

Research Article

A Warehouse Imperfect Fuzzified Production Model with Shortages under Inflationary Conditions

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We develop a two-warehouse production model with imperfect items. Production rate is taken as the linear combination of on-hand inventory and demand, while demand rate is taken as function of time. Most of the researchers consider that the production rate is independent from the demand rate. In this paper we assume production rate as being dependent on the demand rate, and this assumption is more realistic. Shortages are allowed and partially backlogged with time-dependent backlogging rate. Due to different preservation facilities we consider that the deterioration rate is time dependent in own warehouse (OW) and Weibull distribution deterioration in rented warehouse (RW). Holding cost in RW is greater than in OW. We developed a fuzzy model with fuzzifying all the costs of the model as triangular fuzzy numbers. The present model is developed in both crisp and fuzzy senses. Finally, numerical example is shown, and sensitivity is also illustrated.

1. Introduction

One of the weaknesses of some production-inventory models is the unrealistic assumption that all items produced are of good quality. But production of defective units is a natural phenomenon in a production process. Defective items should be treated as a result of imperfect quality production. The effect of an imperfect process on production run time and EPQ was initially studied by Rosenblatt and Lee [1]. In their study, the elapsed time until the process shift was assumed to be exponentially distributed. The optimal production run was found to be shorter than that of classical EPQ model. In recent years, numerous research efforts have been undertaken to extend the work of Rosenblatt and Lee [1]. Kim and Hong [2] extended the work of Rosenblatt and Lee [1] by assuming that elapsed time

until the process shift was arbitrarily distributed. However, neither of their models took into consideration of any allowable shortages. Chung and Hou [3], however, have generalized the work of Kim and Hong [2] by assuming that shortages were allowed. Nevertheless, the common assumption of all the above-mentioned models was that there were a fixed percentage of defective items produced during the out-of-control period. C. Singh and S. R. Singh [4] developed an imperfect production process with exponential demand rate, Weibull deterioration under inflation.

In some cases, uncertainties can be defined as fuzziness or vagueness, which are characterized by fuzzy numbers of the fuzzy set theory. Zadeh [5] introduced fuzzy set theory to deal with quality-related problems with imprecise demand. Bellman and Zadeh [6] distinguished the difference between randomness and fuzziness by showing that the former deals with uncertainty regarding membership or nonmembership of an element in a set, while the later is concerned with the degree of uncertainty by which an element belongs to a set. Gen et al. [7] expressed the input data by fuzzy numbers, where they used interval mean value concept to solve an inventory problem. Yao and Chiang [8] considered an inventory model with total demand and storing cost as triangular fuzzy numbers. They performed the defuzzification by centroid and signed distance methods. Mondal and Maiti [9] applied genetic algorithms (GAs) to solve a multi-item fuzzy EOQ model. M. K. Maiti and M. Maiti [10] dealt with a fuzzy inventory model with two warehouses under possibility constraints. Mahapatra and Maiti [11] formulated a multi-item, multiobjective inventory model for deteriorating items with stock- and time-dependent demand rate over a finite time horizon in fuzzy stochastic environment. Halim et al. [12] developed a fuzzy inventory model for perishable items with stochastic demand, partial backlogging, and fuzzy deterioration rate. The model is further extended to consider fuzzy partial backlogging factor. Gani and Maheswari [13] discussed the retailer's ordering policy under two levels of delay payments considering the demand and the selling price as triangular fuzzy numbers. They used graded mean integration representation method for defuzzification. Lee and Yao [14] developed an economic production quantity (EPQ) model in which the demand and the production quantity are assumed to be fuzzy. Halim et al. [15] addressed the lot sizing problem in an unreliable production system with stochastic machine breakdown and fuzzy repair time. They defuzzified the cost-per-unit time using the signed distance method. Production of defective items in any manufacturing industry is a natural phenomenon. The number of defectives may have a change from one lot to another that cannot be assessed by a crisp value. Chen and Chang [16] developed a fuzzy economic production quantity (EPQ) model with defective productions that cannot be repaired. In this model, they considered a fuzzy opportunity cost and trapezoidal fuzzy costs under crisp production quantity or fuzzy production.

Since most decision makers think that the inflation does not have significant influence on the inventory policies, the effects of inflation are not considered in some inventory models. However, from a financial point of view, an inventory represents a capital investment and must compete with other assets for a firm's limited capital funds. Thus, it is necessary to consider the effects of inflation on the inventory system. Buzacott [17], Bierman and Thomas [18], and Misra [19] investigated the inventory decisions under an inflationary condition for the EOQ model. Chandra and Bahner [20], Moon and Lee [21], Lo et al. [22], and Singh et al. [23, 24] are amongst the few who studied the concept of inflation with regard to inventory. Singh and Jain [25] explored a deterministic inventory model for a deteriorating item in an inflation-induced environment.

In this paper, we assumed the following assumptions.

- (i) Production rate is greater than the demand rate.
- (ii) Production rate is the linear combination of on-hand inventory and demand rate.
- (iii) Demand rate is exponentially an increasing function of time.
- (iv) Model is considered for imperfect items.
- (v) Deterioration is taken as time dependent for OW, while, Weibull distribution for RW.
- (vi) Inflation is also taken in this model.
- (vii) Costs are considered as a triangular fuzzy numbers.
- (viii) Model is presented in both fuzzy and crisp senses.

2. Assumptions and Notations

2.1. Assumptions

The following assumptions have been used throughout the study.

