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Research Article

An Optimal Returned Policy for a Reverse Logistics Inventory Model with Backorders

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Environmental responsibility plays a significant role in the firm's agendas nowadays. In this paper, we address the environmental operations of reverse logistics. Here we developed an integrated supply chain model with coordinated production and remanufacturing due to time-dependent rates. To study the problem we consider the demand to be satisfied with newly manufactured (produced) and the remanufactured products, so there is no difference between manufactured and remanufactured items. The shortages are allowed and excess demand is backlogged as well. The returned items are collected from the end user to be remanufactured. Optimal expression is obtained for the acceptable returned quantity, maximum inventory level, production and remanufacturing scheduling period, and the total average cost. Illustrative examples, which explain the application of the theoretical results as well as their numerical verifications, are given. The sensitivity of these solutions to change in underling parameter values is also discussed.

1. Introduction

Environmental issues are gaining reasonable attention among society, worldwide. Consumer demand for clean manufacturing and recycling is increasing. Consumers expect to trade in an old product when they buy a new one. Hence, for the past few decades the reverse flow of products from consumers to upstream businesses has received much interest. Due to the governmental regulations and consumer concerns regarding these environmental issues, an increasing number of companies have focused on reduction efforts in the amount of waste stream, diversion of the discarded products, and disposition of the retired products properly whereas reverse logistics is the process of retrieving the product from the end consumer for the purposes of proper disposal. To facilitate the reverse flow of used products from consumers to manufacturers in an efficient manner, the most appropriate approach is to create a reverse supply chain network. Wherein reverse logistics can take place through the original forward channel, through a separate reverse channel, or through combinations of the forward and the reverse channel. Generally, companies focus on setting up a reverse supply chain either because of environmental regulations or to reduce their operating cost by remanufactured products or components. For companies that utilise a reverse supply chain deals with handling and reprocess of repairable used products withdrawn from production and consumption process. Such a reuse is, for example, recycling or repair of spare parts. It has an advantage from economic point of view, as reduction of environmental load through return of used items in the manufacturing process.

In the recent years researchers paid much attention to reverse logistics inventory models. There have been numerous studies and research on reverse logistics. In the past one approach adopted by many authors to the study of recovery systems is the use of Economic Ordering Quantity (EOQ) technique. The EOQ models are simple and they usually lead to closed-form solutions. The first reverse logistic model was investigated by Schrady [1]. He analyzed the problem in the EOQ model for repairable items which assumes that the production and repairing rates are instantaneous without disposal cost. Nahmias and Rivera [2] considered the model of Schrady [1] for the case of finite repair rate and limited storage in the repair and production shops. A very good review on quantitative models for recovery production planning and inventory control is given by Fleischmann et al. [3]. In this review they subdivided the field into three main areas, namely, distribution planning, inventory control, and production planning for each of these they discuss the implication of the emerging reuse efforts. There is a multiproduct generalization of EOQ-type reverse logistics models published by Mabini et al. [4]. They have extended the basic model of Schrady [1] with capital budget restriction. Richter [5, 6] investigated a modified version of the model of Schrady [1] by assuming multiple production and multiple repair cycles within a time interval. Most of the models investigated earlier are governed by two extreme (bang-bang) strategies, that is, "dispose all" or "recover all" [7]. In a similar work to Richter [5–7], Teunter [8] developed a deterministic EOQ inventory model with disposal option where recoverable and manufactured items have different holding costs and obtained a general finding similar to Richter [7]. Koh et al. [9] generalized the model of Nahmias and Rivera [2] by assuming a limited repair capacity. In a later study, several researchers have developed models along the same lines as Schrady and Richter, but with different assumptions, for example, Teunter [10], Inderfurth et al. [11], Dobos, and Richter [12, 13]. Dobos and Richter [14] explored their previous model by assuming that the quality of collected used items is not always suitable for further recycling. Konstantaras and Papachristos [15] have investigated an inventory model for stability. In his next work [16] he extended the work of Koh et al. [9] and followed a different analysis to obtain closed form expressions for both optimal number of set up in the recovery and the ordering processes. Konstantaras and Papachristos [17] extended the work of Teunter [10] by introducing the exact solution method for the same model. Jaber and El Saadany [18] extend the work of Richter [5, 6] by assuming the newly produced and remanufactured items are perceived differently by customers. Omar and Yeo [19] developed a production model that satisfies a continuous time varying demand for finished goods over a known and finite planning horizon by supplying either new products or repaired used products. The extended version of Dobos and Richter [12, 13] is made by El Saadany and Jaber [20] by assuming the returned rate of used items follows a demand like function depend on the purchasing price and acceptance quality level of returns. Alamri [21] derives a general reverse logistics inventory model for deteriorating items taking the returned rate as a decision variable. Another extension of Koh et al. [9] is made by Konstantaras et al. [22] by introducing the inspection and sorting of returned items. Hasanov et al. [23] extended the

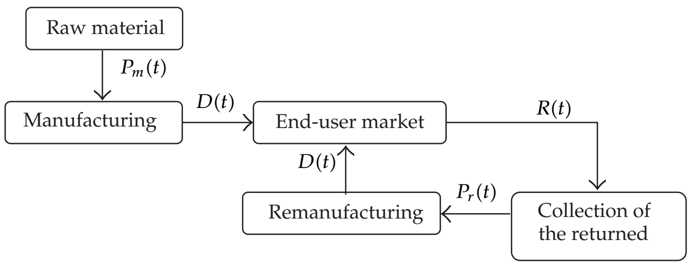


Figure 1: The flow of material in the integrated inventory model.

work Jaber and El Saadany [18] by assuming that unfulfilled demand for remanufacturing and produced items is either fully or partially backordered. This paper also considered the scenario of overlapping of one production and one remanufacturing cycle to minimize the effect of stock outs. A closed-loop supply chain inventory model is developed by Yang et al. [24], (in press). In this paper he considers price-sensitive demand and multiretailer and analysis of the problem with three optimization methods sequential optimization, a centralized optimization without benefit sharing, and a centralized optimization with benefit sharing. The comparative review is given in Table 1 .

Usually in most of the models shortages are not permitted to occur. However, in many practical situations, stock out is unavoidable due to various uncertainties. Therefore, the occurrence of shortages in inventory is a natural phenomenon. In this paper we use this phenomenon. In the proposed model we determined the coordination of reverse manufacturing with the forward supply chain in the inventory management. The reverse logistics operations deal with the collection of returns, cleaning of the collected returns, and remanufacturing of the reusable collected items. The quality of the remanufactured items is assumed to be good as those of new products hence the demand is to be satisfied with newly manufactured (produced) and the remanufactured products. A general framework of the system is shown in Figure 1. In Section 1, a comprehensive literature review and background of the model are presented. Section 2 is for assumption and notations. Section 3 demonstrates the model development. Section 4 presents the solution procedure to solve the optimization problem. Section 4.1 shows three numerical examples to illustrate the model and sensitivity analysis is presented in Section 4.2 . A particular case of the given problem is given in Section 4.3 . Concluding remarks are derived and future research topics are suggested in Section 5 .

