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Research Article

A Global Optimizing Policy for Decaying Items with Ramp-Type Demand Rate under Two-Level Trade Credit Financing Taking Account of Preservation Technology

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An inventory system for deteriorating items, with ramp-type demand rate, under two-level trade credit policy taking account of preservation technology is considered. The objective of this study is to develop a deteriorating inventory policy when the supplier provides to the retailer a permissible delay in payments, and during this credit period, the retailer accumulates the revenue and earns interest on that revenue; also the retailer invests on the preservation technology to reduce the rate of product deterioration. Shortages are allowed and partially backlogged. Sufficient conditions of the existence and uniqueness of the optimal replenishment policy are provided, and an algorithm, for its determination, is proposed. Numerical examples draw attention to the obtained results, and the sensitivity analysis of the optimal solution with respect to leading parameters of the system is carried out.

1. Introduction

In broad spectrum, deterioration is defined as the damage, spoilage, dryness, vaporization, and so forth, that result in the decrease of usefulness of the original one. In the past few decades, inventory problems for deteriorating items have been widely studied. The first attempt to derive optimal policies for deteriorating items was made by Ghare and Schrader [1], who derived a revised form of the economic order quantity (EOQ) model assuming exponential decay. This model was extended to consider the Weibull distribution deterioration by Covert and Philip [2]. Raafat [3] presented a complete survey of the available inventory literature for deteriorating inventory models. Goyal and Giri [4] also provided a detailed review of deteriorating inventory literatures. Teng et al. [5] developed an inventory model for deteriorating items with time varying demand and partial backlogging. Recently, C. Singh and S. R. Singh [6] presented an inventory model considering the Weibull distribution deterioration.

Investing in preservation technology (PT) for reducing deterioration rate has received little attention in the past years. The consideration of PT is important due to rapid social changes and the fact that P can reduce the deterioration rate significantly. Moreover, sales, inventories, and order quantities are very sensitive to the rate of deterioration, especially for fast deteriorating products. The higher rate of deterioration would result in a higher total annual relevant cost and a lower demand rate [7, 8]. Ouyang et al. [9] found that if the retailer can reduce effectively the deteriorating rate of item by improving the storage facility, the total annual relevant inventory cost will be reduced. Many enterprises invest in equipments to reduce the deterioration rate and extend the product expiration date. For example, refrigeration equipments are used to reduce the deterioration rate of fruits, flowers, and sea foods in the supermarket. Murr and Morris [10] showed that a lower temperature will increase the storage life and decrease decay. Wee et al. [11] presented a model using preservation technology.

In global market, supplier uses trade credit as a promotion tool to increase his sale and attract new retailers. Therefore, in practice, the supplier will allow a certain fixed period (credit period) for settling the amount that the supplier owes to retailer for the items supplied. Before the end of the trade credit period, the the retailer can sell the goods, accumulate revenue, and earn interest. However, beyond this period the supplier charges interest on the unpaid balance. Hence, trade credit can play a major role in inventory control for both the supplier and the retailer (see Jaggi et al. [12]). Huang and Chung [13] extended Goyal's [14] model to discuss the replenishment and payment policies to minimize the annual total average cost under cash discount and payment delay from the retailer's point of view. Some relevant models related to this research area are found in the works of Huang [15], Yang [7], Darzanou and Skouri [16], Singh et al. [17], and so forth.

In the literature referring to models with allowable delay in payments, the demand is, mostly, treated either as constant or as continuous differentiable function of time. However, in the case of a new brand of consumer good coming to the market, its demand rate increases in its growth stage (i.e., $[0, \mu]$) and then remains stable in its maturity stage (i.e., $[\mu, T]$). The term "ramp type" is used to represent such demand pattern. Hill [18] proposed an inventory model with variable branch, being any power function of time. Research on this field continues with Mandal and Pal [19] and Wu and Ouyang [20]. In the above cited papers, the optimal replenishment policy requires to determine the decision time (say, t_1) at which the inventory level falls to zero. Consequently, the following two cases should be examined: (1) the inventory level falls to zero before the demand reaches constant (i.e., $t_1 < \mu$) and (2) the inventory level falls to zero after the demand reaches constant (i.e., $t_1 > \mu$). Almost all of the researchers examined only the first case. Deng et al. [21] first reconsidered the inventory models proposed by Mandal and Pal [19] and Wu and Ouyang [20] and discussed both cases. Skouri et al. [22] extend the work of Deng et al. [21] by introducing a general ramp-type demand rate and Weibull deterioration rate.

The present paper is an extension of the inventory system of Darzanou and Skouri [16] in the sense of preservation technology for deteriorating items when the two-level trade credit scheme, $r/M_1/M_2$, (in which the supplier provides r discount off the price if the payment is made within period M_1; otherwise, the full payment is due within period M_2) is considered. The study of this organism requires the inspection of the ordering relations between the time parameters M_1, M_2, μ, T, which, actually, lead to the various models. To reduce the length of the paper, one of these models ($M_1 \le \mu < M_2 < T$) will be presented at this time. The optimal solution of the proposed model not only exists but also is unique. To illustrate the theory of the proposed model the numerical example is provided, and sensitivity analysis with respect to parameters of the system is carried out. The original deterioration rate is assumed to be θ. A reduced deterioration rate of $m(\xi)$ is assumed when the retailer's investment cost of preservation equipments or technology is ξ.

2. Notation and Assumptions

The following notation is used through the paper.

2.1. Notation

T is the constant scheduling period (cycle),

t_1 is the time when the inventory level falls to zero,

C_p is the unit purchase cost,

c_1 is the inventory holding cost per unit per unit time,

c_2 is the shortage cost per unit per unit time,

c_3 is the cost incurred from the deterioration of one unit,

c_4 is the per unit opportunity cost due to the lost sales ($c_4 > C_p$ see Teng et al. [5]),

S is the maximum inventory level at the scheduling period (cycle),

ξ is the preservation technology (PT) cost for reducing deterioration rate in order to preserve the products, $\xi \ge 0$,

$m(\xi)$ is reduced deterioration rate, a function of ξ,

p is the unit selling price,

I_e is the interest rate earned,

I_c is the interest rate charged,

r is cash discount rate, $0 < r < 1$,

M_1 is the period of cash discount in years,

M_2 is the period of permissible delay in payments in years, $M_1 < M_2$,

μ is the parameter of the ramp-type demand function (time point),

$I(t)$ is the inventory level at time t.

2.2. Assumptions. The inventory model is developed under the following assumptions.

(1) The ordering quantity brings the inventory level up to the order level S. Replenishment rate is infinite.

