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Research Article

Cost Analysis for a Supplier in an Inflationary Environment with Stock Dependent Demand Rate for Perishable Items

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The present study is concerned with the cost modeling of an inventory system with perishable multi-items having stock dependent demand rates under an inflationary environment of the market. The concept of permissible delay is taken into account. The study provides the cost analysis of inventory system under the decision criteria of time value of money, inflation, deterioration, and stock dependent demand. Numerical illustrations are derived from the quantitative model to validate the results. The cost of inventory and optimal time are also computed by varying different system parameters. The comparison of these results is facilitated by computing the results with neurofuzzy results.

1. Introduction

Inflation is a very common scenario of a dynamic market which affects a common man and the decision makers equally. The term inflation refers to the increment in the rates of the goods. Most of the inventory models, developed so far, did not include the inflation and time value of the money as parameters of the system. But, during the past two decades, a sudden downfall in the market caused highly inflated rates and decision makers felt the need of considering the inflation an integrated part of an inventory model. There are many items which are subject to decay with respect to time. The concept of deterioration has been incorporated by some researchers in different frameworks (cf. [1–3]). The management of inventory emphasizing on time dependent deterioration with salvage value was discussed by Mishra and Shah [4]. Jain et al. [5] considered the concept of deterioration to develop a deterministic production inventory model with time-varying demand. Manna et al. [6] developed an EOQ model for noninstantaneous deteriorating items. The concept of exponential deterioration was considered by Mahata [7] to develop an EPQ-based inventory model. Xiao and Xu [8] discussed a supply chain with deteriorating items for a vendor managed inventory. Wang et al. [9] optimized a seller's credit period in a supply chain for deteriorating items.

Some notable works in the direction of inventory models with time value of money along with inflation are due to Bose et al. [10], Moon and Lee [11], Chang [12]. Singh and Jain [13] developed a model to study the supplier credits in an inflationary environment when reserve money was available. An inventory model by considering the concepts of inflation, deterioration, and permissible delay in the payments was studied by Jain and Chauhan [14]. Sarkar and Moon [15] analyzed an EPQ model by incorporating the effect of inflation for an imperfect production system. Lubik and Teo [16] presented their views on inflation dynamics. Recently, Gilding [17] discussed inflation to analyze the optimal inventory replenishment schedule.

In various inventory models, it is assumed that the demand rate is independent of the internal factors. But, in real world problems, the demand may be influenced by many internal factors such as price, availability, and season. This phenomenon is known as elasticity of the demand. A few researchers have considered stock dependent demand in their works [18–21]. Stock along with price sensitive demand rates was considered by Jain et al. [22] to develop an EPQ model with shortages. Some notable aspects related to EOQ model with stock and price sensitive demand was explored by Mo et al. [23]. The inventory model for deteriorating items was discussed by Ouyang et al. [24], Min et al. [25], and Soni [26] by considering stock dependent demand. Recently, Yang [27] analyzed an inventory model with stock dependent demand rate and holding cost.

In today's business tractions, it is more and more common to see that the buyers are allowed some grace period before they settle the account with the supplier instead of paying for the product just after receiving it. This grace period is known as credit period. After crossing the limit of the grace period, the buyer is likely to pay an interest also. Some notable works in this direction are due to Mandal and Phaujdar [28], Hwang and Shinn [29], Chung and Huang [30], and Chen and Kang [31]. Singhal et al. [32] incorporated the concept of delay in payments to obtain an optimal pricing and ordering policy of retailers incorporating variable holding cost. Liang and Zhou [33] considered the concept of permissible delay in payment for an inventory model with deteriorating items. The joint control of inventory with permissible delay in payments was obtained by Maihami and Abadi [34]. Soni [35] presented optimal replenishment policies for the noninstantaneous deteriorating items considering permissible delay in payment. Recently, a two-warehouse inventory model with permissible delay in payment was developed by Bhunia et al. [36].

The neurofuzzy technique came into existence due to the combination of two commonly used soft computing approaches: fuzzy logic and neural network. Recently, a brief review on applications of neurofuzzy systems was given by Kar et al. [37]. The technique is an emerging powerful soft computing technique which has successfully covered many areas including the performance modeling of electronic goods, hardware and software systems, automobiles, manufacturing/production systems, telecommunication systems, and many more. This hybrid soft computing technique plays an important role for the performance modeling of inventory system but only a very few researchers have employed this technique in the field of inventory control till now. Gumus and Guneri [38] discussed stochastic and fuzzy supply chains. Further, Gumus et al. [39] employed neurofuzzy technique to develop a new methodology for multiechelon inventory system in stochastic and neurofuzzy environments.

This present investigation deals with the credits of the supplier in an inflationary environment when the demand rate for the perishable items is stock dependent. The shortages cause an indispensable loss to the suppliers. In this study, we aim at determining the optimal time when the shortages start so that the undesirable shortages can be avoided within time. The phenomenon of credit period in an inflationary environment for multi-items inventory system makes our study different from others. We have employed neurofuzzy technique to compare our analytical results which is one of the special features of our study. The rest of the paper is organized as follows. In Section 2, model description along with the notations and assumptions is given. The formulation of cost functions along with the computational procedure for obtaining the optimal time and minimum cost is given in Section 3. In Section 4, numerical results are presented to validate the computational procedure. Finally, conclusion is drawn in Section 5.

