_\mu(e_1) &= \left\{ \left(\frac{x_1}{0.7}, 0.4 \right), \left(\frac{x_2}{0.7}, 0.6 \right), \left(\frac{x_3}{0.6}, 0.6 \right) \right\}, \\
F_\mu(e_2) &= \left\{ \left(\frac{x_1}{0.4}, 0.6 \right), \left(\frac{x_2}{0.8}, 0.5 \right), \left(\frac{x_3}{0.3}, 0.8 \right) \right\},
\end{aligned}$$

$$F_\mu(e_3) = \left\{\left(\frac{x_1}{0.2}, 0.9\right), \left(\frac{x_2}{0.8}, 0.8\right), \left(\frac{x_3}{0.3}, 0.6\right)\right\},$$
$$G_\delta(e_1) = \left\{\left(\frac{x_1}{0.6}, 0.4\right), \left(\frac{x_2}{0.3}, 0.5\right), \left(\frac{x_3}{0.3}, 0.5\right)\right\},$$
$$G_\delta(e_2) = \left\{\left(\frac{x_1}{0.7}, 0.7\right), \left(\frac{x_2}{0.5}, 0.6\right), \left(\frac{x_3}{0.4}, 0.7\right)\right\},$$
$$G_\delta(e_3) = \left\{\left(\frac{x_1}{0.3}, 0.9\right), \left(\frac{x_2}{0.4}, 0.4\right), \left(\frac{x_3}{0.6}, 0.5\right)\right\}. \tag{4.8}$$

By using the basic fuzzy intersection, we have $F_\mu \tilde{\cap} G_\delta = H_\nu$, where

$$\begin{aligned} H_\nu(e_1) &= \left\{\left(\frac{x_1}{\min(0.7, 0.6)}, \min(0.4, 0.4)\right), \left(\frac{x_2}{\min(0.7, 0.3)}, \min(0.6, 0.5)\right),\right. \\ &\qquad \left.\left(\frac{x_3}{\min(0.6, 0.3)}, \min(0.6, 0.5)\right)\right\} \\ &= \left\{\left(\frac{x_1}{0.6}, 0.4\right), \left(\frac{x_2}{0.3}, 0.5\right), \left(\frac{x_3}{0.3}, 0.5\right)\right\}. \end{aligned} \tag{4.9}$$

Similarly we get

$$\begin{aligned} H_\nu(e_2) &= \left\{\left(\frac{x_1}{0.4}, 0.6\right), \left(\frac{x_2}{0.5}, 0.5\right), \left(\frac{x_3}{0.3}, 0.7\right)\right\}, \\ H_\nu(e_3) &= \left\{\left(\frac{x_1}{0.2}, 0.9\right), \left(\frac{x_2}{0.4}, 0.4\right), \left(\frac{x_3}{0.3}, 0.5\right)\right\}. \end{aligned} \tag{4.10}$$

In matrix notation, we write

$$H_\nu(e) = \begin{pmatrix} 0.6, 0.4 & 0.3, 0.5 & 0.3, 0.5 \\ 0.4, 0.6 & 0.5, 0.5 & 0.3, 0.7 \\ 0.2, 0.9 & 0.4, 0.4 & 0.3, 0.5 \end{pmatrix}. \tag{4.11}$$

Proposition 4.5. *Let F_μ, G_δ, and H_ν be any three PFSSs over (U, E). Then the following results hold:*

(i) $F_\mu \tilde{\cup} G_\delta = G_\delta \tilde{\cup} F_\mu$,

(ii) $F_\mu \tilde{\cap} G_\delta = G_\delta \tilde{\cap} F_\mu$,

(iii) $F_\mu \tilde{\cup} (G_\delta \tilde{\cup} H_\nu) = (F_\mu \tilde{\cup} G_\delta) \tilde{\cup} H_\nu$,

(iv) $F_\mu \tilde{\cap} (G_\delta \tilde{\cap} H_\nu) = (F_\mu \tilde{\cap} G_\delta) \tilde{\cap} H_\nu$.

Proof. The proof is straightforward by using the fact that fuzzy sets are commutative and associative. □

Proposition 4.6. *Let F_μ be a PFSS over (U, E). Then the following results hold:*

(i) $F_\mu \tilde{\cup} F_\mu = F_\mu$,

(ii) $F_\mu \tilde{\cap} F_\mu = F_\mu$,

(iii) $F_\mu \tilde{\cup} A_\mu = A_\mu$,

(iv) $F_\mu \tilde{\cap} A_\mu = F_\mu$,

(v) $F_\mu \underset{\sim}{\cup} \varphi_\mu = F_\mu$,

(vi) $F_\mu \tilde{\cap} \varphi_\mu = \varphi_\mu$.

