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Research Article

A Data Envelopment Analysis Approach to Supply Chain Efficiency

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Supply chain management is an important competitive strategies used by modern enterprises. Effective design and management of supply chains assists in the production and delivery of a variety of products at low costs, high quality, and short lead times. Recently, data envelopment analysis (DEA) has been extended to examine the efficiency of supply chain operations. Due to the existence of intermediate measures, the usual procedure of adjusting the inputs or outputs, as in the standard DEA approach, does not necessarily yield a frontier projection. The current paper develops a DEA model for measuring the performance of suppliers and manufacturers in supply chain operations. Additive efficiency decomposition for suppliers and manufacturers in supply chain operations is proposed.

1. Introduction

Data envelopment analysis (DEA), originated from the work of Charnes et al. [1], is a linear programming, nonparametric technique used to measure the relative efficiency of peer decision making units with multiple inputs and outputs. This methodology has been applied in a wide range of applications over the last three decades, in setting that include banks, hospitals, and maintenance. See for instances Amirteimoori and Emrouznejad [2], Amirteimoori and Kordrostami [3], Amirteimoori [4], and Cooper et al. [5]. Recently, a number of studies have looked at production processes that have two-stage network structure, as supply chain operations. Due to the existence of intermediate measures, the usual procedure of adjusting the inputs or outputs, as in the standard DEA approach, does not necessarily yield a frontier projection. Many researchers have applied standard DEA models to measure the performance of supply chain members. See for instances, Weber and Desai [6], Easton et al. [7], Talluri and Baker [8], Liang et al. [9], Chen et al. [10], and Chen [11].

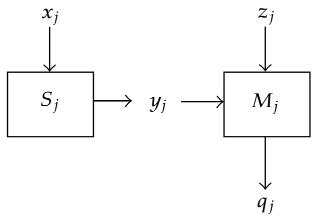

Figure 1: Supplier-manufacturer supply chain.

Weber and Desai [6] employed DEA to construct an index of relative supplier performance. Färe and Grosskopf [12] developed a network DEA approach to model the general multistage processes. Easton et al. [7] suggested a DEA model to compare the purchasing efficiency of firms in the petroleum industry. Talluri and Baker [8] proposed a multiphase mathematical programming approach for effective supply chain design. Their methodology applies a combination of multicriteria efficiency models, based on game theory concepts, and linear and integer programming methods. Liang et al. [9] and Chen et al. [10] developed several DEA-based approaches for characterizing and measuring supply chain efficiency when intermediate measures are incorporated into the performance evaluation. Chen [11] proposed a structured methodology for supplier selection and evaluation in a supply chain, to help enterprises establish a systematic approach for selecting and evaluating potential suppliers in a supply chain. Feng et al. [13] defined two types of supply chain production possibility sets, which are proved to be equivalent to each other.

Some of the above-mentioned studies treat the supply chain as a black-box and do not consider the intermediate measures. However, in all studies, the treatment to the intermediate measures is ambiguous. Moreover, some of the above-mentioned studies are not applicable in a more complex supply chain or a network structured case.

If we treat the supply chain operation as a black-box, ignoring the intermediate measure may yield an efficient supply chain with inefficient supplier and/or manufacturer. In the proposed model in this paper, the intermediate measure is considered as a free variable, and it will be reduced to make the whole supply chain as efficient.

The paper develops a DEA model for measuring the performance of suppliers and manufacturers in supply chain operations. Additive efficiency decomposition for suppliers and manufacturers in supply chain operations is proposed and the DEA frontier points for inefficient supply chain members are determined.

The structure of this paper is the following. In Sections 2 and 3, the problem statement and axiomatic foundation are, respectively, presented. In Section 4, the proposed approach is presented. Section 5 applies the proposed model on a numerical example taken from Liang et al. [9]. The last section summarizes and concludes.

2. Problem Statement

Consider a two-stage supply chain, for example, supplier-manufacturer supply chain as shown in Figure 1.

Suppose we have n homogeneous supply chain operations. Each supply chain observation is considered to be a DMU. It is assumed that each supplier S_j in $\text{DMU}_j : j = 1, \ldots, n$ has m inputs $x_{ij} : i = 1, \ldots, m$ and s outputs $y_{rj} : r = 1, \ldots, s$. These s outputs can become the inputs to the manufacturer M_j. The manufacturer M_j has its own inputs $z_{dj}: \ d = 1, \ldots, D$. The final outputs from manufacturer are $q_{lj}: \ l = 1, \ldots, L$.

3. Axiomatic Foundation

Let T_S be the production possibility set of technology under consideration for the supplier S. We postulate the following:

P1: Feasibility of Observed Data

$(x_j, y_j) \in T_S$ for any $j = 1, \ 2, \ldots, n$.

P2: Unbounded Ray

$(x, y) \in T_S$ implies $\lambda(x, y) \in T_S$ for any $\lambda \geq 0$.

P3: Convexity

Let $(x', y') \in T_S$ and $(x'', \ y'') \in T_S$. Then, for any $\lambda \in [0, 1]$, the unit $\lambda(x', y') + (1 - \lambda)(x'', y'') \in T_S$.