(2) Shortages are backlogged at a rate $\beta(x)$ which is a nonincreasing function of x with $0 < \beta(x) \le 1$, $\beta(0) = 1$, and x is the waiting time up to the next replenishment. Moreover, it is assumed that $\beta(x)$ satisfies the relation $c_2\beta(x) + c_2 x\beta'(x) - c_4\beta'(x) \ge 0$, where $\beta'(x)$ is the derivate of $\beta(x)$. The case with $\beta(x) = 1$ corresponds to complete backlogging model.

(3) The supplier offers cash discount if payment is paid within M_1; otherwise, the full payment is paid within M_2 (see Huang [15]).

(4) The on-hand inventory deteriorates at a constant rate θ $(0 < \theta < 1)$ per time unit.
The deteriorated items are withdrawn immediately from the warehouse, and there is no provision for repair or replacement.

(5) The demand rate $D(t)$ is a ramp-type function of time given by

$$D(t) = \begin{cases} f(t), & t < \mu, \\ f(\mu), & t \ge \mu, \end{cases} \tag{1}$$

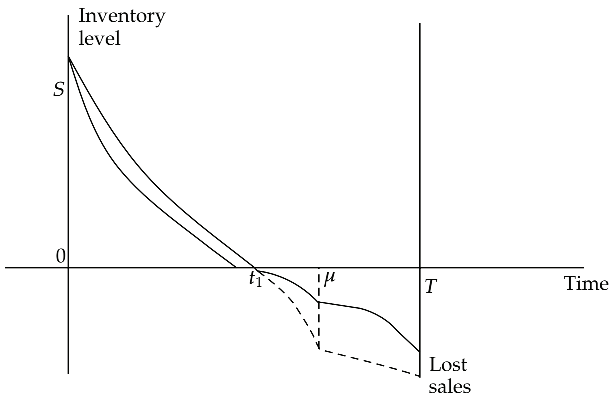

Figure 1: Inventory level for the model starting with no shortage over the cycle (case $t_1 < \mu$).

where $f(t)$ is a positive, differentiable function of $t \in (0, T]$.

3. Deriving the Common Quantities for the Inventory Models

The inventory level $I(t)$, $0 \le t \le T$ satisfies the following differential equations:

$$I'(t) + (\theta - m(\xi)) I(t) = -D(t), \quad 0 \le t \le t_1 \tag{2}$$

with boundary condition $I(t_1) = 0$, and

$$I'(t) = -D(t) \beta(T - t), \quad t_1 \le t \le T, \tag{3}$$

with boundary condition $I(t_1) = 0$.

From the two possible relations between parameters t_1 and μ, (i) $t_1 \le \mu$ and (ii) $t_1 > \mu$, the sum of holding, deterioration, shortages, and lost sales cost is obtained as

$$C(t_1) = \begin{cases} C_1(t_1) & \text{if } t_1 \le \mu, \\ C_2(t_1) & \text{if } t_1 > \mu, \end{cases} \tag{4}$$

where, $C_1(t_1)$ and $C_2(t_1)$ are calculated as follows.

Case 1 (($t_1 \le \mu$) (see Figure 1)). In this section, the inventory model starting with no shortages is studied. The replenishment at the beginning of the cycle brings the inventory level up to S. Due to demand and deterioration, the inventory level gradually depletes during the period $(0, t_1)$ and falls to zero at $t = t_1$. Thereafter, shortages occur during the period (t_1, T), which are partially backlogged.

The backlogged demand is satisfied at the next replenishment. The inventory level, $I(t)$, $0 \le t \le T$ satisfies the following differential equations.

In this case, (2) becomes

$$I'(t) + (\theta - m(\xi)) I(t) = -f(t), \quad 0 \le t \le t_1, \ I(t_1) = 0. \tag{5}$$

Equation (3) leads to the following two equations:

$$I'(t) = -f(t) \beta(T - t), \quad t_1 \le t \le \mu, \ I(t_1) = 0, \tag{6}$$

$$I'(t) = -f(\mu) \beta(T - t), \quad \mu \le t \le T, \ I(\mu_-) = I(\mu_+). \tag{7}$$

The solutions of (5)–(7) are, respectively,

$$I(t) = e^{-(\theta - m(\xi))t} \int_t^{t_1} f(x) e^{(\theta - m(\xi))x} dx, \quad 0 \le t \le t_1, \tag{8}$$

$$I(t) = -\int_{t_1}^{t} f(x) \beta(T - x) dx, \quad t_1 \le t \le \mu, \tag{9}$$

$$\begin{aligned} I(t) = &-f(\mu) \int_\mu^t \beta(T - x) dx \\ &- \int_{t_1}^{\mu} f(x) \beta(T - x) dx, \quad \mu \le t \le T. \end{aligned} \tag{10}$$

The total amount of deteriorated items during $[0, t_1]$ is

$$D = \int_0^{t_1} f(x) e^{(\theta - m(\xi))x} dx - \int_0^{t_1} f(t) dt. \tag{11}$$

The cumulative inventory carried in the interval $[0, t_1]$ is found from (8) and is

$$I_1 = \int_0^{t_1} I(t) dt = \int_0^{t_1} e^{-(\theta - m(\xi))t} \left[\int_t^{t_1} f(x) e^{(\theta - m(\xi))x} dx \right] dt. \tag{12}$$

Due to (9) and (10), the time-weighted backorders due to shortages during the interval $[t_1, T]$ are

$$\begin{aligned} I_2 &= \int_{t_1}^{T} [-I(t)] dt = \int_{t_1}^{\mu} [-I(t)] dt + \int_\mu^T [-I(t)] dt \\ &= \int_{t_1}^{\mu} (\mu - t) f(t) \beta(T - t) dt \\ &\quad + f(\mu) \int_\mu^T \left[\int_\mu^t \beta(T - x) dx \right] dt \\ &\quad + \int_\mu^T \left[\int_{t_1}^{\mu} f(x) \beta(T - x) dx \right] dt. \end{aligned} \tag{13}$$

The amount of lost sales during $[t_1, T]$ is

$$L = \int_{t_1}^{\mu} [1 - \beta(T - t)] f(t) dt + f(\mu) \int_\mu^T [1 - \beta(T - t)] dt. \tag{14}$$

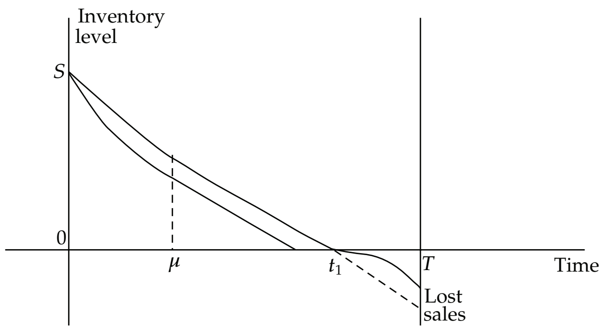

Figure 2: Inventory level for the model starting with no shortage over the cycle (case $t_1 > \mu$).