2. Assumptions and Notations

In this paper the subscript "m" is used to indicate the quantity corresponds to the remanufactured stock, we will use the subscript "r" to indicate the quantity corresponds to the remanufactured stock and the subscript "R" to indicate the quantity corresponds to the returned stock.

The model is developed with the following assumptions and notations.

(i) New products are produced at a rate of $P_m(t)$.

(ii) Repairable used products are collected at a rate of $R(t)$ and then remanufactured at a rate of $P_r(t)$. All the returned items are remanufactured.

Table 1: The comparative review in the tabular form.

References	Production rate	Remanufacturing/ repairing rate	Demand rate	Returned rate	The quality of the remanufactured items	Shortages
Schrady [1]	Not considering	Constant	Constant	Constant	As-good-as new	Not allowed
Nahmias and Rivera [2]	Not considering	Constant	Constant	Constant	As-good-as new	Not allowed
Richter [5–7]	Constant	Constant	Constant	Constant	As-good-as new	Not allowed
Dobos and Richter [12–14]	Demand dependent	Demand Dependent	Constant	Demand dependent	As-good-as new	Not allowed
Konstantaras and Papachristos [15]	Constant	Constant	Constant	Constant	As-good-as new	Not allowed
Konstantaras and Papachristos [16], Teunter [10]	Constant	Constant	Constant	Demand dependent	As-good-as new	Not allowed
Konstantaras and Papachristos [17], Koh et al., [9]	Not considering	Constant	Constant	Constant	As-good-as new	Not allowed
Konstantaras and Skouri, [25]	Constant	Constant	Constant	Constant	As-good-as new	Allowed
Konstantaras et al. [22]	Not considering	Constant	Constant	Constant	Different from the new items	Not allowed
Jaber and El Saadany, [18]	Demand dependent	Demand Dependent	Constant	Demand dependent	Different from the newly produced	Not allowed
El Saadany and Jaber [20]	Demand dependent	Demand Dependent	Constant	Price and quality dependent	As-good-as new	Not allowed
Jaber and El Saadany, [26]	Demand dependent	Demand Dependent	Constant	Demand dependent	As-good-as new	Not allowed
Alamri, [21]	Arbitrary function of time	Arbitrary function of time	Arbitrary function of time	Arbitrary function of time	As-good-as new	Not allowed
Hasanov et al. [23]	Constant	Constant	Constant	Constant	Different from the newly produced	Allowed
Yang et al., (in press) [24]	Constant	Constant	Price sensitive	Constant	As-good-as new	Not allowed

Table 1: Continued.

References	Production rate	Remanufacturing/ repairing rate	Demand rate	Returned rate	The quality of the remanufactured items	Shortages
The present paper	Arbitrary function of time	Arbitrary function of time	Arbitrary function of time	Arbitrary function of time	As-good-as new	Allowed

(iii) Demand is satisfied from the newly produced and remanufactured items at a rate of $D(t)$.

(iv) $P_m(t), R(t), P_r(t)$, and $D(t)$ are assumed to be arbitrary functions of time.

(v) We will require that

(a) $P_r(t) > D(t),\ P_m(t) > D(t),\ D(t) > R(t)$,
(b) $P_r(t) > R(t),\ D(t) \neq 0,\ R(t) \neq 0$.

(vi) Shortage is allowed and c_s is the unit shortage cost.

(vii) Cost parameters related to manufactured products are as follows.

$I_m(t)$ is the inventory level at time t related to manufactured products.
q_2 is the maximum inventory level in the production process.
c_m is the unit procurement cost.
s_m is the unit production cost.
h_m is the unit holding cost per unit time.
k_m is the setup cost per production cycle.

(viii) The cost parameters related to remanufactured products are as follows.

$I_r(t)$ is the inventory level at time t related to remanufactured products.
q_1 is the maximum inventory level in the remanufacturing process.
s_r is the unit remanufacturing cost.
h_r is the unit holding cost per unit time.
k_r is the setup cost per remanufacturing cycle.

(ix) The cost parameters related to returns are as follows.

$I_R(t)$ is the inventory level at time t related to returns.
c_R is the unit acquisition cost.
h_R is the unit holding cost per unit time.
k_R is the setup cost per returned cycle.

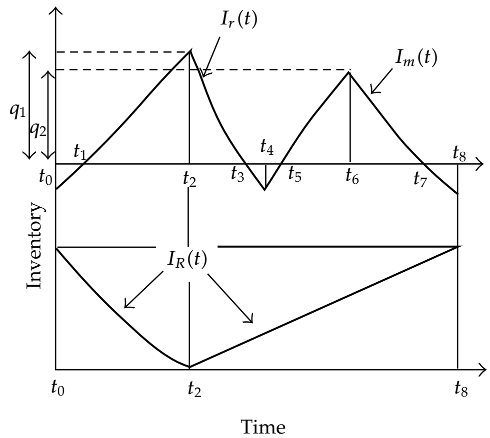


Figure 2: Inventory variation of an EPQ model for Reverse logistics system.

3. Mathematical Modelling and Analysis

The cycle now starts at time t_0 with backorders. At this instant of time, remanufacturing starts to clear the backlog by the time t_1 and the inventory level $I_r(t)$ increases at a rate $P_r(t) - D(t)$ until the time t_2 where stock level reaches its maximum value. Then the remanufacturing is stopped and hence the demand depletes the inventory level $I_r(t)$ during the period (t_2, t_3) and falls to zero at $t = t_3$, thereafter shortages occur during the period (t_3, t_4) due to the absence of stock. At this instant of time, fresh production starts to clear the backlog by the time t_5. Production raises the inventory level $I_m(t)$ at a rate $P_m(t) - D(t)$ and reaches its maximum at time $t = t_6$. Then the production is stopped and hence the demand depletes the inventory level $I_m(t)$ until the time t_7 by which it becomes zero. Now shortages start developing and accumulate to their maximum (equal to the shortage level at time $t = t_0$) at the time $t = t_8$. However for each returned cycle the inventory level $I_R(t)$ is affected by the returned rate and the remanufacturing rate, as the remanufacturing process starts at t_0, the stock level declines at a rate $R(t) - P_r(t)$ and falls to zero at $t = t_2$ by which the remanufacturing stops. Now the stock level increases at a rate $R(t)$ by the time $t = t_8$. This is depicted in the Figure 2.