P4: Free Disposability

(a) $(x, y) \in T_S$, $x' \geq x$ and $y' \leq y$, implies $(x', y') \in T_S$

or

(b) $(x, y) \in T_S$, $x' \geq x$ and $y' \geq y$, implies $(x', y') \in T_S$.

P5: Minimal Extrapolation

For each T' satisfying in the axioms P1–P4, we have $T_S \subseteq T'$.

Now, an Algebraic representation of the PPS of the technology T_S, satisfying the axioms P1–P5, is given.

Theorem 3.1. *The PPS T_S, which satisfies the axioms P1–P5, is defined as*

$$T_S = \left\{ (x, \ y) : \ x \geq \sum_{j=1}^{n} \lambda_j \ x_j, \left(\ y \leq \sum_{j=1}^{n} \lambda_j \ y_j \ \text{or} \ y \geq \sum_{j=1}^{n} \lambda_j \ y_j \right), \ \lambda_j \geq 0, \ j = 1, 2, \ldots, n \right\}. \tag{3.1}$$

Proof. The proof is clear. □

Also, let T_M be the production possibility set of technology under consideration for the manufacturer M. Again, to determine the technology of the manufacturer M, we postulate the following:

P'1: Feasibility of Observed Data

$(y_j, z_j, q_j) \in T_M$ for any $j = 1, 2, \ldots, n$.

P'2: Unbounded Ray

$(y, z, q) \in T_M$ implies $\lambda\ (y, z, q) \in T_M$ for any $\lambda\ \geq 0$.

P'3: Convexity

Let $(y', z', q') \in T_M$ and $(y'', z'', q'') \in T_M$. Then, for any $\lambda \in [0, 1]$ the unit $\lambda(y', z', q') + (1 - \lambda)(y'', z'', q'') \in T_M$.

P'4: Free Disposability

(a) $(y, z, q) \in T_M$, $y' \geq y$, $z' \geq z$ and $q' \leq q$, implies $(y', z', q') \in T_M$

or

(b) $(y, z, q) \in T_M$, $y' \leq y$, $z' \geq z$ and $q' \leq q$, implies $(y', z', q') \in T_M$.

P'5: Minimal Extrapolation

For each T' satisfying in the axioms P1–P4, we have $T_M \subseteq T'$.

Similarly, an Algebraic representation of the PPS of the technology T_M, satisfying the axioms P'1–P'5, is given.

Theorem 3.2. *The PPS T_M, which satisfies the axioms P'1–P'5, is defined as*

$$\begin{aligned} T_M = \Bigg\{ (y, z, q) : \left(y \geq \sum_{j=1}^{n} \lambda_j\ y_j\ or\ y \leq \sum_{j=1}^{n} \lambda_j\ y_j \right), \\ z \geq \sum_{j=1}^{n} \lambda_j z_j,\ q \leq \sum_{j=1}^{n} \lambda_j\ q_j,\ \lambda_j \geq 0,\ j = 1, 2, \ldots, n \Bigg\}. \end{aligned} \tag{3.2}$$

Proof. The proof is clear. □

In the definition of T_M and T_M, the intermediate measure y can increase or decrease. In the proposed model, this measure is simultaneously reduced for supplier and manufacturer to achieve a steady state.

4. The Proposed Model

In applying the model described herein, attention is paid to additive model. Consider the assessment of DMU_o (supplier S_o and manufacturer M_o) in additive form. In the assessment of supplier S_o, the output measure y should be increased. On the other hand, this measure is considered as input to the manufacturer M_o and it should be decreased. If we treat the supply chain operation as a black-box, ignoring the intermediate measure y may yield an efficient supply chain with inefficient supplier and/or manufacturer. In the model we proposed, the intermediate measure y is considered as a free variable, and it will be reduced (increased or decreased) to make the overall system as efficient.

Based on the comments made above, and taking in to account Theorems 3.1 and 3.2, the o-th supply chain's performance can be obtained as the optimal value of the following two models:

$$\begin{aligned} e_o = \text{Min} \ & s^{(1)} + s^{(2)} + s^{(3)} + s^{(4)} + s^{(5)} \\ \text{s.t.} \ & \sum_{j=1}^{n} \lambda_j \ x_j + s^{(1)} = \ x_o, \\ & \sum_{j=1}^{n} \lambda_j \ y_j + s^{(2)} = \ y_o, \\ & \sum_{j=1}^{n} \mu_j \ z_j + s^{(3)} = \ z_o, \\ & \sum_{j=1}^{n} \mu_j \ q_j - s^{(4)} = \ q_o, \\ & \lambda_j, \mu_j \geq 0, \quad j = 1, 2, \ldots, n, \\ & s^{(1)}, s^{(3)}, s^{(4)} \geq 0, \\ & s^{(2)} \text{ is unrestricted in sign}. \end{aligned} \tag{4.1}$$

Obviously, this problem is feasible and the optimal objective value to this problem is bounded. We, therefore, provide an alternative definition for supply chain efficiency as follows.