The cost $C_1(t_1)$ in the time interval $[0, T]$ is the sum of holding, shortage, deterioration, and opportunity costs and is given by

$$\begin{aligned} C_1(t_1) &= \Bigg[\xi + c_1 \left\{\int_0^{t_1} e^{-(\theta - m(\xi))t} \left[\int_t^{t_1} f(x) e^{(\theta - m(\xi))x} dx\right] dt\right\} \\ &\quad + c_3 \left\{\int_0^{t_1} f(x) e^{(\theta - m(\xi))x} dx - \int_0^{t_1} f(t)\, dt\right\} \\ &\quad + c_2 \left\{\int_{t_1}^{\mu} (\mu - t) f(t) \beta(T - t)\, dt \right. \\ &\qquad + f(\mu) \int_\mu^T \left[\int_\mu^t \beta(T - x)\, dx\right] dt \\ &\qquad \left. + \int_\mu^T \left[\int_{t_1}^{\mu} f(x) \beta(T - x)\, dx\right] dt\right\} \\ &\quad + c_4 \left\{\int_{t_1}^{\mu} (1 - \beta(T - t)) f(t)\, dt \right. \\ &\qquad \left. + f(\mu) \int_\mu^T (1 - \beta(T - t))\, dt\right\}\Bigg]. \end{aligned} \tag{15}$$

Case 2 (($t_1 > \mu$) (see Figure 2)). In this case, (2) reduces to the following two equations:

$$\begin{aligned} I'(t) + (\theta - m(\xi)) I(t) &= -f(t), \quad 0 \le t \le \mu, \\ I(\mu^-) &= I(\mu^+), \\ I'(t) + (\theta - m(\xi)) I(t) &= -f(\mu), \quad \mu \le t \le t_1,\ I(t_1) = 0. \end{aligned} \tag{16}$$

Equation (3) becomes

$$I'(t) = -f(\mu) \beta(T - t), \quad t_1 \le t \le T,\ I(t_1) = 0. \tag{17}$$

Their solutions are, respectively,

$$\begin{aligned} I(t) &= e^{-(\theta - m(\xi))t} \\ &\quad \times \left[\int_t^{\mu} f(x) e^{(\theta - m(\xi))x} dx + f(\mu) \int_\mu^{t_1} e^{(\theta - m(\xi))x} dx\right], \\ &\qquad 0 \le t \le \mu, \end{aligned} \tag{18}$$

$$I(t) = f(\mu) e^{-(\theta - m(\xi))t} \int_t^{t_1} e^{(\theta - m(\xi))x} dx, \quad \mu \le t \le t_1, \tag{19}$$

$$I(t) = -f(\mu) \int_t^{t_1} \beta(T - x)\, dx, \quad t_1 \le t \le T. \tag{20}$$

The total amount of deteriorated items during $[0, t_1]$ is

$$\begin{aligned} D &= \int_0^{\mu} f(x) e^{(\theta - m(\xi))x} dx \\ &\quad + f(\mu) \int_\mu^{t_1} e^{(\theta - m(\xi))x} dx - \int_0^{\mu} f(t)\, dt - \int_\mu^{t_1} f(\mu)\, dt. \end{aligned} \tag{21}$$

The total inventory carried in the interval $[0, t_1]$ is found from (18) and (19) and is

$$\begin{aligned} I_1 &= \int_0^{t_1} I(t)\, dt = \int_0^{\mu} I(t)\, dt + \int_\mu^{t_1} I(t)\, dt \\ &= \int_0^{\mu} e^{-(\theta - m(\xi))t} \\ &\quad \times \left[\int_t^{\mu} f(x) e^{(\theta - m(\xi))x} dx + f(\mu) \int_\mu^{t_1} e^{(\theta - m(\xi))x} dx\right] dt \\ &\quad + f(\mu) \int_\mu^{t_1} e^{-(\theta - m(\xi))t} \left[\int_t^{t_1} e^{(\theta - m(\xi))x} dx\right] dt. \end{aligned} \tag{22}$$

The time-weighted backorders due to shortages during the interval $[t_1, T]$ are

$$\begin{aligned} I_2 &= \int_{t_1}^{T} [-I(t)]\, dt = f(\mu) \int_{t_1}^{T} \left[\int_{t_1}^{t} \beta(T - x)\, dx\right] dt \\ &= f(\mu) \int_{t_1}^{T} (T - x) \beta(T - x)\, dx. \end{aligned} \tag{23}$$

The amount of lost sales during $[t_1, T]$ is

$$L = f(\mu) \int_{t_1}^{T} [1 - \beta(T - t)]\, dt. \tag{24}$$

The cost $C_2(t_1)$ in the time interval $[0, T]$ is the sum of holding, shortage, deterioration and opportunity costs and is given by

$$
\begin{aligned}
C_2(t_1) &= \Bigg[\xi + c_1 \Bigg\{\int_0^{\mu} e^{-(\theta-m(\xi))t} \\
&\quad \times \Bigg[\int_t^{\mu} f(x) e^{(\theta-m(\xi))x} dx \\
&\quad + f(\mu) \int_{\mu}^{t_1} e^{(\theta-m(\xi))x} dx\Bigg] dt \\
&\quad + f(\mu) \int_{\mu}^{t_1} e^{-(\theta-m(\xi))t} \Bigg[\int_t^{t_1} e^{(\theta-m(\xi))x} dx\Bigg] dt\Bigg\} \\
&\quad + c_3 \Bigg\{\int_0^{\mu} f(x) e^{(\theta-m(\xi))x} dx + f(\mu) \\
&\quad \times \int_{\mu}^{t_1} e^{(\theta-m(\xi))x} dx - \int_0^{\mu} f(t)\, dt - \int_{\mu}^{t_1} f(\mu)\, dt\Bigg\} \\
&\quad + c_2 \Bigg\{f(\mu) \int_{t_1}^{T} (T-x) \beta(T-x)\, dx\Bigg\} \\
&\quad + c_4 \Bigg\{f(\mu) \int_{t_1}^{T} (1-\beta(T-t))\, dt\Bigg\}\Bigg].
\end{aligned} \tag{25}
$$

4. The Inventory Model When $M_1 \le \mu < M_2 < T$

In order to obtain the total cost for this model, the purchasing cost, interest charges for the items kept in stock, and the interest earned should be taken into account. Since the supplier offers cash discount if payment is paid within M_1, there are two payment policies for the buyer. Either the payment is paid at time M_1 to receive the cash discount (Case 1) or the payment is paid at time M_2 so as not to receive the cash discount (Case 2). Then, these two cases will be discussed.

Case 1 (payment is made at time M_1). In this case, the following subcases should be considered.