- (i) The demand rate is exponentially increasing and is represented by $\lambda(t) = \lambda_0 e^{\delta t}$, where $0 \leq \delta \leq 1$ is a constant inflation rate, and λ_0 is the initial demand rate.
- (ii) Lead time is zero, and no replenishment or repair of deteriorated items is made during a given cycle.
- (iii) Production rate is the linear combination of on-hand inventory and demand rate at any time subject to

$$P(t) = [I(t) + \eta\lambda(t)](1 - e^{-\Delta t}), \quad 0 \leq \Delta \leq 1, \eta > 1, P(t) > \lambda(t). \quad (2.1)$$

- (iv) Shortages are allowed and the backlogging rate is exponentially, a decreasing function of time when inventory is in shortages, the backlogging parameter s is a positive constant s.t. $0 < s < 1$.
- (v) A single item is considered over the prescribed period T units of time, which is subject to variable deterioration rate.
- (vi) The owned warehouse (OW) has a fixed capacity of W units, and the rented warehouse (RW) has unlimited capacity.
- (vii) The goods of OW are consumed only after consuming the goods kept in RW.
- (viii) The unit inventory costs (including holding cost) per unit time in RW are higher than those in OW.
- (ix) Deterioration-rate of the items is considered to be different in different warehouses. In OW time-dependent deterioration rate is $\theta(t) = \theta t$, while in RW Weibull distribution deterioration rate is $\alpha\beta t^{\beta-1}$, where $\alpha, \beta > 0, t > 0$.
- (x) The model is developed in both fuzzy and crisp environment.
- (xi) Finite planning horizon is considered.

2.2. Notations

The following notations have been used throughout the study.

λ : Demand rate, $\lambda(t) = \lambda_0 e^{\delta t}$, increases with time, where λ_0 is the initial demand rate and $0 \leq \delta \leq 1$.

$P(t)$: Production rate $P(t) = [I(t) + \eta\lambda(t)](1 - e^{-\Delta t})$, which is the linear combination of on-hand inventory and demand rate at any time and $P(t) > \lambda(t)$, where $0 \leq \Delta \leq 1$, $\eta > 1$.

B : Backlogging rate, $B = e^{-st}$, where the backlogging parameter s is a positive constant s.t. $0 < s < 1$.

θ : Deterioration rate, where $\theta = \theta t$ for OW and $\theta = \alpha\beta t^{\beta-1}$ for RW, where $\alpha, \beta > 0$, $t > 0$.

Δ : Rate of imperfect production.

r : Inflation rate.

W : Fixed capacity level of OW.

C_1 : Set up cost per production run.

C_{RW} : Carrying cost per inventory unit held in RW per unit time.

C_{OW} : Carrying cost per inventory unit held in OW per unit time.

C_2 : Deterioration cost per unit time.

C_3 : Shortage cost for backlogged items.

C_4 : Lost sale cost per unit time.

C_5 : Rework cost per unit time.

I_{i1} : Inventory level in OW at time t with $t \in [0, t_1]$.

I_{i2} : Inventory level in RW at time t with $t \in [t_1, t_2]$.

I_{i3} : Inventory level in RW at time t with $t \in [0, t_3]$.

I_{i4} : Inventory level in OW at time t with $t \in [0, t_4]$.

I_{i5} : Inventory level in OW at time t with $t \in [0, t_2 + t_3]$.

I_{i6} : Inventory level in OW at time t with $t \in [0, t_5]$.

I_{i7} : Inventory level in OW at time t with $t \in [0, T]$.

\tilde{C}_1 : Fuzzy set up cost per unit time.

\tilde{C}_2 : Fuzzy deterioration cost per unit time.

\tilde{C}_3 : Fuzzy shortage cost per unit time.

\tilde{C}_4 : Fuzzy lost sale cost per unit time.

\tilde{C}_5 : Fuzzy rework cost per unit time.

\tilde{C}_{OW} : Fuzzy carrying cost per unit time FOR OW.

\tilde{C}_{RW} : Fuzzy carrying cost per unit time for RW.

t_1, t_2 : The production periods for OW and RW.

t_3, t_4 : The non production periods.

t_5 : The shortage period.

T : Total cycle time.

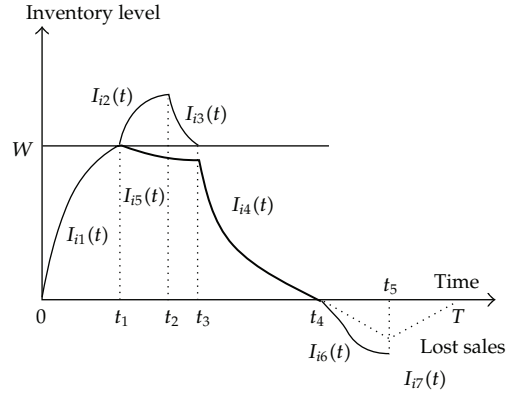


Figure 1: Inventory level in production system for deteriorating items with inflation-induced demand.

3. Formulation and Solution of the Model

Figure 1 shows the inventory level during a production cycle in which both OW and RW are used. Initially, the inventory level is zero. The production starts at time $t = 0$, and items accumulate from 0 up to W units in OW and in t_1 units of time. After time t_1 any production quantity exceeding W will be stored in RW. After this production stopped and the inventory level in RW begins to decrease at t_2 and will reach 0 units at t_3 because of demand and deterioration. The inventory level in OW comes to decrease at t_1 and then falls below W at $t_2 + t_3$ due to deterioration. The remaining stocks in OW will be fully exhausted at t_4 owing to demand and deterioration: the inventory becomes zero. At this time shortage starts developing, and at time t_5 it reaches to maximum shortage level : at this time fresh production starts to clear the backlog by the time T .