Proof. The proof is straightforward by using the definitions of union and intersection. □

Proposition 4.7. *Let F_μ, G_δ, and H_ν be any three PFSSs over (U, E). Then the following results hold:*

(i) $F_\mu \tilde{\cup} (G_\delta \tilde{\cap} H_\nu) = (F_\mu \tilde{\cup} G_\delta) \tilde{\cap} (F_\mu \tilde{\cup} H_\nu)$,

(ii) $F_\mu \tilde{\cap} (G_\delta \tilde{\cup} H_\nu) = (F_\mu \tilde{\cap} G_\delta) \tilde{\cup} (F_\mu \tilde{\cap} H_\nu)$.

Proof. For all $x \in E$,

$$\begin{aligned}
\lambda_{F(x) \tilde{\cup} (G(x) \tilde{\cap} H(x))}(x) &= \max\left\{\lambda_{F(x)}(x), \lambda_{(G(x) \tilde{\cap} H(x))}(x)\right\} \\
&= \max\left\{\lambda_{F(x)}(x), \min\left(\lambda_{G(x)}(x), \lambda_{H(x)}(x)\right)\right\} \\
&= \min\left\{\max\left(\lambda_{F(x)}(x), \lambda_{G(x)}(x)\right), \max\left(\lambda_{F(x)}(x), \lambda_{H(x)}(x)\right)\right\} \\
&= \min\left\{\lambda_{(F(x) \tilde{\cap} G(x))}(x), \lambda_{(F(x) \tilde{\cap} H(x))}(x)\right\} \\
&= \lambda_{(F(x) \tilde{\cap} G(x)) \tilde{\cup} (F(x) \tilde{\cap} H(x))}(x), \\
\gamma_{\mu(x) \tilde{\cup} (\delta(x) \tilde{\cap} \nu(x))}(x) &= \max\left\{\gamma_{\mu(x)}(x), \gamma_{(\delta(x) \tilde{\cap} \nu(x))}(x)\right\} \\
&= \max\left\{\gamma_{\mu(x)}(x), \min\left(\gamma_{\delta(x)}(x), \gamma_{\nu(x)}(x)\right)\right\} \\
&= \min\left\{\max\left(\gamma_{\mu(x)}(x), \gamma_{\delta(x)}(x)\right), \max\left(\gamma_{\mu(x)}(x), \gamma_{\nu(x)}(x)\right)\right\} \\
&= \min\left\{\gamma_{(\mu(x) \tilde{\cap} \delta(x))}(x), \gamma_{(\mu(x) \tilde{\cap} \nu(x))}(x)\right\} \\
&= \gamma_{(\mu(x) \tilde{\cap} \delta(x)) \tilde{\cup} (\mu(x) \tilde{\cap} \nu(x))}(x).
\end{aligned} \tag{4.12}$$

We can use the same method in (i). □

5. AND and OR Operations on PFSS with Applications in Decision Making

In this section, we introduce the definitions of AND and OR operations on possibility fuzzy soft sets. Applications of possibility fuzzy soft sets in decision-making problem are given.

Definition 5.1. If (F_μ, A) and (G_δ, B) are two PFSSs then "(F_μ, A) AND (G_δ, B)", denoted by $(F_\mu, A) \wedge (G_\delta, B)$ is defined by

$$(F_\mu, A) \wedge (G_\delta, B) = (H_\lambda, A \times B), \tag{5.1}$$

where $H_\lambda(\alpha,\beta) = (H(\alpha,\beta)(x),\ v(\alpha,\beta)(x))$, forall $(\alpha,\beta) \in A \times B$, such that $H(\alpha,\beta) = t(F(\alpha),\ G(\beta))$ and $v(\alpha,\beta) = t(\mu(\alpha),\delta(\beta))$, forall $(\alpha,\beta) \in A \times B$.