Subcase 1.1 ($t_1 \le M_1 \le \mu < T$). The purchasing cost is

$$
\begin{aligned}
C_{A1,1}(t_1) &= C_p (1-r) \\
&\quad \times \Bigg[\int_0^{t_1} f(x) e^{(\theta-m(\xi))x} dx + f(\mu) \\
&\quad \times \int_{\mu}^{T} \beta(T-x)\, dx + \int_{t_1}^{\mu} f(x) \beta(T-x) dx\Bigg].
\end{aligned} \tag{26}
$$

The interest earned, $I_{T1,1}$, during the period of positive inventory level is

$$
\begin{aligned}
I_{T1,1}(t_1) &= pI_e \int_0^{t_1} \Bigg[\int_0^{t} f(x)\, dx\Bigg] dt + pI_e (M_1 - t_1) \int_0^{t_1} f(x)\, dx.
\end{aligned} \tag{27}
$$

Since $t_1 \le \mu$, the total cost in the time interval $[0, T]$ is calculated using (15), (26), and (27):

$$
TC_{1,1}(t_1) = C_1(t_1) + C_{A1,1}(t_1) - I_{T1,1}(t_1). \tag{28}
$$

Subcase 1.2 ($M_1 < t_1 \le \mu < T$). The purchasing cost is $C_{A1,1}$ (relation (26)).

The interest payable for the inventory not being sold after the due date M_1 is

$$
\begin{aligned}
P_{T2,1}(t_1) &= C_p (1-r) I_c \int_{M_1}^{t_1} e^{-(\theta-m(\xi))t} \Bigg[\int_t^{t_1} f(x) e^{(\theta-m(\xi))x} dx\Bigg] dt.
\end{aligned} \tag{29}
$$

The interest earned, $I_{T2,1}$, is

$$
I_{T2,1}(t_1) = pI_e \int_0^{M_1} \Bigg[\int_0^{t} f(x)\, dx\Bigg] dt. \tag{30}
$$

Since again $t_1 \le \mu$, the total cost over $[0, T]$ is calculated using the relations (15), (26), (29), and (30) and is

$$
TC_{1,2}(t_1) = C_1(t_1) + C_{A1,1}(t_1) + P_{T2,1}(t_1) - I_{T2,1}(t_1). \tag{31}
$$

Subcase 1.3 ($M_1 \le \mu \le t_1 \le T$). The purchasing cost is

$$
\begin{aligned}
C_{A2,1}(t_1) &= C_p (1-r) \\
&\quad \times \Bigg[\int_0^{\mu} f(x) e^{(\theta-m(\xi))x} dx + f(\mu) \\
&\quad \times \int_{\mu}^{t_1} e^{(\theta-m(\xi))x} dx + f(\mu) \int_{t_1}^{T} \beta(T-x)\, dx\Bigg].
\end{aligned} \tag{32}
$$

The interest earned, $I_{T3,1}$, is

$$
I_{T3,1}(t_1) = pI_e \int_0^{M_1} \Bigg[\int_0^{t} f(x)\, dx\Bigg] dt. \tag{33}
$$

The interest payable for the inventory not being sold after the due date M_1 is

$$
\begin{aligned}
P_{T3,1}(t_1) &= C_p (1-r) I_c \\
&\quad \times \Bigg[\int_{M_1}^{\mu} e^{-(\theta-m(\xi))t} \Bigg[\int_t^{\mu} f(x) e^{(\theta-m(\xi))x} dx\Bigg] dt \\
&\quad + f(\mu) \int_{M_1}^{\mu} e^{-(\theta-m(\xi))t} \Bigg[\int_{\mu}^{t_1} e^{(\theta-m(\xi))x} dx\Bigg] dt \\
&\quad + f(\mu) \int_{\mu}^{t_1} e^{-(\theta-m(\xi))t} \Bigg[\int_t^{t_1} e^{(\theta-m(\xi))x} dx\Bigg] dt\Bigg].
\end{aligned} \tag{34}
$$

Since $\mu < t_1$, the total cost over $(0, T)$ is again calculated from (25), (32)–(34) and is

$$TC_{1,3}(t_1) = C_2(t_1) + C_{A3,1}(t_1) + P_{T3,1}(t_1) - I_{T3,1}(t_1). \tag{35}$$

The results obtained lead to the following total cost function:

$$TC_1(t_1) = \begin{cases} TC_{1,1}(t_1), & t_1 \le M_1 \le \mu < T, \\ TC_{1,2}(t_1), & M_1 < t_1 \le \mu < T, \\ TC_{1,3}(t_1), & M_1 \le \mu \le t_1 \le T. \end{cases} \tag{36}$$

So the problem is

$$\min_{t_1} TC_1(t_1). \tag{37}$$

Its solution requires, separately, studying each of the three branches and then combining the results to obtain the optimal policy. It is easy to check that $TC_1(t_1)$ is continuous at the points M_1 and μ.

The first-order condition for a minimum of $TC_{1,1}(t_1)$ is

$$\begin{aligned} \frac{dTC_{1,1}(t_1)}{dt_1} &= \Bigg\{ \frac{\{c_1 + c_3(\theta - m(\xi))\}}{(\theta - m(\xi))} \left(e^{(\theta - m(\xi))t_1} - 1\right) \\ &\quad - c_2(T - t_1)\beta(T - t_1) \\ &\quad - c_4(1 - \beta(T - t_1)) \\ &\quad - pI_e(M_1 - t_1) + C_p(1 - r) \\ &\quad \times \left(e^{(\theta - m(\xi))t_1} - \beta(T - t_1)\right) \Bigg\} f(t_1) = 0. \end{aligned} \tag{38}$$

Since $dTC_{1,1}(0)/dt_1 < 0$ and $dTC_{1,1}(T)/dt_1 > 0$, (38) has at least one root. So if $t_{1,1}$ is the root of (38) and if $c_2\beta(x) + c_2x\beta'(x) - c_4\beta'(x) \ge 0$, then this corresponds to minimum as

$$\begin{aligned} &\left(\frac{d^2TC_{1,1}(t_1)}{dt_1^2}\right)_{t_1 = t_{1,1}} \\ &= \Big\{ \{c_1 + c_3(\theta - m(\xi))\} e^{(\theta - m(\xi))t_{1,1}} \\ &\quad + \{c_2\beta(T - t_{1,1}) + c_2(T - t_{1,1}) \\ &\qquad \times\beta'(T - t_{1,1}) - c_4\beta'(T - t_{1,1})\} \\ &\quad + pI_e + C_p(1 - r) \\ &\quad \times \left\{(\theta - m(\xi)) e^{(\theta - m(\xi))t_{1,1}} + \beta'(T - t_1)\right\}\Big\} f(t_{1,1}) > 0. \end{aligned} \tag{39}$$

Consequently, $t_{1,1}$ is the unique unconstrained minimum of $TC_{1,1}(t_1)$.