The differential equations governing the stock level during the period $t_0 \le t \le t_8$ can be written as

$$\begin{aligned}
\frac{dI_r(t)}{dt} &= P_r(t) - D(t) \quad I_r(t_1) = 0,\ t_0 \le t \le t_2, \\
\frac{dI_r(t)}{dt} &= -D(t) \quad I_r(t_3) = 0,\ t_2 \le t \le t_4, \\
\frac{dI_m(t)}{dt} &= P_m(t) - D(t) \quad I_m(t_5) = 0,\ t_4 \le t \le t_6, \\
\frac{dI_m(t)}{dt} &= -D(t) \quad I_m(t_7) = 0,\ t_6 \le t \le t_8, \\
\frac{dI_R(t)}{dt} &= R(t) - P_r(t) \quad I_R(t_2) = 0,\ t_0 \le t \le t_2, \\
\frac{dI_R(t)}{dt} &= R(t) \quad I_R(t_2) = 0,\ t_2 \le t \le t_8.
\end{aligned} \tag{3.1}$$

Solutions of the above differential equations using their boundary conditions are

$$
\begin{aligned}
I_r(t) &= -\int_t^{t_1} (P_r(u) - D(u))du, \quad t_0 \le t \le t_1, \\
I_r(t) &= \int_{t_1}^{t} (P_r(u) - D(u))du, \quad t_1 \le t \le t_2, \\
I_r(t) &= \int_t^{t_3} D(u)du, \quad t_2 \le t \le t_3, \\
I_r(t) &= -\int_{t_3}^{t} D(u)du, \quad t_3 \le t \le t_4,
\end{aligned}
\tag{3.2}
$$

$$
I_m(t) = -\int_t^{t_5} (P_m(u) - D(u))du, \quad t_4 \le t \le t_5, \tag{3.3}
$$

$$
\begin{aligned}
I_m(t) &= \int_{t_5}^{t} (P_m(u) - D(u))du, \quad t_5 \le t \le t_6, \\
I_m(t) &= \int_t^{t_7} D(u)du, \quad t_6 \le t \le t_7, \\
I_m(t) &= -\int_{t_7}^{t} D(u)du, \quad t_7 \le t \le t_8,
\end{aligned}
\tag{3.4}
$$

$$
\begin{aligned}
I_R(t) &= \int_t^{t_2} (P_r(u) - R(u))du, \quad t_0 \le t \le t_2, \\
I_R(t) &= \int_{t_2}^{t} R(u)du, \quad t_2 \le t \le t_8.
\end{aligned}
$$

Let $I(x_1, x_2) = \int_{x_1}^{x_2} I(u)du$, then from (3.3)–(3.6) we have

$$
\begin{aligned}
I_r(t_0, t_1) &= -\int_{t_0}^{t_1} u(P_r(u) - D(u))du, \\
I_r(t_1, t_2) &= \int_{t_1}^{t_2} (t_2 - u)(P_r(u) - D(u))du, \\
I_r(t_2, t_3) &= \int_{t_2}^{t_3} (u - t_2)D(u)du,
\end{aligned}
\tag{3.5}
$$

$$
\begin{aligned}
I_r(t_3, t_4) &= \int_{t_3}^{t_4} (u - t_4)D(u)du, \\
I_m(t_4, t_5) &= \int_{t_4}^{t_5} (t_4 - u)(P_m(u) - D(u))du, \\
I_m(t_5, t_6) &= \int_{t_5}^{t_6} (t_6 - u)(P_m(u) - D(u))du, \\
I_m(t_6, t_7) &= \int_{t_6}^{t_7} (u - t_6)D(u)du,
\end{aligned}
\tag{3.6}
$$

$$I_m(t_7,t_8)=\int_{t_7}^{t_8}(u-t_8)D(u)du,$$

$$I_R(t_0,t_2)=\int_{t_0}^{t_2}u(P_r(u)-R(u))du,$$

$$I_R(t_2,t_8)=\int_{t_2}^{t_8}(t_8-u)R(u)du. \tag{3.7}$$

Without loss of generality let us assume $t_0 = 0$ then the per cycle cost components for the given inventory model are as follows:

$$\text{Procurement and acquisition cost} = c_m\int_{t_4}^{t_6}P_m(u)du + c_R\int_0^{t_8}R(u)du,$$

$$\text{Production and remanufacturing cost} = s_m\int_{t_4}^{t_6}P_m(u)du + s_r\int_0^{t_2}P_r(u)du,$$

$$\text{Holding cost} = h_r[I_r(t_1,t_2)+I_r(t_2,t_3)] + h_m[I_m(t_5,t_6)+I_m(t_6,t_7)] + h_R[I_R(0,t_2)+I_R(t_2,t_8)],$$

$$\text{Shortage cost} = c_s[-I_r(0,t_1)-I_r(t_3,t_4)-I_m(t_4,t_5)-I_m(t_7,t_8)]. \tag{3.8}$$

Hence the total cost per unit time of the given inventory model during the cycle $[0,t_8]$ as a function of $t_1,\ t_2,\ t_3,\ t_4,\ t_5,\ t_6,\ t_7,$ and t_8 say $Z(t_1,\ t_2,\ t_3,\ t_4,\ t_5,\ t_6,\ t_7,\ t_8)$ is given by

$$\begin{aligned}
Z(t_1,t_2,\ldots,t_8) = \frac{1}{t_8}\Bigg\{ &(c_m+s_m)\int_{t_4}^{t_6}P_m(u)du + c_R\int_0^{t_8}R(u)du + s_r\int_0^{t_2}P_r(u)du \\
&+ h_r\left[\int_{t_1}^{t_2}(t_2-u)(P_r(u)-D(u))du + \int_{t_2}^{t_3}(t-t_2)D(u)du\right] \\
&+ h_m\left[\int_{t_5}^{t_6}(t_6-u)(P_m(u)-D(u))du + \int_{t_6}^{t_7}(u-t_6)D(u)du\right] \\
&+ h_R\left[\int_0^{t_2}u(P_r(u)-R(u))du + \int_{t_2}^{t_8}(t_8-u)R(u)du\right] \\
&+ c_s\Bigg[\int_0^{t_1}u(P_r(u)-D(u))du - \int_{t_3}^{t_4}(u-t_4)D(u)du - \int_{t_4}^{t_5}(t_4-u) \\
&\qquad \times(P_m(u)-D(u))du - \int_{t_7}^{t_8}(u-t_8)D(u)du\Bigg] + k_r + k_m + k_R\Bigg\}.
\end{aligned} \tag{3.9}$$

Here we have a cost function of the system in terms of $t_1,\ t_2,\ t_3,\ t_4,\ t_5,\ t_6,\ t_7$, and t_8. To find the optimum solution we have to find the optimum value of $t_1,\ t_2,\ t_3,\ t_4,\ t_5,\ t_6,\ t_7$, and t_8 that minimize $Z(t_1, t_2, \ldots, t_8)$ but we have some relations between the variables as follows.