The differential equations stating the inventory levels within the cycle are given as follows:

$$\frac{dI_{i1}(t)}{dt} + \theta I_{i1}(t) = P(t) - \lambda(t), \quad 0 \leq t \leq t_1. \tag{3.1}$$

$$\frac{dI_{i2}(t)}{dt} + \theta I_{i2}(t) = P(t) - \lambda(t), \quad t_1 \leq t \leq t_2. \tag{3.2}$$

$$\frac{dI_{i3}(t)}{dt} + \theta I_{i3}(t) = -\lambda(t), \quad t_2 \leq t \leq t_3. \tag{3.3}$$

$$\frac{dI_{i4}(t)}{dt} + \theta I_{i4}(t) = -\lambda(t), \quad t_3 \leq t \leq t_4. \tag{3.4}$$

$$\frac{dI_{i5}(t)}{dt} + \theta I_{i5}(t) = 0, \quad t_1 \leq t \leq t_3. \tag{3.5}$$

$$\frac{dI_{i6}(t)}{dt} = -B\lambda(t), \quad t_4 \leq t \leq t_5. \tag{3.6}$$

$$\frac{dI_{i7}(t)}{dt} = P(t) - \lambda(t), \quad t_5 \leq t \leq T. \tag{3.7}$$

With the boundary conditions $I_{i1}(0) = 0, I_{i2}(t_1) = 0, I_{i3}(t_3) = 0, I_{i4}(t_4) = 0, I_{i5}(t_1) = W, I_{i6}(t_4) = 0$, and $I_{i7}(T) = 0$, respectively, the above equations can be solved successively as follows:

$$I_{i1}(t) = (\eta - 1)\lambda_0 \left[\frac{\Delta t^2}{2} + \frac{\delta \Delta t^3}{3} - \frac{(\theta - \Delta)\Delta t^4}{4} \right] - \lambda_0 \left[t + \frac{\delta t^2}{2} - (\theta - \Delta)\frac{t^3}{2} \right], \quad (3.8)$$

$$I_{i2}(t) = (\eta - 1)\lambda_0 \left[\frac{\Delta}{2}(t^2 - t_1^2) - \frac{\Delta \alpha t^\beta}{2}(t^2 - t_1^2) + \frac{\delta \Delta}{3}(t^3 - t_1^3) \right] \\ - \lambda_0 \left[(t - t_1) - \alpha t^\beta(t - t_1) + \frac{\delta}{2}(t^2 - t_1^2) - \frac{\delta \alpha t^\beta}{2}(t^2 - t_1^2) \right], \quad (3.9)$$

$$I_{i3}(t) = \lambda_0 \left[(t_3 - t) - \alpha t^\beta(t_3 - t) + \frac{\delta}{2}(t_3^2 - t^2) - \frac{\delta \alpha t^\beta}{2}(t_3^2 - t^2) \right], \quad (3.10)$$

$$I_{i4}(t) = \lambda_0 \left[(t_4 - t) - \frac{\theta t^2}{2}(t_4 - t) + \frac{\delta}{2}(t_4^2 - t^2) - \frac{\delta \theta t^2}{4}(t_4^2 - t^2) \right], \quad (3.11)$$

$$I_{i5}(t) = W e^{(-\theta/2)(t_1^2 - t^2)}, \quad (3.12)$$

$$I_{i6}(t) = \lambda_0 \left[(t_4 - t) + \left(\frac{\delta - s}{2} \right) (t_4^2 - t^2) - \frac{\delta s}{3} (t_4^3 - t^3) \right], \quad (3.13)$$

$$I_7(t) = (\eta - 1)\lambda_0 \left[(t - T) + \frac{\Delta t^2}{2}(t - T) + \frac{\delta}{2}(t^2 - T^2) + \frac{\delta \Delta t^2}{4}(t^2 - T^2) \right] \\ - \eta \lambda_0 \left[(T - t) + \frac{\Delta t^2}{2}(T - t) + \frac{(\delta - \Delta)}{2}(T^2 - t^2) \right]. \quad (3.14)$$

Therefore, we find out some costs such as the following.

3.1. Present Worth Ordering Cost

Consider the following:

$$\text{Ordering cost per cycle} = C_1 e^{-rt_5}. \quad (3.15)$$

3.2. Present Worth Inventory Level in RW

The total inventory level in RW can be derived as

$$I_{RW} = \int_{t_1}^{t_2} I_2(t) e^{-rt} dt + \int_{t_2}^{t_3} I_3(t) e^{-rt} dt \\ = \lambda_0 \left[(\eta - 1) \left(\frac{\Delta}{2} \left(\frac{t_2^3}{3} + \frac{2t_1^3}{3} \right) - \frac{\Delta r}{3} \left(\frac{t_2^4}{4} + \frac{t_1^4}{2} \right) + \frac{\Delta \delta}{3} \left(\frac{t_2^4}{4} + \frac{3t_1^4}{4} \right) - \frac{r \Delta \delta}{3} \left(\frac{t_2^5}{5} + \frac{3t_1^5}{10} \right) \right) \right] \\ - \left[\left(\frac{t_1^2}{2} + \frac{t_3^2}{2} \right) - r \left(\frac{t_1^3}{6} + \frac{t_3^3}{6} \right) + \frac{\delta}{2} \left(\frac{2t_1^3}{3} + \frac{2t_3^3}{3} \right) - \frac{\delta r}{2} \left(\frac{t_1^4}{4} + \frac{t_3^4}{2} \right) \right]. \quad (3.16)$$