The first-order condition for a minimum of $TC_{1,2}(t_1)$ is

$$\begin{aligned} \frac{dTC_{1,2}(t_1)}{dt_1} &= \Bigg(\frac{\{c_1 + c_3(\theta - m(\xi))\}}{(\theta - m(\xi))} \left(e^{(\theta - m(\xi))t_1} - 1\right) \\ &\quad - c_2(T - t_1)\beta(T - t_1) - c_4\{1 - \beta(T - t_1)\} \\ &\quad + C_p(1 - r)\left(e^{(\theta - m(\xi))t_1} - \beta(T - t_1)\right) \\ &\quad + \frac{C_p(1 - r)I_c}{(\theta - m(\xi))}\left(e^{(\theta - m(\xi))(t_1 - M_1)} - 1\right)\Bigg) f(t_1) = 0. \end{aligned} \tag{40}$$

If $t_{1,2}$ is the root of (40) (this may or may not exist) and further if $c_2\beta(x) + c_2x\beta'(x) - c_4\beta'(x) \ge 0$, then

$$\begin{aligned} &\left(\frac{d^2TC_{1,2}(t_1)}{dt_1^2}\right)_{t_1 = t_{1,2}} \\ &= \Big(\{c_1 + c_3(\theta - m(\xi))\} e^{(\theta - m(\xi))t_{1,2}} \\ &\quad + C_p(1 - r) I_c e^{(\theta - m(\xi))(t_{1,2} - M_1)} \\ &\quad + \{c_2\beta(T - t_{1,2}) + c_2(T - t_{1,2}) \\ &\qquad \times \beta'(T - t_{1,2}) - c_4\beta'(T - t_{1,2})\} + C_p(1 - r) \\ &\quad \times \left\{(\theta - m(\xi)) e^{(\theta - m(\xi))t_{1,2}} + \beta'(T - t_{1,2})\right\}\Big) f(t_{1,2}) > 0, \end{aligned} \tag{41}$$

and this $t_{1,2}$ corresponds to unconstrained minimum of $TC_{1,2}(t_1)$.

The first-order condition for a minimum of $TC_{1,3}(t_1)$ is

$$\begin{aligned} \frac{dTC_{1,3}(t_1)}{dt_1} &= \Bigg(\frac{\{c_1 + c_3(\theta - m(\xi))\}}{(\theta - m(\xi))} \left(e^{(\theta - m(\xi))t_1} - 1\right) \\ &\quad - c_2(T - t_1)\beta(T - t_1) - c_4(1 - \beta(T - t_1)) \\ &\quad + C_p(1 - r)\left(e^{(\theta - m(\xi))t_1} - \beta(T - t_1)\right) \\ &\quad + \frac{C_p(1 - r)I_c}{(\theta - m(\xi))}\left(e^{(\theta - m(\xi))(t_1 - M_1)} - 1\right)\Bigg) f(\mu) = 0. \end{aligned} \tag{42}$$

If $t_{1,3}$ is the root of (42) (this may or may not exist) and $c_2\beta(x) + c_2 x\beta'(x) - c_4\beta'(x) \geq 0$, then

$$\left(\frac{d^2 TC_{1,3}(t_1)}{dt_1^2}\right)_{t_1=t_{1,3}} = \left(\{c_1 + c_3(\theta - m(\xi))\} e^{(\theta-m(\xi))t_{1,3}} + C_p(1-r) I_c e^{(\theta-m(\xi))(t_{1,3}-M_1)} + C_p(1-r)\left((\theta - m(\xi)) e^{(\theta-m(\xi))t_{1,3}} + \beta'(T - t_{1,3})\right) + \{c_2\beta(T-t_{1,3}) + c_2(T-t_{1,3})\beta'(T-t_{1,3}) - c_4\beta'(T-t_{1,3})\}\right) f(\mu) > 0. \quad (43)$$

This $t_{1,3}$ corresponds to unconstrained minimum of $TC_{1,3}(t_1)$.

Remark 1. The function $TC_1(t_1)$ is not differentiable in M_1.

Then, the following procedure summarizes the previous results for the determination of the optimal replenishment policy, when payment is made at time M_1.

Step 1. Find the global minimum of $TC_{1,1}(t_1)$, say $t^*_{1,1,M_1}$, as follows.

Substep 1.1. Compute $t_{1,1,M_1}$ from (38); if $t_{1,1,M_1} < M_1$, then set $t^*_{1,1,M_1} = t_{1,1,M_1}$ and compute $TC_{1,1}(t^*_{1,1,M_1})$; else go to Substep 1.2.

Substep 1.2. Find the $\min\{TC_{1,1}(0), TC_{1,1}(M_1)\}$ and accordingly set $t^*_{1,1,M_1}$.

Step 2. Find the global minimum of $TC_{1,2}(t_1)$, say $t^*_{1,2,M_1}$, as follows.

Substep 2.1. Compute $t_{1,2,M_1}$ from (40); if $M_1 < t_{1,2,M_1} < \mu$, then set $t^*_{1,2,M_1} = t_{1,2,M_1}$ and compute $TC_{1,2}(t^*_{1,2,M_1})$; else go to Substep 2.2.

Substep 2.2. Find the $\min\{TC_{1,2}(M_1), TC_{1,2}(\mu)\}$ and accordingly set $t^*_{1,2,M_1}$.

Step 3. Find the global minimum of $TC_{1,3}(t_1)$, say $t^*_{1,3,M_1}$, as follows.

Substep 3.1. Compute $t_{1,3,M_1}$ from (42); if $\mu < t_{1,3,M_1}$, then set $t^*_{1,3,M_1} = t_{1,3,M_1}$ and compute $TC_{1,3}(t^*_{1,3,M_1})$; else go to Substep 3.2.

Substep 3.2. Find the $\min\{TC_{1,3}(\mu), TC_{1,3}(T)\}$ and accordingly set $t^*_{1,3,M_1}$.

Step 4. Find the $\min\{TC_{1,1}(t^*_{1,1,M_1}), TC_{1,2}(t^*_{1,2,M_1}), TC_{1,3}(t^*_{1,3,M_1})\}$ and accordingly select the optimal value for t_1, say t^*_{1,M_1}, with optimal cost $TC_1(t^*_{1,M_1})$.

Case 2 (payment is made at time M_2). When the payment is made at time M_2 the following cases should be considered.