Example 5.2. Suppose the universe consists of three machines x_1, x_2, x_3, that is, $U = \{x_1, x_2, x_3\}$, and there are three parameters $E = \{e_1, e_2, e_3\}$ which describe their performances according to certain specific task. Suppose a firm wants to buy one such machine depending on any two of the parameters only. Let there be two observations F_μ and G_δ by two experts defined as follows:

$$\begin{aligned}
F_\mu(e_1) &= \left\{\left(\frac{x_1}{0.6}, 0.4\right), \left(\frac{x_2}{0.7}, 0.3\right), \left(\frac{x_3}{0.7}, 0.5\right)\right\},\\
F_\mu(e_2) &= \left\{\left(\frac{x_1}{0.7}, 0.5\right), \left(\frac{x_2}{0.8}, 0.6\right), \left(\frac{x_3}{0.4}, 0.5\right)\right\},\\
F_\mu(e_3) &= \left\{\left(\frac{x_1}{0.4}, 0.4\right), \left(\frac{x_2}{0.6}, 0.6\right), \left(\frac{x_3}{0.9}, 0.2\right)\right\}.\\
G_\delta(e_1) &= \left\{\left(\frac{x_1}{0.7}, 0.4\right), \left(\frac{x_2}{1}, 0.4\right), \left(\frac{x_3}{0.5}, 1\right)\right\},\\
G_\delta(e_2) &= \left\{\left(\frac{x_1}{0.8}, 0.3\right), \left(\frac{x_2}{0.5}, 0.7\right), \left(\frac{x_3}{0.9}, 1\right)\right\},\\
G_\delta(e_3) &= \left\{\left(\frac{x_1}{0.4}, 0.7\right), \left(\frac{x_2}{0}, 0.4\right), \left(\frac{x_3}{0.6}, 0.4\right)\right\}.
\end{aligned} \tag{5.2}$$

Then $(F_\mu, A) \wedge (G_\delta, B) = (H_\lambda, A \times B)$ where

$$\begin{aligned}
H_\lambda(e_1, e_1) &= \left\{\left(\frac{x_1}{\min(0.6, 0.7)}, \min(0.4, 0.4)\right), \left(\frac{x_2}{\min(0.7, 1)}, \min(0.3, 0.4)\right),\right.\\
&\qquad \left.\left(\frac{x_3}{\min(0.6, 0.5)}, \min(0.5, 1)\right)\right\}\\
&= \left\{\left(\frac{x_1}{0.6}, 0.4\right), \left(\frac{x_2}{0.7},\ 0.3\right), \left(\frac{x_3}{0.5}, 0.5\right)\right\}.
\end{aligned} \tag{5.3}$$

Similarly we get

$$\begin{aligned}
H_\lambda(e_1, e_2) &= \left\{\left(\frac{x_1}{0.6}, 0.3\right), \left(\frac{x_2}{0.5}, 0.3\right), \left(\frac{x_3}{0.6}, 0.5\right)\right\},\\
H_\lambda(e_1, e_3) &= \left\{\left(\frac{x_1}{0.4}, 0.4\right), \left(\frac{x_2}{0}, 0.3\right), \left(\frac{x_3}{0.6}, 0.4\right)\right\},
\end{aligned}$$

$$H_\lambda(e_2,e_1)=\left\{\left(\frac{x_1}{0.7},0.4\right),\left(\frac{x_2}{0.8},0.4\right),\left(\frac{x_3}{0.4},0.5\right)\right\},$$
$$H_\lambda(e_2,e_2)=\left\{\left(\frac{x_1}{0.7},0.3\right),\left(\frac{x_2}{0.5},0.6\right),\left(\frac{x_3}{0.4},0.5\right)\right\},$$
$$H_\lambda(e_2,e_3)=\left\{\left(\frac{x_1}{0.4},0.5\right),\left(\frac{x_2}{0},0.4\right),\left(\frac{x_3}{0.4},0.4\right)\right\},$$
$$H_\lambda(e_3,e_1)=\left\{\left(\frac{x_1}{0.4},0.4\right),\left(\frac{x_2}{0.6},0.4\right),\left(\frac{x_3}{0.5},0.2\right)\right\},$$
$$H_\lambda(e_3,e_2)=\left\{\left(\frac{x_1}{0.4},0.3\right),\left(\frac{x_2}{0.5},0.6\right),\left(\frac{x_3}{0.9},0.2\right)\right\},$$
$$H_\lambda(e_3,e_3)=\left\{\left(\frac{x_1}{0.4},0.4\right),\left(\frac{x_2}{0},0.4\right),\left(\frac{x_3}{0.6},0.2\right)\right\}. \tag{5.4}$$