Definition 4.1. Supplier o is said to be additive efficient if and only if $s^{(1)} + s^{(2)} = 0$.

Definition 4.2. Manufacturer o is said to be additive efficient if and only if $s^{(2)} + s^{(3)} + s^{(4)} = 0$.

Clearly, o-th supply chain is said to be overall efficient if and only if $e_o = 0$.

For an inefficient supplier $S_o(x_o, y_o)$, we have

$$\begin{aligned} x_o &= \sum_{j=1}^{n} \lambda_j x_j - s^{(1)}, \\ y_o &= \sum_{j=1}^{n} \lambda_j y_j + s^{(2)}. \end{aligned} \tag{4.2}$$

On the other hand, for an inefficient manufacturer $M_o(y_o, z_o, q_o)$, we have

$$\begin{aligned} y_o &= \sum_{j=1}^{n} \mu_j y_j + s^{(2)}, \\ z_o &= \sum_{j=1}^{n} \mu_j z_j - s^{(3)}, \\ q_o &= \sum_{j=1}^{n} \mu_j q_j + s^{(4)}. \end{aligned} \tag{4.3}$$

Table 1: Data set.

j	x_1	x_2	x_3	y_1	y_2	z_1	q_1	q_2
1	9	50	1	20	10	8	100	25
2	10	40	4	10	15	10	80	10
3	9	30	3	8	20	8	96	30
4	8	25	1	20	20	10	80	20
5	10	40	5	15	20	15	85	15
6	7	35	2	35	10	5	90	35
7	7	30	3	10	25	10	100	30
8	12	40	4	20	25	8	120	10
9	9	25	2	10	10	15	110	15
10	10	50	1	20	15	10	80	20

Table 2: Efficiency of suppliers and manufacturers.

j	Overall efficiency	θ_1	θ_2
1	35.8679	32.7547	3.1132
2	67.1818	11.7273	55.4545
3	11.9808	10.9423	1.0385
4	67.1818	6.7273	60.4545
5	102.7632	0	102.7632
6	0	0	0
7	32.7171	10.25	22.4671
8	33.28	5.28	28
9	22.5	0	22.5
10	84.1818	28.7273	55.4545

The supplier S_o and manufacturer M_o can be improved and become efficient by deleting the input excess and augmenting the output shortfalls. We point out that the intermediate measure y_o may be increased or decreased to make the overall system as efficient. These operations are called supply chain projection.

5. Numerical Example

To illustrate the proposed approach consider a simple example involving ten supply chains taken from Liang et al. [9]. The suppliers use three inputs x_1, x_2, and x_3 to produce two outputs y_1, y_2. On the other hand, the manufacturer uses y_1, y_2, and z_1 to generate two outputs q_1 and q_2. The data are summarized in Table 1. The results from model 1 are listed in Table 2. As the table indicates, only one supply chain, 6, is efficient in aggregate sense. Clearly, a DMU may be efficient in supplier or manufacturer only, such as in the case for members 5 and 9. The projection points are listed in Table 3.

The interpretation of our model can be illustrated by considering a specific supply chain, say supply chain 1. Both, the supplier and manufacturer in this supply chain are inefficient. The projection point to this supply chain is

$$(x_1, x_2, x_3, y_1, y_2, z_1, q_1, q_2) = (6.19, 19.34, 0.77, 24.53, 4.53, 8, 100, 29.06). \tag{5.1}$$

Table 3: Projection points.

j	x_1	x_2	x_3	y_1	y_2	z_1	q_1	q_2
1	6.19	19.34	0.77	24.53	4.53	8	100	29.06
2	7.28	22.73	0.91	1.82	11.82	10	122.73	34.10
3	6.38	19.95	0.80	0.04	24.04	8	100.96	30
4	7.27	22.73	0.91	21.82	21.82	10	122.73	34.10
5	9.73	30.41	1.22	5.68	15.68	15	171.44	44.97
6	7	35	2	35	10	5	90	35
7	7	21.88	0.87	2.5	32.5	10	119.80	32.66
8	8.88	35.1	1.74	8	32	8	120	43
9	6.05	18.92	0.76	4.86	4.87	15	132.03	25.74
10	7.27	22.73	0.91	21.82	11.82	10	122.73	34.09

It is to be noted that the first intermediate measure should be increased from 20 to 24.53, whereas the second one should be decreased from 10 to 4.53. These reductions make the supply chain as efficient.

We used GAMS software on a Pentium 4, 512 Mbytes RAM, 2 GHz PC.

6. Conclusions

A supply chain's performance can be overestimated if we treat the supply chain operation as a black box. Moreover, due to the existence of intermediate measures, the usual procedure of adjusting the inputs or outputs, as in the standard DEA approach, does not necessarily yield to a frontier projection. The current paper introduced the technologies used in supply chain operations. A method for determining the DEA frontier points for inefficient supplier and manufacturers in supply chain operations has been shown. The paper developed a DEA model for measuring the performance of suppliers and manufacturers in supply chain operations.

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