The noble feature of our investigation is to develop the inventory model with deterioration and stock dependent demand to facilitate the cost and optimal policy by incorporating the concepts of time value of money along with inflation. The concept of controlled deterioration can be realized in many electronic and machining systems. It is noticed that the deterioration can be controlled up to some extent with some preventive maintenance (e.g., by controlling the temperature/humidity of ware houses) or some preservation technology (e.g., polishing and oiling) in case of machining systems. The controlled deterioration rate included in the present inventory model makes our study different from the existing research.

2. Model Description

Consider an inventory model with a single supplier who can supply several items to satisfy the customers' demands. The demand of nth items is directly proportional to the available stock of each type of item. All the items are prone to deteriorate with a constant rate but the deterioration can be controlled by some preservation technology. The planning horizon is finite and the shortages are completely backlogged. In order to increase the business, the customers are allowed to take a grace period for payment.

2.1. Notations. In order to formulate the mathematical model of the present problem, we use the following notations:

α_n: The initial demand for the nth item

β_n: Constant of the inventory at time t for the nth item

$Q_n(t)$: The stock available for the nth item at time t

m: Total number of items

f: Inflation factor

r: Discount rate representing the time value of money

R: Present value of the nominal inflation; that is, $R = f - r$

I_{en}: Interest rate earned at time t for the nth item per rupee/unit time

i_{en}: $I_{en} - r$ where I_{en} is nominal interest at time $t = 0$ for nth item

$I_{pn}(t)$: Interest rate paid at time t for the nth item per rupee/unit time

i_p: $I_{pn} - r$ where I_{pn} is nominal interest paid at time $t = 0$ for the nth item

I_{Tn}: Total interest earned per cycle with inflation for the nth item

P_{Tn}: Total interest paid per cycle with inflation for the nth item

i_n: Inventory carrying rate for the nth item

A_n: Cost of ordering per order for nth item

T: Length of inventory cycle

T_1: Length of period with positive stock

θ_n: Rate of deterioration of the nth item per unit time

$D_n(t)$: Amount of deterioration of the nth item per cycle

ε: Deterioration control rate

M_n: Permissible delay in settling the account for the nth item

c_n: Unit cost per item at time $t = 0$ for the nth item

c_{bn}: Backorder cost at time $t = 0$ for the nth item

C_{bn}: Present value of inflated backorder cost c_b for the nth item

C_{Dn}: Total cost of deterioration for the nth item per cycle per unit

C_{Hn}: Total holding cost per cycle with inflation for the nth item.

2.2. Assumptions. In this subsection, the instantaneous differential equations for the present problem are formulated and solved. For the sake of formulation, some assumptions are considered which are as follows.

(i) The inventory model involves multiple (m) items.

(ii) There is a single supplier in the market.

(iii) The demand of the nth ($n = 1, 2, 3, \ldots, m$) item from the supplier is deterministic and directly proportional to the available stock of the nth item in hand of the supplier. Then

$$\alpha_n + \beta_n Q_n(t); \quad n = 1, 2, \ldots, m, \text{ where } \alpha_n > 0,\ 0 < \beta_n < 1. \tag{1}$$

(iv) Shortages are allowed and completely backlogged.

(v) Time horizon is finite.

(vi) Backorder starts after time T_1 which is a decision variable.

(vii) The items deteriorate at a constant rate. The deteriorated items can be neither repaired nor replaced. The items will be withdrawn immediately from the warehouse by the suppliers as they are found to be deteriorated.

(viii) The deterioration of an item can be reduced by using preservation technology.

(ix) $Q_m(Q_0)$ denotes the maximum (minimum) inventory level.

Figure 1 represents the inventory model with shortage. The time horizon is finite denoted by T and the shortages start at time T_1.

The following differential equations represent the instantaneous state of the inventory at any instant of time t for the nth ($n = 1, 2, 3, \ldots, m$) item:

$$\frac{dQ_n(t)}{dt} + (\theta_n - \varepsilon) I_n(t) = -(\alpha_n + \beta_n Q_n(t)); \quad 0 \le t \le T_1. \tag{2}$$

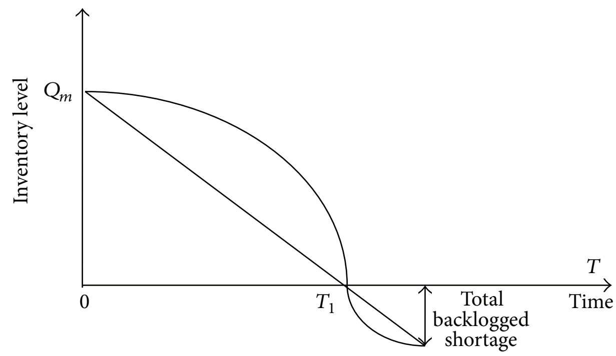

Figure 1: Graphical representation of the inventory system.