Subcase 2.1 ($t_1 \leq \mu < M_2 < T$). The purchasing cost is

$$C_{A1,2}(t_1) = \frac{C_{A1,1}(t_1)}{(1-r)}. \quad (44)$$

The interest earned during the period of positive inventory level is

$$I_{T1,2}(t_1) = pI_e \int_0^{t_1}\left[\int_0^t f(x)\,dx\right] dt + pI_e(M_2 - t_1)\int_0^{t_1} f(x)\,dx. \quad (45)$$

Since $t_1 \leq \mu$, the total cost in the time interval $[0, T]$ is calculated using (15), (44), and (45):

$$TC_{2,1}(t_1) = C_1(t_1) + C_{A1,2}(t_1) - I_{T1,2}(t_1). \quad (46)$$

Subcase 2.2 ($\mu < t_1 \leq M_2 < T$). The purchasing cost is

$$C_{A2,2}(t_1) = \frac{C_{A2,1}(t_1)}{(1-r)}. \quad (47)$$

The interest earned, $I_{T2,2}(t_1)$, is

$$I_{T2,2}(t_1) = pI_e\left[\int_0^{\mu}\left[\int_0^t f(x)\,dx\right] dt + \int_{\mu}^{M_2}\left[\int_0^{\mu} f(x)\,dx\right] dt + \int_{\mu}^{t_1}\left[\int_{\mu}^{t} f(\mu)\,dx\right] dt + \int_{t_1}^{M_2}\left[\int_{\mu}^{t_1} f(\mu)\,dx\right] dt\right]. \quad (48)$$

Since again $\mu \leq t_1$, the total cost over $[0, T]$ is calculated using the relations (25), (47), and (48) and is

$$TC_{2,2}(t_1) = C_2(t_1) + C_{A2,2}(t_1) - I_{T2,2}(t_1). \quad (49)$$

Subcase 2.3 ($\mu \leq M_2 \leq t_1 \leq T$). The purchasing cost is $C_{A2,2}(t_1)$.

The interest earned, $I_{T3,2}$, is

$$I_{T3,2}(t_1) = pI_e\left[\int_0^{\mu}\left[\int_0^t f(x)\,dx\right] dt + \int_{\mu}^{M_2}\left[\int_0^{\mu} f(x)\,dx\right] dt\right]. \quad (50)$$

The interest payable for the inventory not being sold after the due date M_2 is

$$P_{T3,2}(t_1) = C_p I_c f(\mu)\left[\int_{M_2}^{t_1} e^{-(\theta-m(\xi))t}\left[\int_t^{t_1} e^{(\theta-m(\xi))x}\,dx\right] dt\right]. \quad (51)$$

Since $\mu < t_1$, the total cost over $[0, T]$ is again calculated from (25), (47), (50), and (51) and is

$$TC_{2,3}(t_1) = C_2(t_1) + C_{A2,2}(t_1) + P_{T3,2}(t_1) - I_{T3,2}(t_1). \tag{52}$$

The results obtained lead to the following total cost function:

$$TC_2(t_1) = \begin{cases} TC_{2,1}(t_1), & t_1 \le \mu < M_2 < T, \\ TC_{2,2}(t_1), & \mu < t_1 \le M_2 < T, \\ TC_{2,3}(t_1), & \mu \le M_2 \le t_1 \le T. \end{cases} \tag{53}$$

So the problem is

$$\min_{t_1} TC_2(t_1). \tag{54}$$

Its solution, as in the previous case, requires, separately, studying each of the three branches and then combining the results to obtain the optimal policy. It is easy to check that $TC_2(t_1)$ is continuous at the points M_2 and μ.

The first-order condition for the minimum for $TC_{2,1}(t_1)$ is

$$\begin{aligned} \frac{dTC_{2,1}(t_1)}{dt_1} &= \left\{ \frac{\{c_1 + c_3(\theta - m(\xi))\}}{(\theta - m(\xi))} \left(e^{(\theta - m(\xi))t_1} - 1\right) \right. \\ &\quad - c_2(T - t_1)\beta(T - t_1) \\ &\quad - c_4(1 - \beta(T - t_1)) - pI_e(M_2 - t_1) \\ &\quad \left. + C_p\left\{e^{(\theta - m(\xi))t_1} - \beta(T - t_1)\right\} \right\} f(t_1) = 0. \end{aligned} \tag{55}$$

Since $dTC_{2,1}(0)/dt_1 < 0$ and $dTC_{2,1}(T)/dt_1 > 0$, (55) has at least one root. So, if $t_{1,1}$ is the root of (55), this corresponds to minimum if $c_2\beta(x) + c_2x\beta'(x) - c_4\beta'(x) \ge 0$ as

$$\begin{aligned} &\left(\frac{d^2TC_{2,1}(t_1)}{dt_1^2}\right)_{t_1 = t_{1,1}} \\ &= \left\{\{c_1 + c_3(\theta - m(\xi))\} e^{(\theta - m(\xi))t_{1,1}} \right. \\ &\quad + \left\{c_2\beta(T - t_{1,1}) + c_2(T - t_{1,1})\beta'(T - t_{1,1}) \right. \\ &\qquad \left. - c_4\beta'(T - t_{1,1})\right\} + pI_e \\ &\quad \left. + C_p\left\{(\theta - m(\xi)) e^{(\theta - m(\xi))t_{1,1}} + \beta'(T - t_1)\right\}\right\} f(t_{1,1}) > 0. \end{aligned} \tag{56}$$

So, $t_{1,1}$ is the unconstrained minimum of $TC_{2,1}(t_1)$.

The first-order condition for a minimum of $TC_{2,2}(t_1)$ is

$$\begin{aligned} &\frac{dTC_{2,2}(t_1)}{dt_1} \\ &= \left(\frac{\{c_1 + c_3(\theta - m(\xi))\}}{(\theta - m(\xi))} \left(e^{(\theta - m(\xi))t_1} - 1\right) \right. \\ &\quad - c_2(T - t_1)\beta(T - t_1) \\ &\quad - c_4(1 - \beta(T - t_1)) + C_p\left\{e^{(\theta - m(\xi))t_1} - \beta(T - t_1)\right\} \\ &\quad \left. - pI_e(M_2 - t_1) \right) f(\mu) = 0. \end{aligned} \tag{57}$$

If $t_{1,2}$ is the root of (57) (this may or may not exist), this corresponds to unconstrained minimum of $TC_{2,2}(t_1)$ if $c_2\beta(x) + c_2x\beta'(x) - c_4\beta'(x) \ge 0$ as

$$\begin{aligned} &\left(\frac{d^2TC_{2,2}(t_1)}{dt_1^2}\right)_{t_1 = t_{1,2}} \\ &= \left(\{c_1 + c_3(\theta - m(\xi))\} e^{(\theta - m(\xi))t_{1,2}} \right. \\ &\quad + \left\{c_2\beta(T - t_{1,2}) + c_2(T - t_{1,2}) \right. \\ &\qquad \left. \times \beta'(T - t_{1,2}) - c_4\beta'(T - t_{1,2})\right\} + pI_e \\ &\quad \left. + C_p\left\{(\theta - m(\xi)) e^{(\theta - m(\xi))t_{1,2}} + \beta'(T - t_{1,2})\right\}\right) f(\mu) > 0. \end{aligned} \tag{58}$$