In matrix notation, we have

$$(F_\mu,A)\wedge(G_\delta,B)=\begin{pmatrix} 0.6,0.4 & \underline{0.7},0.3 & 0.5,0.5 \\ \underline{0.6},0.3 & 0.5,0.3 & \underline{0.6},0.5 \\ 0.4,0.4 & 0,0.3 & \underline{0.6},0.4 \\ 0.7,0.4 & \underline{0.8},0.4 & 0.4,0.5 \\ 0.7,0.3 & 0.5,0.6 & \underline{0.9},0.2 \\ \underline{0.4},0.5 & 0,0.4 & \underline{0.4},0.4 \\ 0.4,0.4 & \underline{0.6},0.4 & 0.5,0.2 \\ 0.4,0.3 & 0.5,0.6 & \underline{0.9},0.2 \\ 0.4,0.4 & 0,0.4 & \underline{0.6},0.2 \end{pmatrix}. \tag{5.5}$$

Now to determine the best machine, we first mark the highest numerical grade (values with underline mark) in each row. Now the score of each of such machines is calculated by taking the sum of the products of these numerical grades with the corresponding possibility λ. The machine with the highest score is the desired machine. We do not consider the numerical grades of the machine against the pairs (e_i,e_i), $i=1,2,3$, as both the parameters are the same.

Then the firm will select the machine with the highest score. Hence, they will buy machine x_3 (see Table 1).

Definition 5.3. If (F_μ,A) and (G_δ,B) are two PFSSs then "(F_μ,A) OR (G_δ,B)", denoted by $(F_\mu,A)\vee(G_\delta,B)$, is defined by

$$(F_\mu,A)\vee(G_\delta,B)=(H_\lambda,A\times B), \tag{5.6}$$

where $H_\lambda(\alpha,\beta)=(H(\alpha,\beta)(x),\ v(\alpha,\beta)(x))$ forall $(\alpha,\beta)\in A\times B$, such that $H(\alpha,\beta)=s(F(\alpha),\ G(\beta))$ and $v(\alpha,\beta)=s(\mu(\alpha),\delta(\beta))$, forall $(\alpha,\beta)\in A\times B$.

Table 1: Grade table.

H	x_i	Highest numerical grade	λ_i
(e_1, e_1)	x_2	$\times$	$\times$
(e_1, e_2)	x_1, x_3	0.6	0.3, 0.5
(e_1, e_3)	x_3	0.6	0.4
(e_2, e_1)	x_2	0.8	0.4
(e_2, e_2)	x_3	$\times$	$\times$
(e_2, e_3)	x_1, x_3	0.4	0.5, 0.4
(e_3, e_1)	x_2	0.6	0.4
(e_3, e_2)	x_3	0.9	0.2
(e_3, e_3)	x_3	$\times$	$\times$

Score$(x_1) = (0.6 \times 0.3) + (0.4 \times 0.5) = 0.38$.
Score$(x_2) = (0.8 \times 0.4) + (0.6 \times 0.4) = 0.56$.
Score$(x_3) = (0.6 \times 0.5) + (0.6 \times 0.4) + (0.4 \times 0.4) + (0.9 \times 0.2) = 0.88$.

Example 5.4. Let $U = \{x_1, x_2, x_3\}$, $E = \{e_1, e_2, e_3\}$; consider F_μ and G_δ as in Example 5.2. suppose now the firm wants to buy a machine depending on any one of two parameters. Then we have $(F_\mu, A) \vee (G_\delta, B) = (H_\lambda, A \times B)$ where

$$\begin{aligned} H_\lambda(e_1, e_1) &= \left\{ \left(\frac{x_1}{\max(0.6, 0.7)}, \max(0.4, 0.4) \right), \left(\frac{x_2}{\max(0.7, 1)}, \max(0.3, 0.4) \right), \right. \\ &\qquad \left. \left(\frac{x_3}{\max(0.6, 0.5)}, \max(0.5, 1) \right) \right\} \\ &= \left\{ \left(\frac{x_1}{0.7}, 0.4 \right), \left(\frac{x_2}{1}, 0.4 \right), \left(\frac{x_3}{0.6}, 1 \right) \right\}. \end{aligned} \tag{5.7}$$