$$0 \le t_1 \le t_2 \le t_3 \le t_4 \le t_5 \le t_6 \le t_7 \le t_8, \tag{3.10}$$

$$\int_{t_1}^{t_2} (P_r(t) - D(t))dt = \int_{t_2}^{t_3} D(t)dt, \tag{3.11}$$

$$\int_{t_3}^{t_4} D(t)dt = \int_{t_4}^{t_5} (P_m(t) - D(t))dt, \tag{3.12}$$

$$\int_{t_5}^{t_6} (P_m(t) - D(t))dt = \int_{t_6}^{t_7} D(t)dt, \tag{3.13}$$

$$\int_{0}^{t_1} (P_r(t) - D(t))dt = \int_{t_7}^{t_8} D(t)dt, \tag{3.14}$$

$$\int_{0}^{t_8} R(t)dt = \int_{0}^{t_2} P_r(t)dt. \tag{3.15}$$

Equation (3.10) is an essential condition for the existence of the model. Equation (3.11) show that the inventory levels $I_r(t)$ are same at the time $t = t_2$, similarly (3.12) and (3.13) show that the inventory level $I_m(t)$ and $I_r(t)$ are same at the time $t = t_4$ and $t = t_6$, respectively, depicted in Figure 2. Equation (3.14) shows that the backorders at the time $t = 0$ and $t = t_8$ are alike as we have already assumed in the formulation of model. According to the presented model all the units buyback from the market will be remanufactured (no disposal) hence the total remanufacturing during the cycle is equal to the total returned items in the complete cycle which is demonstrated in (3.15).

4. Solution Procedure

Let Q be the acceptable returned quantity for used items in the interval $[0, t_8]$:

$$Q = \int_{0}^{t_8} R(u)du. \tag{4.1}$$

And let we have the maximum inventory level q_1 and q_2 in remanufacturing and production process, respectively, so that from (3.11) and (3.13) we get

$$\int_{t_1}^{t_2} (P_r(t) - D(t))dt = \int_{t_2}^{t_3} D(t)dt = q_1, \tag{4.2}$$

$$\int_{t_5}^{t_6} (P_m(t) - D(t))dt = \int_{t_6}^{t_7} D(t)dt = q_2. \tag{4.3}$$

Therefore from (3.11), (3.12),..., (4.2), and (4.3) the value of $t_1,\ t_2,\ t_3,\ t_4,\ t_5,\ t_6,\ t_7$, and t_8 can be determined as a function of Q, q_1, and q_2.

Therefore the total variable cost function will be the function of three variables Q, q_1, and q_2 which is $Z(Q, q_1, q_2)$:

$$\begin{aligned} Z(Q,q_1,q_2) = \frac{1}{t_8}\Bigg\{ &(c_m+s_m)\int_{t_4}^{t_6} P_m(u)du + c_R\int_0^{t_8} R(u)du + s_r\int_0^{t_2} P_r(u)du \\ &+ h_r\left[\int_{t_1}^{t_2}(-u)(P_r(u)-D(u))du + \int_{t_2}^{t_3} uD(u)du\right] \\ &+ h_m\left[\int_{t_5}^{t_6}(-u)(P_m(u)-D(u))du + \int_{t_6}^{t_7} uD(u)du\right] \\ &+ h_R\left[\int_0^{t_2} u(P_r(u)-R(u))du + \int_{t_2}^{t_8}(f_8-u)R(u)du\right] \\ &+ c_s\Bigg[\int_0^{t_1} u(P_r(u)-D(u))du - \int_{t_3}^{t_4} uD(u)du + \int_{t_4}^{t_5} u(P_m(u)-D(u))du \\ &\quad - \int_{t_7}^{t_8} uD(u)du\Bigg] + k_r + k_m + k_R \Bigg\}, \end{aligned} \tag{4.4}$$

where the values of t_1, t_2, t_3, t_4, t_5, t_6, t_7, and t_8 are given in the next section.

To obtain the optimal solution of the proposed problem, we minimize the functions $Z(Q, q_1, q_2)$ with respect to Q, q_1, and q_2 Taking the first order derivatives of $Z(Q, q_1, q_2)$ with respect to Q, q_1, and q_2. Thereafter setting it equal to zero gives

$$\begin{aligned} \frac{\partial Z(Q,q_1,q_2)}{\partial Q} &= 0, \\ \frac{\partial Z(Q,q_1,q_2)}{\partial q_1} &= 0, \\ \frac{\partial Z(Q,q_1,q_2)}{\partial q_2} &= 0. \end{aligned} \tag{4.5}$$

The optimal value of Q, q_1, and q_2 can be derived from the above equations when the following conditions of Hessian matrix are satisfied. The Hessian matrix is

$$H = \begin{pmatrix} \dfrac{\partial^2 Z}{\partial Q^2} & \dfrac{\partial^2 Z}{\partial Q q_1} & \dfrac{\partial^2 Z}{\partial Q q_2} \\ \dfrac{\partial^2 Z}{\partial q_1 Q} & \dfrac{\partial^2 Z}{\partial q_1^2} & \dfrac{\partial^2 Z}{\partial q_1 q_2} \\ \dfrac{\partial^2 Z}{\partial q_2 Q} & \dfrac{\partial^2 Z}{\partial q_2 q_1} & \dfrac{\partial^2 Z}{\partial q_2^2} \end{pmatrix}. \tag{4.6}$$

The first principal minor determinant of H, $|H_{11}| > 0$. The second principal minor determinant of H, $|H_{22}| > 0$ and the third principal minor determinant of H, $|H_{33}| > 0$.

4.1. Illustrative Examples with Numerical Analysis

Example 4.1. The model is developed with linearly time-dependent demand, production, remanufacturing, and returned rates, in this example we consider

$$P_m(t) = a_m + b_m t, \qquad P_r(t) = a_r + b_r t, \qquad R(t) = c + dt, \qquad D(t) = \alpha + \beta t. \tag{4.7}$$

On the basis of these above demand, production, remanufacturing, and returned rates we calculate the theoretical results and the total cost function as defined in the previous section.