3.3. Present Worth Inventory Level in OW

The total inventory level in OW can be derived as

$$\begin{aligned}
 I_{OW} &= \int_0^{t_1} I_1(t)e^{-rt} dt + \int_{t_1}^{t_3} I_5(t)e^{-rt} dt + \int_{t_3}^{t_4} I_4(t)e^{-rt} dt \\
 &= (\eta - 1)\lambda_0 \left[\frac{\Delta t_1^3}{6} - \frac{r\Delta t_1^4}{8} + \frac{\delta\Delta t_1^4}{12} \right] - \lambda_0 \left[\frac{t_1^2}{2} - \frac{t_4^2}{2} - \frac{t_3^2}{2} - \frac{rt_1^3}{6} - \frac{rt_3^3}{3} - \frac{\delta t_4^3}{6} \right] \\
 &\quad + W \left[(t_3 - t_1) - \frac{r}{2}(t_3^2 - t_1^2) + \frac{\theta t_1^3}{2} \right].
 \end{aligned} \tag{3.17}$$

3.4. Present Worth Deteriorated Items

The total quantity of deteriorated items during the period $(0, T)$ is given by

$$\begin{aligned}
 I_D &= \int_0^{t_1} \theta t I_{i1}(t)e^{-rt} dt + \int_{t_1}^{t_3} \theta t I_{i5}(t)e^{-rt} dt \\
 &\quad + \int_{t_3}^{t_4} \theta t I_{i4}(t)e^{-rt} dt + \int_{t_1}^{t_2} \alpha \beta t^{\beta-1} I_{i2}(t)e^{-rt} dt + \int_{t_2}^{t_3} \alpha \beta t^{\beta-1} I_{i3}(t)e^{-rt} dt \\
 &= \left[\theta \lambda_0 \left\{ (\eta - 1) \left(\frac{\Delta t_1^4}{8} + \frac{\Delta \delta t_1^5}{15} + \frac{\Delta r t_1^5}{10} \right) - \left(\frac{t_1^3}{3} + \frac{\delta t_1^4}{8} + \frac{r t_1^4}{4} \right) \right\} \right. \\
 &\quad + W \theta \left\{ \frac{t_3^2}{2} - \frac{t_1^2}{2} + \frac{\theta t_3^4}{8} - \frac{\theta t_1^4}{8} - \frac{r t_3^3}{3} + \frac{r t_1^3}{3} \right\} \\
 &\quad + \theta \lambda_0 \left\{ \left(-\frac{t_3^3}{6} - \frac{t_4^3}{3} \right) - r \left(-\frac{t_3^4}{12} - \frac{t_4^4}{4} \right) + \frac{\delta r}{2} \left(-\frac{2t_3^5}{15} - \frac{t_4^5}{5} \right) \right\} \\
 &\quad + \lambda_0 \alpha \beta \left\{ \left\{ (\eta - 1) \Delta \left(\frac{t_2^{\beta+2}}{2(\beta+2)} + \frac{t_1^{\beta+2}}{2\beta(\beta+2)} \right) \right\} - \left(\frac{t_2^{\beta+1}}{(\beta+1)} + \frac{t_1^{\beta+1}}{\beta(\beta+1)} \right) \right\} \\
 &\quad \left. \times \lambda_0 \alpha \beta \left\{ \left(\frac{t_2^{\beta+1}}{(\beta+1)} + \frac{t_3^{\beta+1}}{\beta(\beta+1)} \right) - r \left(\frac{t_2^{\beta+2}}{(\beta+2)} + \frac{t_3^{\beta+2}}{(\beta+2)(\beta+1)} \right) \right\} \right].
 \end{aligned} \tag{3.18}$$

3.5. Present Worth Shortage Cost

Total quantity of shortage units (I_S) during the period $(0, T)$ is given by

$$\begin{aligned}
 I_S &= - \int_{t_4}^{t_5} I_{i6}(t)e^{-rt} dt - \int_{t_5}^T I_{i7}(t)e^{-rt} dt \\
 &= s \left[\lambda_0 \left\{ \left(\frac{t_4^2}{2} + \frac{t_5^2}{2} \right) - r \left(\frac{t_4^3}{6} + \frac{t_5^3}{3} \right) - \frac{\delta s}{3} \left(\frac{3t_4^4}{4} + \frac{t_5^4}{4} \right) \right\} \right. \\
 &\quad \left. + (\eta - 1)\lambda_0 \left\{ \left(\frac{T^2}{2} + \frac{t_5^2}{2} \right) - r \left(\frac{T^3}{6} + \frac{t_5^3}{3} \right) \right\} + \eta\lambda_0 \left\{ \left(\frac{T^2}{2} + \frac{t_5^2}{2} \right) - r \left(\frac{T^3}{6} + \frac{t_5^3}{3} \right) \right\} \right]. \tag{3.19}
 \end{aligned}$$

3.6. Present Worth Lost Sales Quantity

The total amount of lost sales (I_L) during the period $(0, T)$ can be obtained as

$$I_L = \int_{t_4}^{t_5} \{1 - e^{-st}\} \lambda_0 e^{\delta t} dt = s\lambda_0 \left[\frac{1}{2} (t_5^2 - t_4^2) + \frac{\delta}{3} (t_5^3 - t_4^3) \right]. \tag{3.20}$$

Number of defective items is

$$N = \left\{ \begin{array}{ll} 0, & X \geq t_2 \\ \int_X^{t_2} P dt, & X < t_2 \end{array} \right\}, \quad N = t_2 \left\{ \begin{array}{ll} 0, & X \geq t_2 \\ P(t_2 - X), & X < t_2 \end{array} \right\}. \tag{3.21}$$