Similarly we get

$$\begin{aligned} H_\lambda(e_1, e_2) &= \left\{ \left(\frac{x_1}{0.8}, 0.4 \right), \left(\frac{x_2}{0.7}, 0.7 \right), \left(\frac{x_3}{0.9}, 1 \right) \right\}, \\ H_\lambda(e_1, e_3) &= \left\{ \left(\frac{x_1}{0.6}, 0.7 \right), \left(\frac{x_2}{0.7}, 0.4 \right), \left(\frac{x_3}{0.6}, 0.5 \right) \right\}, \\ H_\lambda(e_2, e_1) &= \left\{ \left(\frac{x_1}{0.7}, 0.5 \right), \left(\frac{x_2}{1}, 0.6 \right), \left(\frac{x_3}{0.5}, 1 \right) \right\}, \\ H_\lambda(e_2, e_2) &= \left\{ \left(\frac{x_1}{0.8}, 0.5 \right), \left(\frac{x_2}{0.8}, 0.7 \right), \left(\frac{x_3}{0.9}, 1 \right) \right\}, \\ H_\lambda(e_2, e_3) &= \left\{ \left(\frac{x_1}{0.7}, 0.7 \right), \left(\frac{x_2}{0.8}, 0.6 \right), \left(\frac{x_3}{0.6}, 0.5 \right) \right\}, \\ H_\lambda(e_3, e_1) &= \left\{ \left(\frac{x_1}{0.7}, 0.4 \right), \left(\frac{x_2}{1}, 0.6 \right), \left(\frac{x_3}{0.9}, 1 \right) \right\}, \end{aligned}$$

$$H_\lambda(e_3,e_2)=\left\{\left(\frac{x_1}{0.8},0.4\right),\left(\frac{x_2}{0.6},0.7\right),\left(\frac{x_3}{0.9},1\right)\right\},$$
$$H_\lambda(e_3,e_3)=\left\{\left(\frac{x_1}{0.4},0.7\right),\left(\frac{x_2}{0.6},0.4\right),\left(\frac{x_3}{0.9},0.4\right)\right\}. \tag{5.8}$$

In matrix notation, we have

$$(F_\mu,A)\vee(G_\delta,B)=\begin{pmatrix} 0.7,0.4 & \underline{1},0.4 & 0.6,\ 1 \\ 0.8,0.4 & 0.7,0.7 & \underline{0.9},\ 1 \\ 0.6,0.7 & \underline{0.7},0.4 & 0.6,0.5 \\ 0.7,0.5 & \underline{1},0.6 & 0.5,\ 1 \\ 0.8,0.5 & 0.8,0.7 & \underline{0.9},\ 1 \\ 0.7,0.7 & \underline{0.8},0.6 & 0.6,0.5 \\ 0.7,0.4 & \underline{1},0.6 & 0.9,\ 1 \\ 0.8,0.4 & 0.6,0.7 & \underline{0.9},\ 1 \\ 0.4,0.7 & 0.6,0.6 & \underline{0.9},0.4 \end{pmatrix}. \tag{5.9}$$

Now to determine the best machine, we first mark the highest numerical grade (value with underline mark) in each row. Now the score of each of such machines is calculated by taking the sum of the products of these numerical grades with the corresponding possibility λ. The machine with the highest score is the desired machine. We do not consider the numerical grades of the machine against the pairs (e_i,e_i), $i=1,2,3$, as both the parameters are the same. Then the firm will select the machine with the highest score. Hence, they will buy the machine x_2 (see Table 2).

6. Similarity between Two Possibility Fuzzy Soft Sets

Similarity measures have extensive application in several areas such as pattern recognition, image processing, region extraction, coding theory, and so forth. We are often interested to know whether two patterns or images are identical or approximately identical or at least to what degree they are identical.

Several researchers have studied the problem of similarity measurement between fuzzy sets, fuzzy numbers, and vague sets. Majumdar and Samanta [12–14] have studied the similarity measure of soft sets, fuzzy soft sets, and generalised fuzzy soft sets.

In this section, we introduce a measure of similarity between two PFSSs. The set theoretic approach has been taken in this regard because it is easier for calculation and is a very popular method too.

Definition 6.1. Similarity between two PFSSs F_μ and G_δ, denoted by $S(F_\mu,G_\delta)$, is defined as follows:

$$S(F_\mu,G_\delta)=M(F(e),G(e))\cdot M(\mu(e),\delta(e)), \tag{6.1}$$

Table 2: Grade table.