From (4.1) we have

$$Q = \int_0^{t_8} R(u)du = \int_0^{t_8} (c + du)du = \frac{dt_8^2}{2} + ct_8. \tag{4.8}$$

From which t_8 is given by

$$t_8 = \frac{-c + \sqrt{c^2 + 2dQ}}{d}. \tag{4.9}$$

From (3.15) and (4.1) we have

$$Q = \int_0^{t_8} R(u)du = \int_0^{t_2} P_r(u)du = \int_0^{t_8} (a_r + b_r u)du. \tag{4.10}$$

Hence t_2 is given by

$$t_2 = \frac{-a_r + \sqrt{a_r^2 + 2b_r Q}}{b_r}. \tag{4.11}$$

From (4.2) we get

$$\begin{gathered} \int_{t_1}^{t_2} (P_r(t) - D(t))dt = q_1 \\ (a_r - \alpha)(t_2 - t_1) + \frac{(b_r - \beta)}{2}\left(t_2^2 - t_1^2\right) = q_1. \end{gathered} \tag{4.12}$$

By which we can find t_1, say

$$t_1 = \frac{-2a_r + 2\alpha + \sqrt{(2a_r - 2\alpha)^2 - 4(b_r - \beta)(2q_1 - 2a_r t_2 - b_r t_2^2 + 2t_2\alpha + t_2^2\beta)}}{2(b_r - \beta)}. \tag{4.13}$$

From (4.2) we have

$$\int_{t_2}^{t_3} D(t)dt = q_1$$
$$\alpha(t_3 - t_2) + \frac{\beta}{2}\left(t_3^2 - t_2^2\right) = q_1. \tag{4.14}$$

From which we can find t_3, say

$$t_3 = \frac{-\alpha + \sqrt{\alpha^2 + 2q_1\beta + 2t_2\alpha\beta + t_2^2\beta^2}}{\beta}. \tag{4.15}$$

From (3.14) we have

$$\int_0^{t_1} (P_r(t) - D(t))dt = \int_{t_7}^{t_8} D(t)dt$$
$$(a_r - \alpha)t_1 + \frac{(b_r - \beta)}{2}t_1^2 = \alpha(t_8 - t_7) + \frac{\beta}{2}\left(t_8^2 - t_7^2\right). \tag{4.16}$$

Hence the value of t_7 is given by

$$t_7 = \frac{-\alpha + \sqrt{\alpha^2 - 2a_r t_1\beta - b_r t_1^2\beta + 2t_1\alpha\beta + 2t_8\alpha\beta + t_1^2\beta^2 + t_8^2\beta^2}}{\beta}. \tag{4.17}$$

From (4.3) we have

$$\int_{t_6}^{t_7} D(t)dt = q_2$$
$$\Longrightarrow \alpha(t_7 - t_6) + \frac{\beta}{2}\left(t_7^2 - t_6^2\right) = q_2. \tag{4.18}$$

From which we can determine the value of t_6, say

$$t_6 = \frac{-\alpha + \sqrt{\alpha^2 - 2q_2\beta + 2t_7\alpha\beta + t_7^2\beta^2}}{\beta}. \tag{4.19}$$

Similarly from (4.3)

$$\int_{t_5}^{t_6} (P_m(t) - D(t))dt = q_2. \tag{4.20}$$

Table 2: The optimal results for the inventory model under the above parametric values as in Example 4.2.

Q^*	q_1^*	q_2^*	t_1^*	t_2^*	t_3^*	t_4^*	t_5^*	t_6^*	t_7^*	t_8^*	$Z(Q^*, q_1^*, q_2^*)$
1712 units	781 units	191 units	0.160528 Month	0.94369 month	1.86346 month	1.92922 Month	2.01177 month	2.28889 Month	2.50498 Month	2.6851 month	7616.1 $

From which we can find t_5

$$t_5 = \frac{-2a_m + 2\alpha + \sqrt{(2a_m - 2\alpha)^2 - 4(b_m - \beta)(2q_2 - 2a_m t_6 - b_m t_6^2 + 2t_6\alpha + t_6^2\beta)}}{2(b_m - \beta)}. \tag{4.21}$$

From (3.12) we have

$$\begin{gathered}\int_{t_3}^{t_4} D(t)dt = \int_{t_4}^{t_5} (P_m(t) - D(t))dt \\ (a_m - \alpha)(t_5 - t_4) + \frac{(b_m - \beta)}{2}\left(t_5^2 - t_4^2\right) = \alpha(t_4 - t_3) + \frac{\beta}{2}\left(t_4^2 - t_3^2\right).\end{gathered} \tag{4.22}$$

From which we can find t_4

$$t_4 = \frac{-a_m + \sqrt{a_m^2 + 2a_m b_m t_5 + b_m^2 t_5^2 + 2b_m t_3\alpha - 2b_m t_5\alpha + b_m t_3^2\beta - b_m t_5^2\beta}}{b_m}. \tag{4.23}$$

Example 4.2. The above theoretical results are illustrated through the numerical verification. To illustrate the proposed model, we have considered the input parameters as given below.

$c_m = 8$ \$/unit, $s_m = 3$ \$/unit, $s_r = 2$ \$/unit, $c_R = 2$ \$/unit, $c_s = 5$ \$/unit, $a_m = 1500$ unit/month, $b_m = 30$ unit/month^2, $\alpha = 800$ unit/month, $\beta = 35$ unit/month^2, $a_r = 1800$ unit/month, $b_r = 30$ unit/month^2, $h_R = 1$ \$/unit/month, $h_m = 1.5$ \$/unit/month, $k_m = 1500$ \$/cycle, $k_R = 1000$ \$/cycle, $h_r = 1$ \$/unit/month, $k_r = 800$ \$/cycle, $c = 600$ unit/month, $d = 28$ unit/month^2.

Applying the solution procedure given in the last section we derive the optimal solution and results are presented in Table 2 .

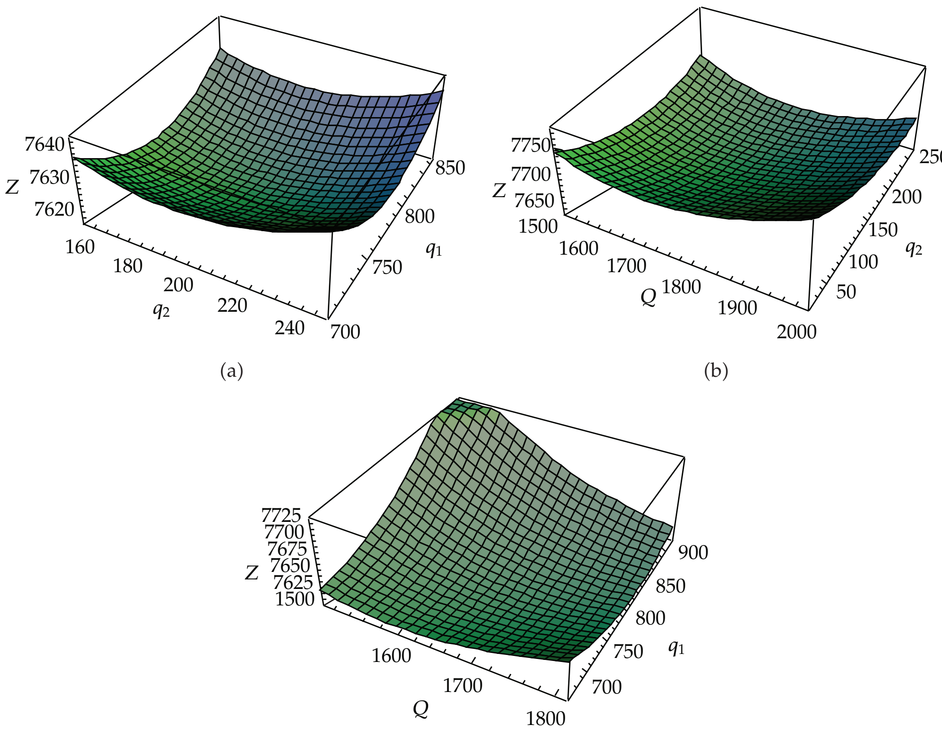
The convexity of the reverse logistics inventory model is shown in Figure 3 . The three dimensional graph shows that the integrated expected total annual cost is convex, and that there exists a unique solution minimizing the integrated expected total annual cost.