Expected number of defective items in a production cycle is

$$E(N) = \int_0^{t_2} P(t_2 - X) f(X) dX. \tag{3.22}$$

$f(X)$ is probability density function of X :

$$f(X) = \mu e^{-\mu X}. \tag{3.23}$$

3.7. Present Worth of Rework Cost

The total amount of rework cost (I_R) during the period $(0, T_2)$ can be obtained as

$$\begin{aligned}
 I_R &= E(N)e^{-rt_2} \\
 &= \left\{ \int_0^{t_2} P(t_2 - X)f(X)dX \right\} e^{-rt_2} \\
 &= \lambda_0 \mu \left[(\eta - 1) \Delta \left\{ \frac{\Delta}{8} \left(\frac{\mu t_2^6}{6} - \frac{t_2^5}{5} \right) + \frac{\Delta \delta}{15} \left(\frac{\mu t_2^7}{7} - \frac{t_2^6}{6} \right) \right\} - \Delta \left\{ \frac{1}{3} \left(\frac{\mu t_2^5}{5} - \frac{t_2^4}{4} \right) + \frac{\delta}{8} \left(\frac{\mu t_2^6}{6} - \frac{t_2^5}{5} \right) \right\} \right. \\
 &\quad + \eta \Delta \left\{ \frac{1}{2} \left(\frac{\mu t_2^4}{4} - \frac{t_2^3}{3} \right) + \frac{\delta}{6} \left(\frac{\mu t_2^5}{5} - \frac{t_2^4}{4} \right) \right\} \\
 &\quad + (\eta - 1) \Delta \left\{ \frac{\Delta}{2} \left(\frac{t_2^5}{4} - \frac{\mu t_2^6}{8} \right) - \frac{\Delta^2 \alpha}{2} \left(\frac{t_2^{\beta+5}}{\beta+4} - \frac{\mu t_2^{\beta+6}}{2(\beta+4)} \right) \right\} \\
 &\quad \left. - \left\{ \left(\frac{t_2^4}{3} - \frac{\mu t_2^5}{6} \right) - \alpha \left(\frac{t_2^{\beta+4}}{\beta+3} - \frac{\mu t_2^{\beta+5}}{2(\beta+3)} \right) \right\} + \eta \Delta \left(\frac{t_2^3}{2} + \frac{\delta t_2^4}{3} - \frac{\mu t_2^4}{4} \right) \right] (1 - rt_2). \tag{3.24}
 \end{aligned}$$

3.8. Present Worth of Total Cost

The total average cost (TC) of the inventory system is given by

$$TC = \frac{1}{T} [C_1 + C_{RW} I_{RW} + C_{OW} I_{OW} + C_2 I_D + C_3 I_S + C_4 I_L + C_5 I_R]. \tag{3.25}$$

4. Fuzzy Model

In order to develop the model in a fuzzy environment, we consider the costs as the triangular fuzzy numbers where $\tilde{C}_1 = (C_1 - \Delta_1, C_1, C_1 + \Delta_2)$, $\tilde{C}_{RW} = (C_{RW} - \Delta_3, C_{RW}, C_{RW} + \Delta_4)$, $\tilde{C}_{OW} = (C_{OW} - \Delta_5, C_{OW}, C_{OW} + \Delta_6)$, $\tilde{C}_2 = (C_2 - \Delta_7, C_2, C_2 + \Delta_8)$, $\tilde{C}_3 = (C_3 - \Delta_9, C_3, C_3 + \Delta_{10})$, $\tilde{C}_4 = (C_4 - \Delta_{11}, C_4, C_4 + \Delta_{12})$, and $\tilde{C}_5 = (C_5 - \Delta_{13}, C_5, C_5 + \Delta_{14})$ such that $0 < \Delta_1 < C_1$, $0 < \Delta_2$, $0 < \Delta_3 < C_{RW}$, $0 < \Delta_4$, $0 < \Delta_5 < C_{OW}$, $0 < \Delta_6$, $0 < \Delta_7 < C_2$, $0 < \Delta_8$, $0 < \Delta_9 < C_3$, $0 < \Delta_{10}$, $0 < \Delta_{11} < C_4$, $0 < \Delta_{12}$, $0 < \Delta_{13} < C_5$, $0 < \Delta_{14}$, and $\Delta_1, \Delta_2, \Delta_3, \Delta_4, \Delta_5, \Delta_6, \Delta_7, \Delta_8, \Delta_9, \Delta_{10}, \Delta_{11}, \Delta_{12}, \Delta_{13}$, and Δ_{14} are determined by the decision maker based on the uncertainty of the problem. Thus, the costs are considered as the fuzzy numbers with membership function:

$$\begin{aligned}
TC &= \frac{1}{T} \left[\tilde{C}_1 + \tilde{C}_{RW} I_{RW} + \tilde{C}_{OW} I_{OW} + \tilde{C}_2 I_D + \tilde{C}_3 I_S + \tilde{C}_4 I_L + \tilde{C}_5 I_R \right], \\
\tilde{C}_1 &= (C_1 - \Delta_1, C_1, C_1 + \Delta_2), \\
\tilde{C}_{RW} &= (C_{RW} - \Delta_3, C_{RW}, C_{RW} + \Delta_4), \\
\tilde{C}_{OW} &= (C_{OW} - \Delta_5, C_{OW}, C_{OW} + \Delta_6), \\
\tilde{C}_2 &= (C_2 - \Delta_7, C_2, C_2 + \Delta_8), \\
\tilde{C}_3 &= (C_3 - \Delta_9, C_3, C_3 + \Delta_{10}), \\
\tilde{C}_4 &= (C_4 - \Delta_{11}, C_4, C_4 + \Delta_{12}), \\
\tilde{C}_5 &= (C_5 - \Delta_{13}, C_5, C_5 + \Delta_{14}).
\end{aligned} \tag{4.1}$$