The boundary conditions are $Q_n(t) = Q_0$ at time $t = 0$ and $Q_n(t) = Q_0$ at time $t = T_1$ which yield

$$\begin{aligned} Q_0 &= \frac{\alpha_n}{\beta_n + (\theta_n - \varepsilon)} \left(-1 + e^{(\beta_n + (\theta_n - \varepsilon))T_1}\right); \\ Q_n(t) &= \frac{\alpha_n}{\beta_n + (\theta_n - \varepsilon)} \left(-1 + e^{(\beta_n + (\theta_n - \varepsilon))(T_1 - t)}\right); \quad 0 \le t \le T_1. \end{aligned} \tag{3}$$

Amount of the nth item that deteriorates during one cycle is given by

$$D_n(t) = Q_0 - \int_0^{T_1} (\alpha_n + \beta_n Q_n(t))\, dt. \tag{4}$$

It gives

$$D_n(t) = \frac{(\theta_n - \varepsilon)\alpha_n}{(\beta_n + (\theta_n - \varepsilon))} \times \left(\frac{e^{(\beta_n + (\theta_n - \varepsilon))T_1}}{(\beta_n + (\theta_n - \varepsilon))} - \frac{1}{(\beta_n + (\theta_n - \varepsilon))} - T_1\right). \tag{5}$$

For inflation rate f, the continuous time inflation factor for the time period is e^{ft} which means that the nth item costs *Rs.* c_n at time $t = 0$ will cost $(c_n e^{ft})$ at time t. For a discount rate r, representing the time value of money, the present value of an item at time t is $(c_n e^{-rt})$. Hence the present value of the inflated price of an item at time $t = 0$ is given by

$$c_0 = c_n e^{(f-r)t} = c_n e^{Rt}; \quad R = f - r. \tag{6}$$

Similarly, the present value of the inflated backorder cost c_b for nth item is given by C_n^b where

$$C_n^b = c_{bn} e^{(f-r)t} = c_{bn} e^{Rt}. \tag{7}$$

There are two distinct cases in this type of inventory models:

(1) payment at or before the total depletion of inventory ($M \leq T_1 < T$),

(2) payment after depletion ($T_1 < M$).

Case 1 (payment at or before the total depletion of inventory). Now we obtain various costs involved in the system if the payment is made at or before the total depletion of the inventory.

(i) The Fixed Ordering Cost for the nth Item. Consider

$$A_n Rs/\text{order}.$$

As the ordering is made only once at time $t = 0$, the inflation does not affect the ordering cost.

(ii) The Deterioration Cost for the nth Item. Since the initial inventory Q_0 was purchased at the rate c_n without inflation and was sold at a rate c_0 with inflation during the time period $(0, T_1)$, the deterioration cost for nth item is

$$C_{Dn} = c_n Q_0 - \int_0^{T_1} c_0 \left(\alpha_n + \beta_n Q_n(t)\right) \tag{8}$$

which means

$$\begin{aligned} C_{Dn} &= \frac{c_n \alpha}{\beta_n + (\theta_n - \varepsilon)} \left(-1 + e^{(\beta_n + (\theta_n - \varepsilon))T_1}\right) \\ &\quad - \int_0^{T_1} c_n e^{Rt} \left(\alpha_n + \beta_n Q_n(t)\right) \\ C_{Dn} &= \frac{c_n \alpha_n}{\beta_n + (\theta_n - \varepsilon)} \left(-1 + \frac{(\theta_n - \varepsilon)}{R}\right) \\ &\quad \times \left(1 + \frac{R e^{(\beta_n + (\theta_n - \varepsilon))}}{\beta_n + (\theta_n - \varepsilon) - R}\right) - \frac{c_n \alpha_n e^{Rt_1}}{\beta_n + (\theta_n - \varepsilon)} \\ &\quad \times \left(\frac{\beta_n + (\theta_n - \varepsilon)}{R} - \frac{\beta_n}{\beta_n + (\theta_n - \varepsilon) - R}\right). \end{aligned} \tag{9}$$

(iii) The Holding Cost under Inflation for nth Item. It is given by

$$\begin{aligned} C_{Hn} &= i \int_0^{T_1} c_0 Q_n(t)\, dt \\ C_{Hn} &= \frac{i\alpha_n}{\beta_n + (\theta_n - \varepsilon)} \int_0^{T_1} c_n e^{Rt} \left(e^{(\beta_n + (\theta_n - \varepsilon))(T_1 - t)} - 1\right) dt \\ C_{Hn} &= \frac{i\alpha_n c}{\beta_n + (\theta_n - \varepsilon)} \left(\frac{e^{RT_1}}{R - (\beta_n + (\theta_n - \varepsilon))} - \frac{e^{RT_1}}{R}\right. \\ &\quad \left. - \frac{e^{(\beta_n + (\theta_n - \varepsilon))T_1}}{R - (\beta_n + (\theta_n - \varepsilon))} + \frac{1}{R}\right). \end{aligned} \tag{10}$$