H	x_i	Highest numerical grade	λ_i
(e_1, e_1)	x_2	×	×
(e_1, e_2)	x_3	0.9	1
(e_1, e_3)	x_2	0.7	0.4
(e_2, e_1)	x_2	1	0.6
(e_2, e_2)	x_3	×	×
(e_2, e_3)	x_2	0.8	0.6
(e_3, e_1)	x_2	1	0.6
(e_3, e_2)	x_3	0.9	1
(e_3, e_3)	x_3	×	×

Score(x_1) = 0.
Score(x_2) = (0.7 × 0.4) + (1 × 0.6) + (0.8 × 0.6) + (1 × 0.6) = 1.96.
Score(x_3) = (0.9 × 1) + (0.9 × 1) = 1.8.

such that

$$\begin{aligned} M(F(e), G(e)) &= \max_i M_i(F(e), G(e)), \\ M(\mu(e), \delta(e)) &= \max_i M_i(\mu(e), \delta(e)), \end{aligned} \tag{6.2}$$

where

$$\begin{aligned} M_i(F(e), G(e)) &= 1 - \frac{\sum_{j=1}^{n} \left|F_{ij}(e) - G_{ij}(e)\right|}{\sum_{j=1}^{n} \left|F_{ij}(e) + G_{ij}(e)\right|}, \\ M_i(\mu(e), \delta(e)) &= 1 - \frac{\sum_{j=1}^{n} \left|\mu_{ij}(e) - \delta_{ij}(e)\right|}{\sum_{j=1}^{n} \left|\mu_{ij}(e) + \delta_{ij}(e)\right|}. \end{aligned} \tag{6.3}$$

Definition 6.2. Let F_μ and G_δ be two PFSSs over (U, E). We say that F_μ and G_δ are *significantly similar* if $S(F_\mu, G_\delta) \geq 1/2$.

Proposition 6.3. *Let F_μ and G_δ be any two PFSSs over (U, E). Then the following holds:*

(i) $S(F_\mu, G_\delta) = S(G_\delta, F_\mu)$,

(ii) $0 \leq S(F_\mu, G_\delta) \leq 1$,

(iii) $F_\mu = G_\delta \Rightarrow S(F_\mu, G_\delta) = 1$,

(iv) $F_\mu \subseteq G_\delta \subseteq H_\lambda \Rightarrow S(F_\mu, H_\lambda) \leq S(G_\delta, H_\lambda)$,

(v) $F_\mu \tilde{\cap} G_\delta = \varphi \Leftrightarrow S(F_\mu, G_\delta) = 0$.

Proof. The proof is straightforward and follows from Definition 6.1. □

Example 6.4. Consider Example 4.2 where F_μ and G_δ are defined as follows:

$$\begin{aligned} F_\mu(e_1) &= \left\{ \left(\frac{x_1}{0.7}, 0.4 \right), \left(\frac{x_2}{0.7}, 0.6 \right), \left(\frac{x_3}{0.6}, 0.6 \right) \right\}, \\ F_\mu(e_2) &= \left\{ \left(\frac{x_1}{0.4}, 0.6 \right), \left(\frac{x_2}{0.8}, 0.5 \right), \left(\frac{x_3}{0.3}, 0.8 \right) \right\}, \end{aligned}$$

$$F_\mu(e_3) = \left\{ \left(\frac{x_1}{0.2}, 0.9 \right), \left(\frac{x_2}{0.8}, 0.8 \right), \left(\frac{x_3}{0.3}, 0.6 \right) \right\},$$
$$G_\delta(e_1) = \left\{ \left(\frac{x_1}{0.6}, 0.4 \right), \left(\frac{x_2}{0.3}, 0.5 \right), \left(\frac{x_3}{0.3}, 0.5 \right) \right\},$$
$$G_\delta(e_2) = \left\{ \left(\frac{x_1}{0.7}, 0.7 \right), \left(\frac{x_2}{0.5}, 0.6 \right), \left(\frac{x_3}{0.4}, 0.7 \right) \right\},$$
$$G_\delta(e_3) = \left\{ \left(\frac{x_1}{0.3}, 0.9 \right), \left(\frac{x_2}{0.4}, 0.4 \right), \left(\frac{x_3}{0.6}, 0.5 \right) \right\}. \tag{6.4}$$