The first-order condition for a minimum of $TC_{2,3}(t_1)$ is

$$\begin{aligned} &\frac{dTC_{2,3}(t_1)}{dt_1} \\ &= \left(\frac{\{c_1 + c_3(\theta - m(\xi))\}}{(\theta - m(\xi))} \left(e^{(\theta - m(\xi))t_1} - 1\right) \right. \\ &\quad - c_2(T - t_1)\beta(T - t_1) - c_4(1 - \beta(T - t_1)) \\ &\quad + C_p\left(e^{(\theta - m(\xi))t_1} - \beta(T - t_1)\right) \\ &\quad \left. + \frac{C_pI_c}{(\theta - m(\xi))}\left(e^{(\theta - m(\xi))(t_1 - M_2)} - 1\right) \right) f(\mu) = 0. \end{aligned} \tag{59}$$

If $t_{1,3}$ is a root of (59) (this may or may not exist) and this corresponds to unconstrained minimum of $TC_{2,3}(t_1)$

if $c_2\beta(x) + c_2 x\beta'(x) - c_4\beta'(x) \geq 0$ as

$$\left(\frac{d^2TC_{2,3}(t_1)}{dt_1^2}\right)_{t_1=t_{1,3}} = \left(\{c_1 + c_3(\theta - m(\xi))\} e^{(\theta-m(\xi))t_{1,3}} + \{c_2\beta(T - t_{1,3}) + c_2(T - t_{1,3}) \times \beta'(T - t_{1,3}) - c_4\beta'(T - t_{1,3})\} + C_p\{(\theta - m(\xi)) e^{(\theta-m(\xi))t_{1,3}} + \beta'(T - t_{1,3})\} + C_p I_c e^{(\theta-m(\xi))(t_{1,3}-M_2)}\right) f(\mu) > 0. \quad (60)$$

Remark 2. The function $TC_2(t_1)$ is not differentiable in M_2.

Then, the following procedure summarizes the previous results for the determination of the optimal replenishment policy, when payment is made at time M_2.

Step 1. Find the global minimum of $TC_{2,1}(t_1)$, say $t^*_{1,1,M_2}$, as follows.

Substep 1.1. Compute $t_{1,1,M_2}$ from (55); if $t_{1,1,M_2} < \mu$, then set $t^*_{1,1,M_2} = t_{1,1,M_2}$ and compute $TC_{2,1}(t^*_{1,1,M_2})$; else go to Substep 1.2.

Substep 1.2. Find the $\min\{TC_{2,1}(0), TC_{2,1}(\mu)\}$ and accordingly set $t^*_{1,1,M_2}$.

Step 2. Find the global minimum of $TC_{2,2}(t_1)$, say $t^*_{1,2,M_2}$, as follows.

Substep 2.1. Compute $t_{1,2,M_2}$ from (57); if $\mu < t_{1,2,M_2} < M_2$, then set $t^*_{1,2,M_2} = t_{1,2,M_2}$ and compute $TC_{2,2}(t^*_{1,2,M_2})$; else go to Substep 2.2.

Substep 2.2. Find the $\min\{TC_{2,2}(\mu), TC_{2,2}(M_2)\}$ and accordingly set $t^*_{1,2,M_2}$.

Step 3. Find the global minimum of $TC_{2,3}(t_1)$, say $t^*_{1,3,M_2}$, as follows.

Substep 3.1. Compute $t_{1,3,M_2}$ from (59); if $M_2 < t_{1,3,M_2} < T$, then set $t^*_{1,3,M_2} = t_{1,3,M_2}$ and compute $TC_{2,3}(t^*_{1,3,M_2})$; else go to Substep 3.2.

Substep 3.2. Find the $\min\{TC_{2,3}(M_2), TC_{2,3}(T)\}$ and accordingly set $t^*_{1,3,M_2}$.

Step 4. Find the $\min\{TC_{2,1}(t^*_{1,1,M_2}), TC_{2,2}(t^*_{1,2,M_2}), TC_{2,3}(t^*_{1,3,M_2})\}$ and accordingly select the optimal value for t_1 say t^*_{1,M_2}, with optimal cost $TC_2(t^*_{1,M_2})$.

Finally to find the overall optimum t_1 for the problem under consideration, the results obtained for the two presented cases (i.e., payment is made at M_1 and payment is made at M_2) are combined; that is, find $\min\{TC_1(t^*_{1,M_1}), TC_2(t^*_{1,M_2})\}$ and accordingly select the optimal value t^*_1.

5. Numerical Examples and Sensitivity Analysis

In this section, a numerical example is provided to illustrate the results obtained in previous sections. In addition, a sensitivity analysis, with respect to system parameters, is carried out. Here, $m(\xi)$ is a function of the preservation technology cost ξ such that $m(\xi) = \theta(1 - e^{-a\xi})$, $a \geq 0$, and a is the simulation coefficient representing the percentage increase in $m(\xi)$ per euro increase in ξ, which means $m(\xi)$ is an increasing function bounded above by θ.

The input parameters are $c_1 = 3$€ per unit per unit time, $c_2 = 15$€ per unit per unit time, $c_3 = 5$€ per unit, $c_4 = 20$€ per unit per unit time, $r = 0.005$, $\mu = 0.3$ years, $\theta = 0.001$, $a = 0.001$, $\xi = 20$€, $T = 0.5$ years, $f(t) = 3e^{4.5t}$ and $\beta(x) = e^{-0.2x}$, $M_1 = 0.13$ years, $M_2 = 0.43$ years, $p = 15$, $C_p = 10$€, $I_e = 0.12$, and $I_c = 0.15$.

5.1. The Payment Is Made at M_1. From (38), $t_{1,1,M_1} = 0.398924$, which is not feasible as $t_{1,1,M_1} > M_1$. Since $TC_{1,1}(0) = 75.469$ and $TC_{1,1}(M_1) = 71.9633$, it follows that $t^*_{1,1,M_1} = M_1$. From (40), $t_{1,2,M_1} = 0.40288$, which is not valid again as $t_{1,2,M_1} > \mu$. Since $TC_{1,1}(M_1) = TC_{1,2}(M_1) = 71.9633$ and $TC_{1,2}(\mu) = 67.0128$, the optimal value for $t^*_{1,2,M_1} = \mu$. From (42), $t_{1,3,M_1} = 0.412557$; this value for t_1 is valid as $\mu < t_{1,3,M_1} < T$, so $t^*_{1,3,M_1} = t_{1,3,M_1}$ and $TC_{1,3}(t^*_{1,3,M_1}) = 65.5819$.