(iv) The Interest Payable per Cycle. The interest payable rate at time t is $(e^{I_{pn}(t)} - 1)$. Therefore the interest payable per cycle for the inventory of nth item not sold after the due date M is given by

$$\begin{aligned} P_{Tn} &= c_n \int_M^{T_1} \left(e^{I_{pn}(t)} - 1\right) Q_n(t)\, dt; \\ &\quad \text{as } Q_n(t) = 0 \text{ for } T_1 \leq t \leq T \\ P_{Tn} &= \frac{c_n \alpha_n}{\beta_n + (\theta_n - \varepsilon)} \\ &\quad \times \int_M^{T_1} \left(e^{I_{p_n}(t)} - 1\right) \left(e^{(\beta_n + (\theta_n - \varepsilon))(T_1 - t)} - 1\right) dt \\ P_{Tn} &= \frac{c_n \alpha_n}{\beta_n + (\theta_n - \varepsilon)} \left(\frac{e^{i_P}}{i_P - (\beta_n + (\theta_n - \varepsilon))}\right. \\ &\quad + \frac{1}{\beta_n + (\theta_n - \varepsilon)} - \frac{e^{i_P T_1}}{i_P} + T_1 \\ &\quad - \frac{e^{(\beta_n + (\theta_n - \varepsilon))(T_1 - M) + i_P M}}{i_P - (\beta_n + (\theta_n - \varepsilon))} \\ &\quad \left. - \frac{e^{(\beta_n + (\theta_n - \varepsilon))(T_1 - M)}}{\beta_n + (\theta_n - \varepsilon)} + \frac{e^{i_P M}}{i_P} - M\right). \end{aligned} \tag{11}$$

The present value of the interest earned at time t is given by $I_e(t) = (e^{I_e t} - 1)e^{-rt}$. Consider inflated unit cost at time t to be $c_0 = c_n e^{Rt}$. The interest earned per cycle from nth item, I_{Tn}, is the interest earned up to time T_1 and it is given by

$$\begin{aligned} I_{Tn} &= c_0 \int_0^{T_1} I_e(t) \left(\alpha_i + \beta_i I_k(t)\right) dt \\ I_{Tn} &= c_n \int_0^{T_1} \left(e^{I_e(t)} - 1\right) e^{(R - r)t} t \\ &\quad \times \left(\alpha_n + \frac{\alpha_n \beta_n}{\beta_n + (\theta_n - \varepsilon)}\right. \\ &\quad \left. \times \left(-1 + e^{(\beta_n + (\theta_n - \varepsilon))(T_1 - t)}\right)\right) dt \\ I_{Tn} &= \alpha_n c_n \left(\frac{\theta_n - \varepsilon}{\beta_n + (\theta_n - \varepsilon)}\right) \\ &\quad \times \left(\frac{T_1 e^{(R + i_e)T_1}}{(R + i_e)} + \frac{e^{(R + i_e)T_1}}{(R + i_e)^2} - \frac{T_1 e^{(R - r)T_1}}{(R - r)} + \frac{e^{(R - r)T_1}}{(R - r)^2}\right. \\ &\quad \left. + \frac{1}{(R + i_e)^2} - \frac{1}{(R - r)^2}\right) \end{aligned}$$

$$
\begin{aligned}
&+\left(\frac{\alpha_n c_n \beta_n}{\beta_n+(\theta_n-\varepsilon)}\right)\left(\frac{T_1 e^{(R+i_e)T_1}}{(i_e-\beta_n-(\theta_n-\varepsilon))}\right. \\
&\qquad - \frac{T_1 e^{(\beta+\theta_e)T_1+R}}{(i_e-\beta_n-(\theta_n-\varepsilon))} \\
&\qquad - \frac{e^{(R+i_e)T_1}}{(i_e-\beta_n-(\theta_n-\varepsilon))^2} \\
&\qquad + \frac{e^{(\beta+\theta_e)T_1+R}}{(i_e-\beta_n-(\theta_n-\varepsilon))^2} \\
&\qquad - \frac{T_1 e^{(R-r_e)T_1}}{(R-r-\beta_n+(\theta_n-\varepsilon))} \\
&\qquad \left. + \frac{T_1 e^{(R-r_e)T_1}}{(R-r-\beta_n+(\theta_n-\varepsilon))^2}\right).
\end{aligned} \tag{12}
$$

(v) The Backorder Cost per Cycle under Inflation. It is

$$C_{bn} = c_0 \int_0^{T-T_1} c_{bn} e^{R(T_1+t)} \left(\alpha_n + \beta_n Q_n(t)\right) dt. \tag{13}$$

The shortage starts at time T_1 and ends at the end of the cycle, that is, at

$$
\begin{aligned}
C_{bn} = {} & \alpha_n c_{bn}\left(\frac{\theta_n}{\beta_n+(\theta_n-\varepsilon)}\right)\left(\frac{(T-T_1)e^{RT}}{R} - \frac{e^{RT}}{R^2} + \frac{e^{RT_1}}{R^2}\right) \\
& + \left(\frac{\alpha_n c_{bn}\beta_n}{\beta_n+(\theta_n-\varepsilon)}\right)\left(\frac{(T-T_1)e^{RT+\beta_n+(\theta_n-\varepsilon)(2T_1-T)}}{(R-\beta_n-(\theta_n-\varepsilon))}\right. \\
&\qquad - \frac{(T-T_1)e^{RT+\beta_n+(\theta_n-\varepsilon)(2T_1-T)}}{(R-\beta_n-(\theta_n-\varepsilon))^2} \\
&\qquad \left. + \frac{e^{(R+\theta_n-\varepsilon)T_1}}{(R-\beta_n-(\theta_n-\varepsilon))^2}\right).
\end{aligned} \tag{14}
$$