4.2. Sensitivity Analysis

To study the effects of the parameter changes on the optimal solutions derived by the proposed method, this investigation performs a sensitivity analysis by increasing or decreasing the parameters, one at a time.

The main conclusions drawn from the sensitivity analysis given above are as follows.

(i) From Tables 3 and 4 we have observed that as the production rate increases the optimum value of total acceptable returned quantity decreases and hence the total



(a) (b) (c)

Figure 3: Convexity of the reverse logistics inventory model is derived, and when the optimal value of acceptable returned quantity Q, maximum inventory level q_1 and q_2 is taken.

minimum cost slightly decreases. However we observe a little but unexpected increment in the total minimum average cost and this increment happened due to the reduction in total cycle length.

(ii) Similarly from Tables 5 and 6 we have observed that as the remanufacturing rate increases the optimum value of total acceptable returned quantity decreases and hence the total minimum cost slightly decreases. But due to the reduction in total cycle length we observe a little increment in the total minimum average cost.

(iii) From Tables 7 and 8 it is observed that as the return rate increases the optimum value of total acceptable returned quantity increases while the total time required to produce the optimum manufactured quantity decrease. Hence the total procurement and production cost will decrease. This reduction resulted in the decrease in the total minimum average cost.

(iv) Similarly from Table 9 it is observed that as the constant demand rate increases the optimal values of total acceptable returned quantity and the total time required to produce the optimum manufactured quantity increase. Hence the total procurement, acquisition, and production cost increases. This increment resulted in the increase in the total minimum average cost per cycle.

(v) Now from Table 10 it is observed that as the variable demand rate increases the optimal values of total acceptable returned quantity decrease and hence the total

Table 3: For the different constant production rates optimal results for the same set of values as in Example 4.2.

a_m	Q^*	q_1^*	q_2^*	t_1^*	t_2^*	t_3^*	t_4^*	t_5^*	t_6^*	t_7^*	t_8^*	$Z(Q^*,q_1^*,q_2^*)$
1200	1719	785	134	0.1603	0.9475	1.8717	1.9177	2.0198	2.3642	2.5156	2.6954	7605.77
1350	1715	783	165	0.1601	0.9453	1.8673	1.9249	2.0173	2.3233	2.5098	2.6895	7611.48
1500	1712	781	191	0.1605	0.9436	1.8634	1.9292	2.0117	2.2888	2.5049	2.6851	7616.1
1650	1709	780	212	0.1598	0.9420	1.8607	1.9335	2.0086	2.2612	2.5012	2.6806	7619.9
1800	1707	779	229	0.1598	0.9409	1.8585	1.9379	2.0075	2.2389	2.49833	2.6777	7623.08

Table 4: Optimal results for the same set of values as in Example 4.2 for different variable production rates.

b_m	Q^*	q_1^*	q_2^*	t_1^*	t_2^*	t_3^*	t_4^*	t_5^*	t_6^*	t_7^*	t_8^*	$Z(Q^*,q_1^*,q_2^*)$
28	1712	782	190	0.159527	0.94369	1.86462	1.93053	2.01376	2.29116	2.5061	2.6851	7615.98
29	1712	781	191	0.160528	0.94369	1.86346	1.92873	2.01091	2.28889	2.50498	2.6851	7616.04
30	1712	781	191	0.160528	0.94369	1.86346	1.92922	2.01177	2.28889	2.50498	2.6851	7616.1
31	1711	781	191	0.159983	0.943143	1.86293	1.92902	2.01175	2.288	2.5041	2.68362	7616.15
32	1711	781	191	0.159983	0.943143	1.86293	1.92951	2.01261	2.288	2.5041	2.68362	7616.21

minimum cost slightly decreases. But due to the reduction in total cycle length we observe a little increment in the total minimum average cost.

4.3. A Particular Case When q_1 and q_2 Are Constant

In a particular case let us have q_1 and q_2 the maximum inventory level in the remanufacturing and production process, as a constant then from (4.9)–(4.23) the values of $t_1,\ t_2,\ t_3,\ t_4,\ t_5,\ t_6,\ t_7,$ and t_8 can be found in the form of single variable Q, say

$$t_i = f_i(Q), \quad \text{where } 1 \le i \le 8. \tag{4.24}$$

From (4.4) the total cost function will be the function of one variable, say Q and the problem will be converted into the following unconstrained problem with one variable Q:

$$\begin{aligned}
W(Q) = \frac{1}{f_8}\Bigg\{ & (c_m + s_m)\int_{f_4}^{f_6} P_m(u)du + c_R\int_0^{f_8} R(u)du + s_r\int_0^{f_2} P_r(u)du \\
& + h_r\left[\int_{f_1}^{f_2}(-u)(P_r(u) - D(u))du + \int_{f_2}^{f_3} uD(u)du\right] \\
& + h_m\left[\int_{f_5}^{f_6}(-u)(P_m(u) - D(u))du + \int_{f_6}^{f_7} uD(u)du\right] \\
& + h_R\left[\int_0^{f_2} u(P_r(u) - R(u))du + \int_{f_2}^{f_8}(f_8 - u)R(u)du\right]
\end{aligned}$$

Table 5: Optimal results for the same set of values as in Example 4.2 for different constant remanufacturing rates.

a_r	Q^*	q_1^*	q_2^*	t_1^*	t_2^*	t_3^*	t_4^*	t_5^*	t_6^*	t_7^*	t_8^*	$Z(Q^*, q_1^*, q_2^*)$
1440	1823	661	202	0.2109	1.2497	2.0207	2.0903	2.1782	2.4717	2.69856	2.8489	7479.26
1620	1759	730	202	0.1814	1.0751	1.9312	1.9914	2.0671	2.3603	2.5882	2.7546	7556.35
1800	1712	781	191	0.1605	0.9436	1.8634	1.9292	2.0117	2.2888	2.5049	2.6851	7616.1
1980	1676	822	187	0.1430	0.8411	1.8122	1.8772	1.9586	2.2298	2.44188	2.6317	7663.79
2160	1648	855	184	0.1292	0.7589	1.7716	1.8359	1.9161	2.1829	2.39203	2.5901	7702.75