By Centroid Method consider

$$\begin{aligned}
\tilde{C}_1 &= C_1 + \frac{1}{3}(\Delta_2 - \Delta_1) \\
\tilde{C}_{RW} &= C_{RW} + \frac{1}{3}(\Delta_4 - \Delta_3) \\
\tilde{C}_{OW} &= C_{OW} + \frac{1}{3}(\Delta_6 - \Delta_5) \\
\tilde{C}_2 &= C_2 + \frac{1}{3}(\Delta_8 - \Delta_7) \\
\tilde{C}_3 &= C_3 + \frac{1}{3}(\Delta_{10} - \Delta_9) \\
\tilde{C}_4 &= C_4 + \frac{1}{3}(\Delta_{12} - \Delta_{11}) \\
\tilde{C}_5 &= C_5 + \frac{1}{3}(\Delta_{14} - \Delta_{13}),
\end{aligned} \tag{4.2}$$

$$\begin{aligned}
F_1 &= \frac{1}{T} [(C_1 + \Delta_2) + (C_{RW} + \Delta_4) I_{RW} \\
&\quad + (C_{OW} + \Delta_6) I_{OW} + (C_2 + \Delta_8) I_D + (C_3 + \Delta_{10}) I_S + (C_4 + \Delta_{12}) I_L + (C_5 + \Delta_{14}) I_R] \\
F_2 &= \frac{1}{T} [C_1 + C_{RW} I_{RW} + C_{OW} I_{OW} + C_2 I_D + C_3 I_S + C_4 I_L + C_5 I_R], \\
F_3 &= \frac{1}{T} [(C_1 - \Delta_1) + (C_{RW} - \Delta_3) I_{RW} \\
&\quad + (C_{OW} - \Delta_5) I_{OW} + (C_2 - \Delta_7) I_D + (C_3 - \Delta_9) I_S + (C_4 - \Delta_{11}) I_L + (C_5 - \Delta_{13}) I_R].
\end{aligned}$$

By defuzzification, we get

$$\begin{aligned}
 &= \frac{1}{T} \left[C_1 + \frac{1}{4}(\Delta_2 - \Delta_1) + C_{RW}I_{RW} + \frac{1}{4}(\Delta_4 - \Delta_3) + C_{OW}I_{OW} + \frac{1}{4}(\Delta_6 - \Delta_5) + C_2I_D \right. \\
 &\quad \left. + \frac{1}{4}(\Delta_8 - \Delta_7) + C_3I_S + \frac{1}{4}(\Delta_{10} - \Delta_9) + C_4I_L + \frac{1}{4}(\Delta_{12} - \Delta_{11}) + C_5I_R + \frac{1}{4}(\Delta_{14} - \Delta_{13}) \right]. \tag{4.3}
 \end{aligned}$$

5. Numerical Example

To illustrate the theory of the model we consider the following data on the basis of the previous study.

$$\begin{aligned}
 \delta &= 2, & C_{RW} &= 0.6, & W &= 50, & \theta &= 0.03, \\
 C_1 &= 10, & C_{OW} &= 1.2, & \eta &= 5, & C_2 &= 2, \\
 \alpha &= 0.06, & \beta &= 0.01, & C_3 &= 5, & \mu &= 0.005, \\
 C_4 &= 6, & C_5 &= 0.1, & \lambda_0 &= 20, & T &= 10
 \end{aligned} \tag{5.1}$$

in appropriate units. Based on these input data, the findings are as follows So the optimum values of

$$\begin{aligned}
 t_1^* &= 1.42655, & t_2^* &= 2.08592, & t_3^* &= 4.17695, \\
 t_4^* &= 5.72968, & t_5^* &= 8.38866, & TC &= 4007.4.
 \end{aligned} \tag{5.2}$$

6. Fuzzy Numerical

$$\begin{aligned}
 \tilde{C}_1 &= (9, 10, 11), & \tilde{C}_{OW} &= (1.1, 1.2, 1.3), & \eta &= 5, & \lambda_0 &= 20, \\
 \delta &= 2, & \tilde{C}_{RW} &= (0.5, 0.6, 0.7), & W &= 50, & \theta &= 0.03,
 \end{aligned} \tag{6.1}$$

$$\begin{aligned}
 \tilde{C}_2 &= (1.5, 2, 2.5), & \alpha &= 0.06, & \beta &= 0.01, & \tilde{C}_3 &= (4, 5, 6), \\
 \mu &= 0.005, & \tilde{C}_4 &= (5.5, 6, 6.5), & \tilde{C}_5 &= (0.09, 0.1, 0.2), & T &= 10
 \end{aligned}$$

$$\begin{aligned}
 t_1^* &= (1.41419, 1.42655, 1.44654), & t_2^* &= (2.08595, 2.08592, 2.06724), \\
 t_3^* &= (4.16689, 4.17695, 4.23937), & t_4^* &= (5.84037, 5.72968, 5.69694), \\
 t_5^* &= (8.90087, 8.38666, 8.08675), & TC &= (3425.62, 4007.4, 4628.42).
 \end{aligned} \tag{6.2}$$

7. Sensitivity Analysis

See Table 1.