(vi) Total Variable Cost Function. It is

$$C_{VT}(T_1,T) = \sum_{n=1}^{m}\left(A_n + C_{Dn} + C_{Hn} + P_{Tn} - I_{Tn} + C_{bn}\right), \tag{15}$$

where T_1 and T are considered the decision variables. Consider

$$
\begin{aligned}
& C_{VT}(T_1,T) \\
& = \sum_{n=1}^{m}\left(A_n + \frac{c_n\alpha_n}{\beta_n+(\theta_n-\varepsilon)}\left(-1+\frac{(\theta_n-\varepsilon)}{R}\right)\right. \\
&\quad \times\left(1+\frac{Re^{(\beta_n+(\theta_n-\varepsilon))}}{(\beta_n+(\theta_n-\varepsilon))-R}\right) - \frac{c_n\alpha_n e^{Rt_1}}{(\beta_n+(\theta_n-\varepsilon))} \\
&\quad + \left(\frac{(\theta_n-\varepsilon)}{R} - \frac{\beta_n}{(\beta_n+(\theta_n-\varepsilon))-R}\right) \\
&\quad \times \frac{i\alpha_n c}{(\beta_n+(\theta_n-\varepsilon))} \\
&\quad \times\left(\frac{e^{RT_1}}{R-(\beta_n+(\theta_n-\varepsilon))} - \frac{e^{RT_1}}{R}\right. \\
&\qquad \left. - \frac{e^{(\beta_n+(\theta_n-\varepsilon))T_1}}{R-(\beta_n+(\theta_n-\varepsilon))} + \frac{1}{R}\right) \\
&\quad + \frac{c\alpha_n}{(\beta_n+(\theta_n-\varepsilon))} \\
&\quad \times\left(-\frac{e^{RT_1}}{R} + \frac{1}{R} - \frac{e^{(\beta_n+(\theta_n-\varepsilon))T_1}}{R-(\beta_n+(\theta_n-\varepsilon))}\right. \\
&\qquad - \frac{e^{i_P}}{i_P-(\beta_n+(\theta_n-\varepsilon))} + \frac{1}{(\beta_n+(\theta_n-\varepsilon))} \\
&\qquad - \frac{e^{i_P T_1}}{i_P} + T_1 - \frac{e^{(\beta_n+(\theta_n-\varepsilon))(T_1-M)+i_P M}}{i_P-(\beta_n+(\theta_n-\varepsilon))} \\
&\qquad \left. - \frac{e^{(\beta_i+(\theta_i-\varepsilon))(T_1-M)}}{(\beta_i+(\theta_i-\varepsilon))} + \frac{e^{I_P M}}{I_P} - M\right) \\
&\quad - \alpha_i c\left(\frac{(\theta_i-\varepsilon)}{(\beta_i+(\theta_i-\varepsilon))}\right) \\
&\quad \times\left(\frac{T_1 e^{(R+i_e)T_1}}{(R+i_e)} + \frac{e^{(R+i_e)T_1}}{(R+i_e)^2} - \frac{T_1 e^{(R-r)T_1}}{(R-r)} + \frac{e^{(R-r)T_1}}{(R-r)^2}\right. \\
&\qquad \left. + \frac{1}{(R+i_e)^2} - \frac{1}{(R-r)^2}\right) - \left(\frac{\alpha_n c_n \beta_n}{(\beta_n+(\theta_n-\varepsilon))}\right) \\
&\quad - \left(\frac{T_1 e^{(R+i_e)T_1}}{(i_e-(\beta_i+(\theta_i-\varepsilon)))} - \frac{T_1 e^{((\beta_i+(\theta_i-\varepsilon)))T_1+R}}{(i_e-(\beta_i+(\theta_i-\varepsilon)))}\right. \\
&\qquad - \frac{e^{(R+i_e)T_1}}{(i_e-(\beta_i+(\theta_i-\varepsilon)))^2} \\
&\qquad + \frac{e^{((\beta_n+(\theta_n-\varepsilon)))T_1+R}}{(i_e-(\beta_n+(\theta_n-\varepsilon)))^2} \\
&\qquad - \frac{T_1 e^{(R-r)T_1}}{(R-r-(\beta_n+(\theta_n-\varepsilon)))}
\end{aligned}
$$