Table 6: Optimal results for the same set of values as in Example 4.2 for different variable remanufacturing rates.

b_r	Q^*	q_1^*	q_2^*	t_1^*	t_2^*	t_3^*	t_4^*	t_5^*	t_6^*	t_7^*	t_8^*	$Z(Q^*, q_1^*, q_2^*)$
28	1712	781	191	0.160147	0.944177	1.86393	1.92969	2.01223	2.28935	2.50543	2.6851	7615.79
29	1712	781	191	0.160338	0.943934	1.86369	1.92945	2.012	2.28912	2.5052	2.6851	7615.95
30	1712	781	191	0.160528	0.94369	1.86346	1.92922	2.01177	2.28889	2.50498	2.6851	7616.1
31	1711	781	191	0.160173	0.9429	1.8627	1.92831	2.01066	2.28777	2.50387	2.68362	7616.25
32	1711	782	191	0.159362	0.942657	1.86362	1.92922	2.01156	2.28868	2.50477	2.68362	7616.4

$$+ c_s \left[\int_0^{f_1} u(P_r(u) - D(u))du - \int_{f_3}^{f_4} uD(u)du + \int_{f_4}^{f_5} u(P_m(u) - D(u))du \right.$$
$$\left. - \int_{f_7}^{f_8} uD(u)du \right] + K_r + K_m + K_R \biggr\}. \tag{4.25}$$

Now the necessary conditions for having a minimum for the problem

$$\frac{dW}{dQ} = 0. \tag{4.26}$$

To find the solution of (4.26), let $W = w/f_8$ then

$$\frac{dW}{dQ} = \frac{w'_Q f_8 - f'_{8.Q} w}{f_8^2}, \tag{4.27}$$

where, w'_Q and $f'_{8.Q}$ are the derivatives of w and f_8 (w.r.t.) Q, respectively. Hence, (4.26) is equivalent to

$$w'_Q f_8 = f'_{8.Q} w. \tag{4.28}$$

Table 7: Optimal results for the same set of values as in Example 4.2 for different constant return rates.

c	Q^*	q_1^*	q_2^*	t_1^*	t_2^*	t_3^*	t_4^*	t_5^*	t_6^*	t_7^*	t_8^*	$Z(Q^*, q_1^*, q_2^*)$
480	1460	668	315	0.1490	0.8769	1.7353	1.9548	2.2299	2.5963	2.87761	3.0425	8251.8
540	1590	726	252	0.1490	0.8769	1.7353	1.8226	1.9316	2.2971	2.58178	2.7485	7917.36
600	1712	781	191	0.1605	0.9436	1.8634	1.9292	2.0117	2.2888	2.5049	2.6851	7616.1
660	1826	833	133	0.1705	1.0060	1.9833	2.0284	2.0853	2.2783	2.42905	2.6209	7318.71
720	1934	882	77	0.1802	1.0649	2.0961	2.1226	2.1562	2.2680	2.35546	2.5588	7024.89

Table 8: Optimal results for the same set of values as in Example 4.2 for different variable return rates.

d	Q^*	q_1^*	q_2^*	t_1^*	t_2^*	t_3^*	t_4^*	t_5^*	t_6^*	t_7^*	t_8^*	$Z(Q^*, q_1^*, q_2^*)$
26	1697	775	192	0.15836	0.935485	1.84862	1.9157	1.99985	2.2784	2.4957	2.67347	7629.03
27	1704	778	192	0.159172	0.939314	1.85578	1.92156	2.0041	2.28265	2.49992	2.67857	7622.57
28	1712	781	191	0.160528	0.94369	1.86346	1.92922	2.01177	2.28889	2.50498	2.6851	7616.1
29	1719	785	190	0.160338	0.947518	1.87177	1.93732	2.01965	2.29532	2.51023	2.69011	7609.62
30	1711	782	191	0.161149	0.951347	1.87892	1.94426	2.02635	2.30059	2.51432	2.69508	7603.14

Taking the first derivative of both sides of (3.11), (3.12), (3.13), (3.14), and (3.15) with respect to Q. We obtain

$$f'_{2.Q}(P_r(f_2) - D(f_2)) - f'_{1.Q}(P_r(f_1) - D(f_1)) = f'_{3.Q}D(f_3) - f'_{2.Q}D(f_2), \tag{4.29}$$

$$f'_{4.Q}D(f_4) - f'_{3.Q}D(f_3) = f'_{5.Q}(P_m(f_5) - D(f_5)) - f'_{4.Q}(P_m(f_4) - D(f_4)), \tag{4.30}$$

$$\begin{gathered} f'_{6.Q}(P_m(f_6) - D(f_6)) - f'_{5.Q}(P_m(f_5) - D(f_5)) = f'_{7.Q}D(f_7) - f'_{6.Q}D(f_6), \\ f'_{1.Q}(P_r(f_1) - D(f_1)) = f'_{8.Q}D(f_8) - f'_{7.Q}D(f_7), \\ f'_{8.Q}R(f_8) = f'_{2.Q}P_r(f_2). \end{gathered} \tag{4.31}$$

As we have q_1 and q_2 are constant so that from (4.2) and (4.3) both sides of (4.29) and (4.30) will be equal to zero

$$f'_{2.Q}(P_r(f_2) - D(f_2)) - f'_{1.Q}(P_r(f_1) - D(f_1)) = f'_{3.Q}D(f_3) - f'_{2.Q}D(f_2) = 0 \tag{4.32}$$

$$f'_{4.Q}D(f_4) - f'_{3.Q}D(f_3) = f'_{5.Q}(P_m(f_5) - D(f_5)) - f'_{4.Q}(P_m(f_4) - D(f_4)) = 0. \tag{4.33}$$

Form the above equation we can find the values of $f'_{1.Q}$, $f'_{2.Q}$, $f'_{3.Q}$, $f'_{4.Q}$, $f'_{5.Q}$, $f'_{6.Q}$, $f'_{7.Q}$, and $f'_{8.Q}$.

Table 9: Optimal results for the same set of values as in Example 4.2 for different constant demand rates.

α	Q^*	q_1^*	q_2^*	t_1^*	t_2^*	t_3^*	t_4^*	t_5^*	t_6^*	t_7^*	t_8^*	$Z(Q^*, q_1^*, q_2^*)$
640	1645	871	53	0.1544	0.9070	2.1625	2.1845	2.2030	2.2654	2.33903	2.5856	5949.82
720	1683	829	129	0.1582	0.9278	2.0026	2.0517	2.1022	2.2700	2.43081	2.6421	6774.6
800	1712	781	191	0.1605	0.9436	1.8634	1.9292	2.0117	2.2888	2.5049	2.6851	7616.1
880	1735	729	236	0.1614	0.9562	1.7424	1.8180	1.9347	2.3220	2.56647	2.7191	8467.36
960	1756	674	265	0.1626	0.9677	1.638	1.7164	1.8670	2.3675	2.62061	2.7501	9323.28

Table 10: Optimal results for the same set of values as in Example 4.2 for different variable demand rates.