Table 1

Parameter	Change in parameter	t_1^*	t_2^*	t_3^*	t_4^*	t_5^*	TC
C_1	12	1.42651	2.08592	4.17679	5.72944	8.38843	4007.35
	14	1.42648	2.08592	4.17664	5.7292	8.3882	4007.3
	16	1.42644	2.08592	4.17648	5.72897	8.38797	4007.25
	18	1.4264	2.08592	4.17633	5.72873	8.38774	4007.2
	20	1.42637	2.08592	4.17617	5.7285	8.3875	4007.15
α	0.07	1.42891	2.08533	4.17599	5.72978	8.38871	4007.42
	0.08	1.43164	2.08474	4.17505	5.72988	8.38877	4007.43
	0.09	1.43472	2.08414	4.17413	5.72998	8.38833	4007.44
	0.1	1.43816	2.08355	4.17321	5.73009	8.38889	4007.45
	0.2	1.49168	2.07793	4.16471	5.73122	8.38951	4007.46
β	0.02	1.428	2.08244	4.17683	5.72969	8.38866	4007.41
	0.03	1.42945	2.07897	4.17672	5.7297	8.38867	4007.42
	0.04	1.4309	2.07552	4.17662	5.72971	8.38867	4007.43
	0.05	1.43236	2.07209	4.1765	5.72972	8.38868	4007.44
	0.06	1.43381	2.06868	4.17639	5.72973	8.38869	4007.45
λ_0	22	1.34214	2.08549	4.17764	5.72985	8.3888	4407.61
	24	1.27294	2.08518	4.1764	5.7298	8.38881	4807.68
	26	1.21496	2.08495	4.1742	5.72965	8.38876	5207.94
	28	1.16551	2.08476	4.17155	5.72945	8.38868	5608.08
	30	1.12271	2.08462	4.16873	5.72922	8.38857	6008.2
δ	2.1	1.44449	2.08501	4.22808	5.81669	8.46614	4048.1
	2.2	1.46366	2.08411	4.27917	5.90235	8.5434	4089.6
	2.3	1.48418	2.08325	4.33018	5.98681	8.62043	4131.9
	2.4	1.50622	2.08242	4.38107	6.07019	8.69721	4175
	2.5	1.52996	2.08161	4.43178	6.15258	8.77374	4218.9
Δ	0.32	1.34242	1.91621	4.18507	5.73049	8.38911	4007.8
	0.34	1.27138	1.7726	4.19123	5.73111	8.38945	4008.13
	0.36	1.2103	1.65176	4.19603	5.7316	8.38972	4008.4
	0.38	1.15704	1.54628	4.19987	5.73199	8.38994	4008.63
	0.4	1.11006	1.45395	4.20299	5.7323	8.39011	4008.83
C_{OW}	1.4	1.24759	2.08895	4.08125	5.72016	8.3834	4004.78
	1.6	1.12142	2.09152	3.99301	5.70965	8.37761	4002.17
	1.8	1.02623	2.09365	3.85523	5.69852	8.37149	3999.63
	2.0	0.951127	2.09543	3.72869	5.68693	8.36512	3997.18
	2.2	0.889949	2.09691	3.59326	5.67493	8.35856	3994.86
C_{RW}	0.7	1.56693	2.08682	4.31716	5.75917	8.40501	4013.99
	0.8	1.70794	2.08788	4.4499	5.78821	8.42119	4020.78
	0.9	1.85215	2.08912	4.57497	5.81672	8.43716	4027.78
	1.0	2.00219	2.09058	4.69176	5.84457	8.45284	4034.98
	1.1	2.16108	2.0923	4.79913	5.87156	8.46812	4042.38
C_3	5.2	1.42214	2.08589	4.15801	5.70089	8.25883	4101
	5.4	1.4188	2.08586	4.14345	5.67877	8.14299	4198.27
	5.6	1.41632	2.08584	4.13251	5.66213	8.03918	4298.68
	5.8	1.41454	2.08583	4.12454	5.65002	7.94579	4401.77
	6.0	1.41331	2.08582	4.11904	5.64165	7.86147	4507.17

Table 1: Continued.

Parameter	Change in parameter	t_1^*	t_2^*	t_3^*	t_4^*	t_5^*	TC
C_4	6.1	1.42907	2.08594	4.18761	5.74588	8.44763	4036.48
	6.2	1.43177	2.08596	4.19892	5.76306	8.50755	4066.33
	6.3	1.43467	2.08598	4.21088	5.78123	8.5684	4096.98
	6.4	1.43776	2.08601	4.22349	5.8004	8.63019	4128.45
	6.5	1.44106	2.08603	4.23676	5.82055	8.69293	4160.76
C_5	0.12	1.42648	2.08134	4.17695	5.72968	8.38866	4007.41
	0.14	1.42641	2.07681	4.17696	5.72968	8.38866	4007.41
	0.16	1.42634	2.07232	4.17697	5.72968	8.38866	4007.42
	0.18	1.42627	2.06786	4.17698	5.72968	8.38866	4007.42
	0.2	1.4262	2.06345	4.17699	5.72968	8.38866	4007.43
r	0.16	1.3601	2.13665	3.84454	5.20265	7.86865	3662.23
	0.17	1.31431	2.19421	3.55049	4.72879	7.40461	3348.43
	0.18	1.28124	2.26059	3.29027	4.29964	6.98831	3059.69
	0.19	1.25646	2.33895	3.0598	3.90794	6.61292	2791.16
	0.2	1.23733	2.4348	2.85541	3.54734	6.27281	2539.13
s	0.6	1.51085	2.08657	4.48099	6.1919	8.39432	4033.45
	0.7	1.5898	2.08718	4.6929	6.51616	8.39547	4051.61
	0.8	1.66348	2.08777	4.8477	6.75601	8.39463	4064.91
	0.9	1.73232	2.08835	4.96477	6.94047	8.3929	4075.05
	1.0	1.7968	2.08891	5.05568	7.08662	8.39076	4083.01
W	55	1.52163	2.08646	4.1731	5.72929	8.38844	4007.93
	60	1.61945	2.08709	4.16542	5.72852	8.38802	4008.42
	65	1.72024	2.08781	4.15332	5.72731	8.38735	4008.87
	70	1.82396	2.08864	4.13619	5.7256	8.3864	4009.26
	75	1.9302	2.08957	4.11348	5.72336	8.38516	4009.62
η	6	1.14672	1.5716	4.00831	5.44543	8.11066	4627.9
	7	0.988027	1.26651	3.87949	5.23885	7.90997	5256.31
	8	0.881603	1.06271	3.7803	5.08204	7.75838	5889.69
	9	0.803577	0.916472	3.70204	4.95889	7.63982	6526.39
	10	0.743094	0.8063	3.63886	4.85959	7.54456	7165.4