$$+\frac{T_1 e^{(R-r)T_1}}{\left(R-r-\left(\beta_n+\left(\theta_n-\varepsilon\right)\right)\right)^2}\Bigg)$$
$$\times\alpha_n c_b\left(\frac{\theta_n}{\left(\beta_n+\left(\theta_n-\varepsilon\right)\right)}\right)$$
$$\times\left(\frac{\left(T-T_1\right)e^{RT}}{R}-\frac{e^{RT}}{R^2}+\frac{e^{RT_1}}{R^2}\right)$$
$$+\left(\frac{\alpha_n c_b\beta_n}{\left(\beta_n+\left(\theta_n-\varepsilon\right)\right)}\right)$$
$$\times\Bigg(\frac{\left(T-T_1\right)e^{RT+(\beta_n+(\theta_n-\varepsilon))(2T_1-T)}}{\left(R-\left(\beta_n+\left(\theta_n-\varepsilon\right)\right)\right)}$$
$$-\frac{\left(T-T_1\right)e^{RT+(\beta_n+(\theta_n-\varepsilon))(2T_1-T)}}{\left(R-\left(\beta_n+\left(\theta_n-\varepsilon\right)\right)\right)^2}$$
$$+\frac{e^{(R+\theta)T_1}}{\left(R-\left(\beta_n+\left(\theta_n-\varepsilon\right)\right)\right)^2}\Bigg)\Bigg). \quad (16)$$

Case 2 (payment after the total depletion of inventory; that is, $T_1 < M$). The deterioration cost C_D, the holding cost C_H, and the backlog cost C_b are the same as for Case 1. However the interest paid P_T for this case is 0 as the supplier can pay in full at the end of permissible delay, M. The interest earned per cycle is the interest earned during the positive inventory period plus the interest earned from cash invested during the time period (T_1, M) after the inventory is exhausted. Then

$$I_T = c_0\int_0^{T_1} I_e(t)\left(\alpha+\beta I(t)\right)dt$$
$$+\left(e^{i_e(M-T_1)}-1\right)\int_0^{T_1} c_0 t\left(\alpha+\beta I(t)\right)dt$$
$$I_T = \alpha C\left(\frac{\theta}{\beta+\theta}\right)\Bigg(\frac{T_1 e^{(R+i_e)T_1}}{\left(R+i_e\right)}+\frac{e^{(R+i_e)T_1}}{\left(R+i_e\right)^2}-\frac{T_1 e^{(R-r)T_1}}{(R-r)}$$
$$+\frac{e^{(R-r)T_1}}{(R-r)^2}+\frac{1}{\left(R+i_e\right)^2}-\frac{1^1}{(R-r)^2}\Bigg)$$
$$+\left(\frac{\alpha C\beta}{\beta+\theta}\right)\Bigg(\frac{T_1 e^{(R+i_e)T_1}}{\left(R+i_e-\beta-\theta\right)}-\frac{e^{(\beta+\theta_e)T_1 1}}{\left(R+i_e-\beta-\theta\right)^2}$$
$$-\frac{e^{(R+i_e)T_1}}{\left(R+i_e-\beta-\theta\right)^2}+\frac{e^{(\beta+\theta_e)T_1}}{\left(R-r-\beta-\theta\right)^2}$$
$$-\frac{T_1 e^{(R-r_e)T_1}}{\left(R-r-\beta-\theta\right)}+\frac{e^{(R-r_e)T_1}}{\left(R-r-\beta-\theta\right)^2}\Bigg)$$
$$+\alpha C\left(E^{I_E(m-t_1)}-1\right)\left(\frac{T_1 e^{RT_1}}{(R)}-\frac{e^{RT_1}}{(R)^2}+\frac{1}{(R)^2}\right)$$
$$\times\left(\frac{\theta}{\beta+\theta}\right)+\left(\frac{\alpha C\beta}{\beta+\theta}\right)$$
$$\times\left(\frac{T_1 e^{RT_1}}{\left(R-\beta-\theta\right)}-\frac{e^{RT_1}}{\left(R-\beta-\theta\right)^2}+\frac{e^{(\beta+\theta)T_1}}{\left(R-\beta-\theta\right)^2}\right). \quad (17)$$

3. Computational Procedure

We consider the case of multiple cycles per year so that there are N complete cycles during the time horizon T. Hence $NT = 1$. The inflation and time value of money exist for each cycle of replenishment, so we need to consider the effect over the time horizon NT. So the total cost during the total time is given by

$$C_T\left(T_1, T\right) = C_{VT}\times\left(1+e^{RT}+e^{2RT}\cdots+e^{(N-1)RT}\right)$$
$$= C_{VT}\times\left(\frac{1-e^{R}}{1-e^{RT}}\right). \quad (18)$$

Here, the decision variables for these equations are T_1 and T. The optimal value of these decision variables can be determined with the help of the following differential equations:

$$\frac{\partial C_T\left(T_1, T\right)}{\partial T_1} = 0, \qquad \frac{\partial C_T\left(T_1, T\right)}{\partial T} = 0. \quad (19)$$

After determining these values, the cost function can be determined from (16) and (18).

The solutions of (19) will give the optimal value of T_1 and T, that is, T_1^* and T^*. The minimum total cost $C_T^*(T_1, T)$ at $T_1 = T_1^*$ and $T = T^*$ is computed as

$$\frac{\partial^2 C_T\left(T_1, T\right)}{\partial T_1^2} < 0, \qquad \frac{\partial^2 C_T\left(T_1, T\right)}{\partial T^2} < 0,$$
$$\left(\frac{\partial^2 C_T\left(T_1, T\right)}{\partial T_1^2}\right)\left(\frac{\partial^2 C_T\left(T_1, T\right)}{\partial T^2}\right)-\left(\frac{\partial^2 C_T\left(T_1, T\right)}{\partial T_1\partial T}\right)^2 > 0; \quad (20)$$

$T_1 = T_1^*$, $T = T^*$ are satisfied.