Finally, $TC_1(t^*_{1,M_1}) = \min\{TC_{1,1}(M_1), TC_{1,2}(\mu), TC_{1,3}(t^*_{1,3,M_1})\} = 65.5819$, and consequently $t^*_{1,M_1} = 0.412557$.

5.2. The Payment Is Made at M_2. From (55), $t_{1,1,M_2} = 0.424279$ which is not feasible as $t_{1,1,M_2} > \mu$. Since $TC_{2,1}(0) = 75.6719$, $TC_{2,1}(\mu) = 66.2333$, it follows that $t^*_{1,1,M_2} = \mu$. From (57), $t_{1,2,M_2} = 0.424279$ which is valid again as $\mu < t_{1,2} < M_2$ so $t^*_{1,2,M_2} = 0.424279$ and $TC_{2,2}(t^*_{1,2,M_2}) = 64.3494$. From (59), $t_{1,3,M_2} = 0.424197$; this value for t_1 is not valid as $t_{1,3} < M_2$. Since $TC_{2,3}(M_2) = 64.5295$, $TC_{2,3}(T) = 65.2301$ so $t^*_{1,3,M_2} = M_2$ and $TC_{2,3}(t^*_{1,3,M_2}) = 64.5295$.

Finally, $TC_2(t^*_{1,M_2}) = \min\{TC_{2,1}(\mu), TC_{2,2}(t^*_{1,2,M_2}), TC_{2,3}(M_2)\} = 64.3494$, and consequently $t^*_{1,M_2} = 0.424279$.

So, as $TC(t^*_1) = \min\{TC_1(t^*_{1,M_1}), TC_2(t^*_{1,M_2})\} = 64.3494$, the optimal t_1 is $t^*_1 = t^*_{1,M_2} = 0.424279$, which leads to a payment at M_2.

6. Sensitivity Analysis

Using the data of the previous example, a sensitivity analysis is carried out to explore the effect of change on the basic model's parameters to the optimal policy (i.e., t_1 time of payment and optimal total cost). The results are presented in Tables 1 and 2, and some observations are summarized as follows.

Table 1: The effect of changing the parameter (i) while keeping all other parameters unchanged.

Parameter (i)	Percentage of changes (%)	t_1^*	$TC(t_1^*)$
	−20	0.436535	63.8099
c_1	−10	0.430325	64.0842
	+10	0.418390	64.6058
	+20	0.412653	64.8538
	−20	0.412116	64.2352
c_2	−10	0.418648	64.2965
	+10	0.429184	64.3955
	+20	0.433494	64.4360
	−20	0.424279	57.5810
μ	−10	0.424279	60.9523
	+10	0.424279	67.7020
	+20	0.424279	70.9180
	−20	0.424299	64.3486
c_3	−10	0.424289	64.3490
	+10	0.424269	64.3499
	+20	0.424260	64.3503
	−20	0.421352	64.3220
c_4	−10	0.422843	64.3360
	+10	0.425662	64.3624
	+20	0.426995	64.3749
	−20	0.424340	64.3468
θ	−10	0.424310	64.3481
	+10	0.424249	64.3508
	+20	0.424218	64.3521
	−20	0.424278	60.3495
ξ	−10	0.424278	62.3495
	+10	0.424280	66.3494
	+20	0.424280	68.3494
	−20	0.424181	64.5541
I_e	−10	0.424230	64.4518
	+10	0.424327	64.2471
	+20	0.424374	64.1448
	−20	0.424181	64.5541
p	−10	0.424230	64.4518
	+10	0.424327	64.2471
	+20	0.424374	64.1448
	−20	0.425700	55.9219
C_p	−10	0.424996	60.1357
	+10	0.423548	68.5630
	+20	0.422804	72.7765

Table 2: The effect of simulation coefficient (a) while keeping all other parameters unchanged.

Parameter (i)	Changing value	t_1^*	$TC(t_1^*)$
	0.0005	0.424276	64.3496
	0.005	0.424303	64.3484
a	0.01	0.424329	64.3473
	0.02	0.424375	64.3453
	0.05	0.424470	64.3412

(1) It is observed that as the holding cost c_1 decreases or increases the optimal time t_1^* increases or decreases, and the optimal total cost $TC(t_1^*)$ decreases or increases, respectively.

(2) It is seen that as the shortage cost c_2 decreases or increases both the optimal time t_1^* and the optimal total cost $TC(t_1^*)$ decrease or increase, respectively.

(3) It is examined that as the time (at which demand becomes constant) μ decreases or increases there is no effect on the optimal time t_1^* but the optimal total cost $TC(t_1^*)$ decreases or increases, respectively.

(4) It is observed that as the deterioration cost c_3 decreases or increases the optimal time t_1^* slightly increases or decreases, and the optimal total cost $TC(t_1^*)$ also slightly decreases or increases, respectively.

(5) It is seen that as the lost sales cost c_4 decrease or increase both the optimal time t_1^* and the optimal total cost $TC(t_1^*)$ slightly decrease or increase, respectively.

(6) It is seen that as the decay rate θ decreases or increases the optimal time t_1^* increases or decreases and the optimal total cost $TC(t_1^*)$ slightly decreases or increases, respectively.

(7) It is observed that as the preservation cost ξ decreases or increases the optimal time t_1^* and the optimal total cost $TC(t_1^*)$ decrease or increase, respectively.

(8) It is noticed that as the rate of interest earned I_e decreases or increases the optimal time t_1^* decreases or increases, and the optimal total cost $TC(t_1^*)$ slightly increases or decreases, respectively.

(9) It is noticed that as the selling price p decreases or increases the optimal time t_1^* decreases or increases, but the optimal total cost $TC(t_1^*)$ increases or decreases, respectively.

(10) It is seen that as the purchasing cost C_p decreases or increases the optimal time t_1^* increases or decreases, and the optimal total cost $TC(t_1^*)$ decreases or increases, respectively.

(11) It is observed that as the preservation parameter a decreases or increases the optimal time t_1^* decreases or increases, but the optimal total cost $TC(t_1^*)$ increases or decreases, respectively.

7. Concluding Remarks

In this paper, we have developed an inventory model with ramp type demand rate, partially backlogged shortages, preservation technology, and two-level trade credit. The model is discussed under the two cases (1) when the payment is made at time M_1 to get the price discount and (2) when the payment is made at time M_2 with no price discount. The effect of preservation technology is also taken into consideration throughout the model. The presented model is illustrated through numerical experiments with sensitivity. From the sensitivity table, it is observed that the changes in the parameters preservation technology cost (ξ), time at which demand becomes constant (μ), and purchasing cost (C_p) have greater effect on the system than the other parameters of the system. The convexity of the cost function is also exposed analytically.

This model could be inclusive taking into consideration numerous replenishment cycles throughout the planning horizon.

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