8. Observations

- (1) As we increase the ordering cost, the optimum value of $t_1^*, t_3^*, t_4^*, t_5^*$ decreases, and t_2^* remains same. By this effect, our total cost decreases.
- (2) As we increase the deterioration parameter for OW, the optimum value of t_1^*, t_2^* increases and that of t_3^*, t_4^*, t_5^* decreases, so our total cost decreases.
- (3) As we increase the deterioration parameters for RW, the optimum value of t_1^*, t_4^*, t_5^* increases and that of t_2^*, t_3^* decreases then total cost slightly increase decreases. So the total cost is minimum when the deterioration rate of own warehouse is less than the deterioration rate of rented warehouse.

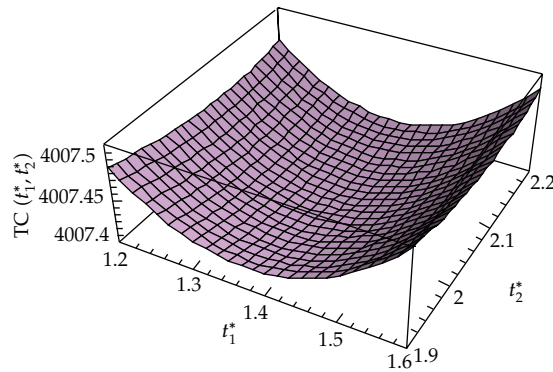


Figure 2: Convexity of total cost with respect to t_1^* and t_2^* .

- (4) As we increase the demand parameter λ_0 , the optimum value of t_1^* , t_2^* , t_3^* , t_4^* , t_5^* decreases, while total cost increases. More demand encourages more production which increases the total cost.
- (5) As we increase the demand parameter δ , the optimum value of t_1^* , t_3^* , t_4^* , t_5^* increases and t_2^* decreases, and then our total cost increases. In both the cases, we see that as we increase the demand, increase in total cost is then obvious.
- (6) On increasing the imperfect rate, the optimum value of t_1^* , t_3^* , t_4^* , t_5^* increases and t_2^* decreases; then our total cost slightly increases. This is very obvious since with an increment in the percentage of the defective items the total cost then increases.
- (7) As we increase the holding cost for OW, the optimum value of t_1^* , t_3^* , t_4^* , t_5^* decreases and t_2^* increases, and then our total cost decreases.
- (8) As we increase the holding cost for RW, the optimum value of t_1^* , t_2^* , t_3^* , t_4^* , t_5^* increases then our total cost increases.
- (9) On increasing the shortage cost, the optimum value of t_1^* , t_2^* , t_3^* , t_4^* , t_5^* decreases, and then our total cost increases.
- (10) As we increase the lost sale cost, the optimum value of t_1^* , t_2^* , t_3^* , t_4^* , t_5^* increases, and then our total cost increases.
- (11) As we increase the rework cost, the optimum value of t_1^* , t_2^* decreases, t_3^* increases, and t_4^* , t_5^* remains the same, and so our total cost slightly increases.
- (12) As we increase the inflation rate the optimum value of t_1^* , t_3^* , t_4^* , t_5^* decreases and t_2^* increases, while total cost decreases.
- (13) As we increase the backlogging parameter, the optimum value of t_1^* , t_2^* , t_3^* , t_4^* , t_5^* and total cost increase.
- (14) As we increase the warehouse capacity, the optimum value of t_1^* , t_2^* increases and t_3^* , t_4^* , t_5^* decreases, and so our total cost increases. As we increase the warehouse capacity, our expenses increases, and so total cost also increases.

9. Conclusion

This paper presented a warehouse imperfect production-inventory models for deteriorating items having time varying demand patterns with Weibull distribution deterioration and partial backlogging under the effect of inflation and time value of money. Inflation plays a very significant role in the inventory models. Demand, production, defective item production, and so forth are natural in any manufacturing system. In this paper, we have developed two models for a manufacturing system. In the first model, all costs were assumed to be stochastic whereas in the second model, the costs were assumed to be fuzzy. The production rate was proportional to the demand rate. Well-known triangular membership function was used for all the fuzzy numbers. Numerical examples are solved by the software MATHEMATICA 8.0. Convexity of the total average cost is also illustrated in Figure 2. However, the advantage of the fuzzy approach is that it relaxes the assumptions such as defective rate, demand rate etc. Also, it eases the difficulties in searching for suitable probability distribution function to represent the random behavior of uncontrollable variables.

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