4. Numerical Results

For the sensitivity analysis of the cost function with respect to various system parameters, we have developed a computer program using Mathematica software. Firstly, we analyse the effects of varying parameters on the cost function as given in Section 3. Further, we compare these numerical results with the neurofuzzy results. Unlike classical analytical approaches, this soft computing approach is capable of dealing with fuzzy information to handle real time problems. We fix the variables as $i = 1$, $R = 0.1$, $c_1 = C = 10$, $M = 2$, $c_{b1} = 20$, $A_1 = 500$, $\alpha_1 = 0.5$, $\beta_1 = 0.06$, $\theta_1 = 0.01$, $c_1 = 10$, $i_e = 0.13$, $\theta = 0.01$, $i_p = 0.15$, $r = 0.2$, $i = 0.8$, and $\varepsilon = 0.3\theta$ and obtain the numerical results.

Table 1: Linguistic values of the membership function for various input parameters.

Input variables	Number of membership functions	Linguistic variables
θ	4	(i) Low (ii) Average (iii) High (iv) Very high

4.1. Effect of Parameters on the Cost Function

(i) Effect of Initial Demand (α). Table 2 depicts that an increased initial demand α causes an earlier end of the positive stock time which consequently decreases the length of the total time considered for the model. As the supplier has to pay a shortage cost after the positive stock time and loss of goodwill in the market, he will place an order earlier and it would lessen the total time T. But this situation will certainly cost more as he will have to first invest for the sufficient stock as the demand is stock dependent and only after that he can earn more profits.

(ii) Effect of Coefficient (β). An increased β means increased demand as β is the coefficient of the inventory level at time t in the demand rate. This means that if the supplier has sufficient inventory, he will have more demand and also increased value of β. But as the supplier has sufficient stock with him, he will order after a longer time and it will result in increment in the cost. This fact is quite clear in Table 3.

(iii) Effect of Inflation Rate (R). Higher inflation rate compels the supplier to stock the items in advance which increases the positive stock times T_1 and T along with the cost. This result is drawn from Table 4.

(iv) Effect of Holding and Backlog Cost. Tables 5 and 6 represent that increased holding cost and backlogging cost result in increased cost for the supplier which matches with real time situations.

(v) Effect of Deterioration Rate (θ). Figures 3 and 4 show that at first small increments in deterioration rate do not affect the supplier so much and he has increment in positive stock time. But as the rate becomes higher his stock replenishes sooner and he has to place order earlier. As deterioration rate increases, the cost also increases which is as we expect.

4.2. Comparison of Analytical Results with ANFIS Results. We have obtained the cost function T, T_1 by varying the parameters, namely, the deterioration rate (θ). Treating θ as linguistic variables in the context of fuzzy systems, the respective inference systems are built up by considering θ to be input values. We use the Gaussian function as the membership functions for these input parameters. The linguistic values of the membership functions are provided in Table 1.

Table 2: Effect of variation in α on cost.

α	T_1	T	C_T
.2	4.9427	12.2787	1117.84
.4	4.2196	10.6359	1678.20
.6	3.9523	10.0286	2238.61
.8	3.8124	9.71079	2799.04
1.0	3.7262	9.51498	3359.16

Table 3: Effect of variation in β on cost.

β	T_1	T	C_T
.01	2.16314	2.98596	118.74
.03	2.83601	5.70964	642.69
.05	3.06597	9.34532	1340.79
.07	4.86386	11.1262	3227.46
.09	4.95471	10.4944	5561.50

Table 4: Effect of variation in R on cost.

R	T_1	T	C_T
.2	—	—	—
.3	3.5352	—	—
.4	5.8345	15.5451	1917.03
.5	6.1272	17.2886	3087.70
.6	7.6404	19.7854	4504.81

Table 5: Effect of variation in c_1 on cost.

c_1	T_1	T	C_T
11	4.1822	10.6766	2096.01
12	4.3222	11.0326	2232.91
13	4.4587	11.3795	2371.45
14	4.5919	11.7182	2508.68
15	4.7222	12.0494	2647.63

Table 6: Effect of variation in c_b on cost.

c_b	T_1	T	C_T
21	3.9391	10.0571	1961.71
22	3.8471	9.8281	1963.89
23	3.7613	9.6025	1966.18
24	3.6811	9.3974	1968.56
25	3.6059	9.2051	1971.01

Figure 2 displays the shape of the corresponding membership function. A comparative study of analytical results and neurofuzzy results is facilitated in Figures 3-4. The figures show almost collinear graphs for both analytical and ANFIS results which imply that our results based on ANFIS are very close to the analytical results and are at par with the analytical results.