β	Q^*	q_1^*	q_2^*	t_1^*	t_2^*	t_3^*	t_4^*	t_5^*	t_6^*	t_7^*	t_8^*	$Z(Q^*, q_1^*, q_2^*)$
33	1732	792	192	0.1613	0.954628	1.88977	1.95572	2.03776	2.31463	2.53282	2.71471	7586.86
34	1722	787	191	0.160409	0.949159	1.87717	1.94361	2.02664	2.30292	2.51948	2.69991	7601.52
35	1712	781	191	0.160528	0.94369	1.86346	1.92922	2.01177	2.28889	2.50498	2.6851	7616.1
36	1702	776	190	0.159656	0.93822	1.85094	1.91713	2.00059	2.27709	2.49157	2.67029	7630.6
37	1692	772	190	0.157792	0.93275	1.83961	1.90504	1.98791	2.26524	2.47926	2.65547	7645.01

From (4.29)–(4.33), and (4.25) we have

$$\begin{aligned} w'_Q = {} & (c_m + s_m)\left\{ f'_{6.Q} P_m(f_6) - f'_{4.Q} P_m(f_4) \right\} + c_R f'_{8.Q} R(f_8) + s_r f'_{2.Q} P_r(f_2) \\ & + h_r \left\{ f'_{1.Q}(f_1 - f_2)(P_r(f_1) - D(f_1)) + f'_{3.Q}(f_3 - f_2) D(f_3) \right\} \\ & + h_m \left\{ f'_{5.Q}(f_5 - f_6)(P_m(f_5) - D(f_5)) - f'_{7.Q}(f_7 - f_6) D(f_7) \right\} \\ & + h_R \left\{ f'_{8.Q} f_2 R(f_8) - f'_{2.Q} f_8 R(f_2) \right\} \\ & + c_s \Big\{ f'_{3.Q}(f_3 - f_4) D(f_3) + f'_{5.Q}(f_5 - f_4)(P_m(f_5) - D(f_5)) \\ & \quad + f'_{8.Q} f_1 D(f_8) + f'_{7.Q}(f_7 - f_8 - f_1) D(f_7) \Big\}. \end{aligned} \tag{4.34}$$

From which and (4.28) we obtain

$$w = \frac{f_8}{f'_{8.Q}} \begin{bmatrix} (c_m + s_m)\left\{ f'_{6.Q} P_m(f_6) - f'_{4.Q} P_m(f_4) \right\} + c_R f'_{8.Q} R(f_8) + s_r f'_{2.Q} P_r(f_2) \\ + h_R \left\{ f'_{8.Q} f_2 R(f_8) - f'_{2.Q} f_8 R(f_2) \right\} \\ + h_r \left\{ f'_{1.Q}(f_1 - f_2)(P_r(f_1) - D(f_1)) + f'_{3.Q}(f_3 - f_2) D(f_3) \right\} \\ + h_m \left\{ f'_{5.Q}(f_5 - f_6)(P_m(f_5) - D(f_5)) - f'_{7.Q}(f_7 - f_6) D(f_7) \right\} \\ + c_s \Big\{ f'_{3.Q}(f_3 - f_4) D(f_3) + f'_{5.Q}(f_5 - f_4)(P_m(f_5) - D(f_5)) \\ + f'_{8.Q} f_1 D(f_8) + f'_{7.Q}(f_7 - f_8 - f_1) D(f_7) \Big\} \end{bmatrix}, \tag{4.35}$$

where W is given by (4.25) and w'_Q is given by (4.34).

Equation (4.35) can now, be used to determine the optimal value of Q. If we get more than one solution we choose the one for which the condition $d^2W/dQ^2 > 0$ is satisfied.

Table 11: The optimal results for the inventory model under the parametric values given in Example 4.2.

Q^*	t_1^*	t_2^*	t_3^*	t_4^*	t_5^*	t_6^*	t_7^*	t_8^*	$Z(Q^*, q_1^*, q_2^*)$
1737 units	0.155131 Month	0.957362 month	1.89856 month	1.95774 Month	2.03215 Month	2.32237 Month	2.5483 Month	2.7221 month	7616.6 $

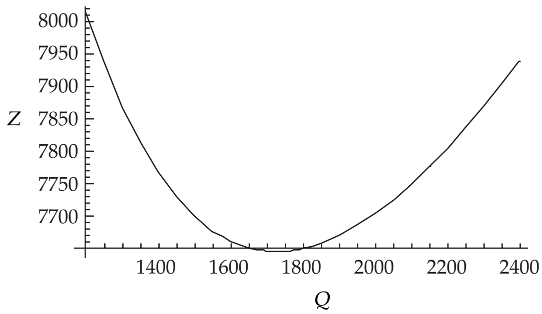


Figure 4: Behavior of the inventory cost function, when the values of q_1 and q_2 are taken to be constant.

Then the optimal values of t_1, t_2, t_3, t_4, t_5, t_6, t_7, and t_8 can be determined by using (4.9)–(4.23). From the above-mentioned results we can determine the value of w'_Q and w. Hence from (4.25) we can get the minimum total average cost.

Example 4.3. The same set of input data are considered as in the Example 4.2 except that the constant values of q_1 and q_2 are as follows: $q_1 = 800$ units, $q_2 = 200$ units.

Applying the solution procedure given in last section we find that the optimal value of the acceptable returned quantity Q is 1737 and hence the corresponding optimal results are presented in Table 11 .

The convexity of the total cost function has been established graphically in Figure 4 it has also been verified numerically that the cost function is convex.

5. Conclusion

In this paper designed model for a supply chain network with reverse flows is proposed. This paper generalises a reverse logistics inventory model for integrated production of new items and remanufacturing of returned items with shortages. In many practical situations, stock out is unavoidable due to various uncertainties. Therefore, the occurrence of shortages in inventory is a natural phenomenon. The developed model is solved by using the Hessian matrix. Results presented herein provide a valuable reference for decision makers in production, storage remanufacturing, and returning planning. A numerical example demonstrates that applying the proposed model can minimize the total average cost. In addition, sensitivity analysis is performed to examine the effect of parameters. According to those results, the presented model is least sensitive with respect to the production and remanufacturing rates while enough sensitive with respect to the returned and demand rates. It can be concluded from the obtained results that the higher returned rate can provide more profit. A future study should incorporate more realistic assumptions into the proposed model, for example, stochastic nature of demand, production, and remanufacturing rates follow learning and forgetting curves, and consider multiple production and remanufacturing batches per interval.

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