Here

$$\begin{aligned} M_1(\mu(e), \delta(e)) &= 1 - \frac{\sum_{j=1}^{3} |\mu_{1j}(e) - \delta_{1j}(e)|}{\sum_{j=1}^{3} |\mu_{1j}(e) + \delta_{1j}(e)|} \\ &= 1 - \frac{|(0.4 - 0.4)| + |(0.6 - 0.5)| + |(0.6 - 0.5)|}{|(0.4 + 0.4)| + |(0.6 + 0.5)| + |(0.6 + 0.5)|} = 0.82. \end{aligned} \tag{6.5}$$

Similarly we get $M_2(\mu(e), \delta(e)) = 0.77$ and $M_3(\mu(e), \delta(e)) = 0.88$. Then

$$\begin{aligned} M(\mu(e), \delta(e)) &= \max(M_1(\mu(e), \delta(e)), M_2(\mu(e), \delta(e)), M_3(\mu(e), \delta(e))) = 0.88, \\ M_1(F(e), G(e)) &= 1 - \frac{\sum_{j=1}^{3} |F_{1j}(e) - G_{1j}(e)|}{\sum_{j=1}^{3} |F_{1j}(e) + G_{1j}(e)|} \\ &= 1 - \frac{|(0.7 - 0.6)| + |(0.7 - 0.3)| + |(0.6 - 0.3)|}{|(0.7 + 0.6)| + |(0.7 + 0.3)| + |(0.6 + 0.3)|} = 0.75. \end{aligned} \tag{6.6}$$

Similarly we get $M_2(F(e), G(e)) = 0.77$ and $M_3(F(e), G(e)) = 0.69$. Then

$$M(F(e), G(e)) = \max(M_1(F(e), G(e)), M_2(F(e), G(e)), M_3(F(e), G(e))) = 0.77. \tag{6.7}$$

Hence, the similarity between the two PFSSs F_μ and G_δ is given by

$$S(F_\mu, G_\delta) = M(F(e), G(e)) \cdot M(\mu(e), \delta(e)) = 0.96 \times 0.77 \cong 0.74. \tag{6.8}$$

7. Application of This Similarity Measure in Medical Diagnosis

In the following example we will try to estimate the possibility that a sick person having certain visible symptoms is suffering from dengue fever. For this we first construct a model possibility fuzzy soft set for dengue fever and the possibility fuzzy soft set of symptoms for the sick person. Next we find the similarity measure of these two sets. If they are significantly similar then we conclude that the person is possibly suffering from dengue fever.

Table 3: Model PFSS for dengue fever.

M_μ	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9	e_{10}	e_{11}
y	1	0	0	1	1	1	1	0	1	1	0
μ_y	1	1	1	1	1	1	1	1	1	1	1
n	0	1	1	0	0	0	0	1	0	0	1
μ_n	1	1	1	1	1	1	1	1	1	1	1

Table 4: PFSS for the sick person.

F_α	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9	e_{10}	e_{11}
y	0.3	0.7	0.5	0.3	0.4	0.1	0	0.7	0	0.4	0.2
μ_y	0.6	0.2	0.1	0.5	0.5	0.8	0.7	0.2	1	0.4	0.5
n	0.6	0.1	0.4	0.5	0.4	0.6	0.7	0.1	0.8	0.5	0.4
μ_n	0.5	0.6	0.4	0.5	0.3	0.6	0.4	0.1	0.5	0.6	0.7

Let our universal set contain only two elements "yes" and "no", that is, $U = (y, n)$. Here the set of parameters E is the set of certain visible symptoms. Let $E = (e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9, e_{10}, e_{11})$, where e_1 is body temperature, e_2 is cough with chest congestion, e_3 is loose motion, e_4 is chills, e_5 is headache, e_6 is low heart rate (bradycardia), e_7 is pain upon moving the eyes, e_8 is breathing trouble, e_9 is a flushing or pale pink rash comes over the face, e_{10} is low blood pressure (hypotension), and e_{11} is Loss of appetite.

Our model possibility fuzzy soft set for dengue fever M_μ is given in Table 3, and this can be prepared with the help of a physician.

After talking to the sick person, we can construct his PFSS G_δ as in Table 4. Now we find the similarity measure of these two sets (as in Example 6.4), here $S(M_\mu, G_\delta) \cong 0.43 < 1/2$. Hence the two PFSSs are not significantly similar. Therefore, we conclude that the person is not suffering from dengue fever.

8. Conclusion

In this paper, we have introduced the concept of possibility fuzzy soft set and studied some of its properties. Applications of this theory has been given to solve a decision-making problem. Similarity measure of two possibility fuzzy soft sets is discussed and an application of this to medical diagnosis has been shown.

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