Summarizing, we can say that if the credit period ends after the complete replenishment, the supplier has to pay less. So it is economical to delay in the settlement of accounts to the last moment of the permissible delay in payments

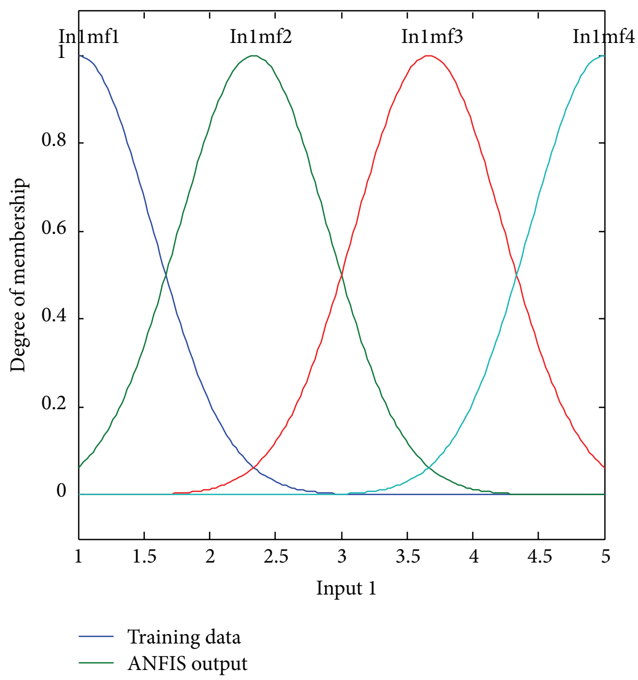

Figure 2: Gaussian membership function for input parameter θ.

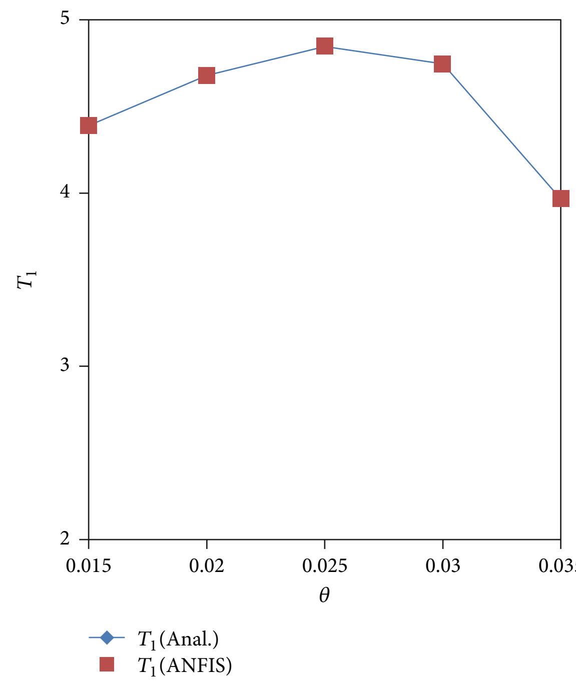

Figure 4: T_1 by varying θ.

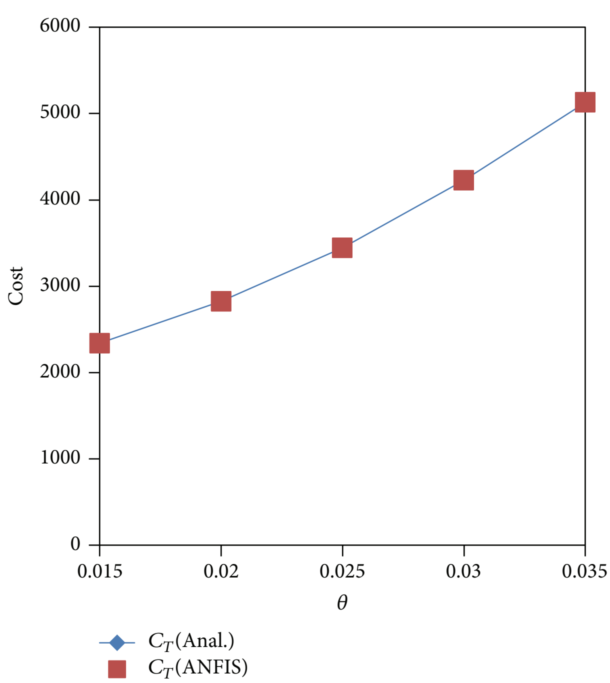

Figure 3: Cost by varying θ.

5. Conclusion

We have developed an inventory model to examine the supplier's credits in an inflationary environment with a stock dependent demand for perishable multi-items. The stock dependent demand rates along with permissible delay in payment are commonly seen in many businesses as such incorporation of such realistic features brings our study closer to real world inventory problems. Deterioration may cause very heavy loss and reduction to the profit incurred because of the credit period. That is why we have considered the controlled deterioration rate. Till now the researches are mainly confined to single item. In the present study, we provide the optimal time to place an order for multi-items so that the undesirable cost of the shortages may be reduced and the supplier may avail the maximum benefit of the credit period and the minimum loss due to unavoidable deterioration. The results derived from this model may prove very helpful to the decision makers, suppliers, and the manufacturer in the present scenario of rising prices.

The research can be further be extended as a two or three level supply chain management may also be a topic of keen interest for the researchers as well as practitioners.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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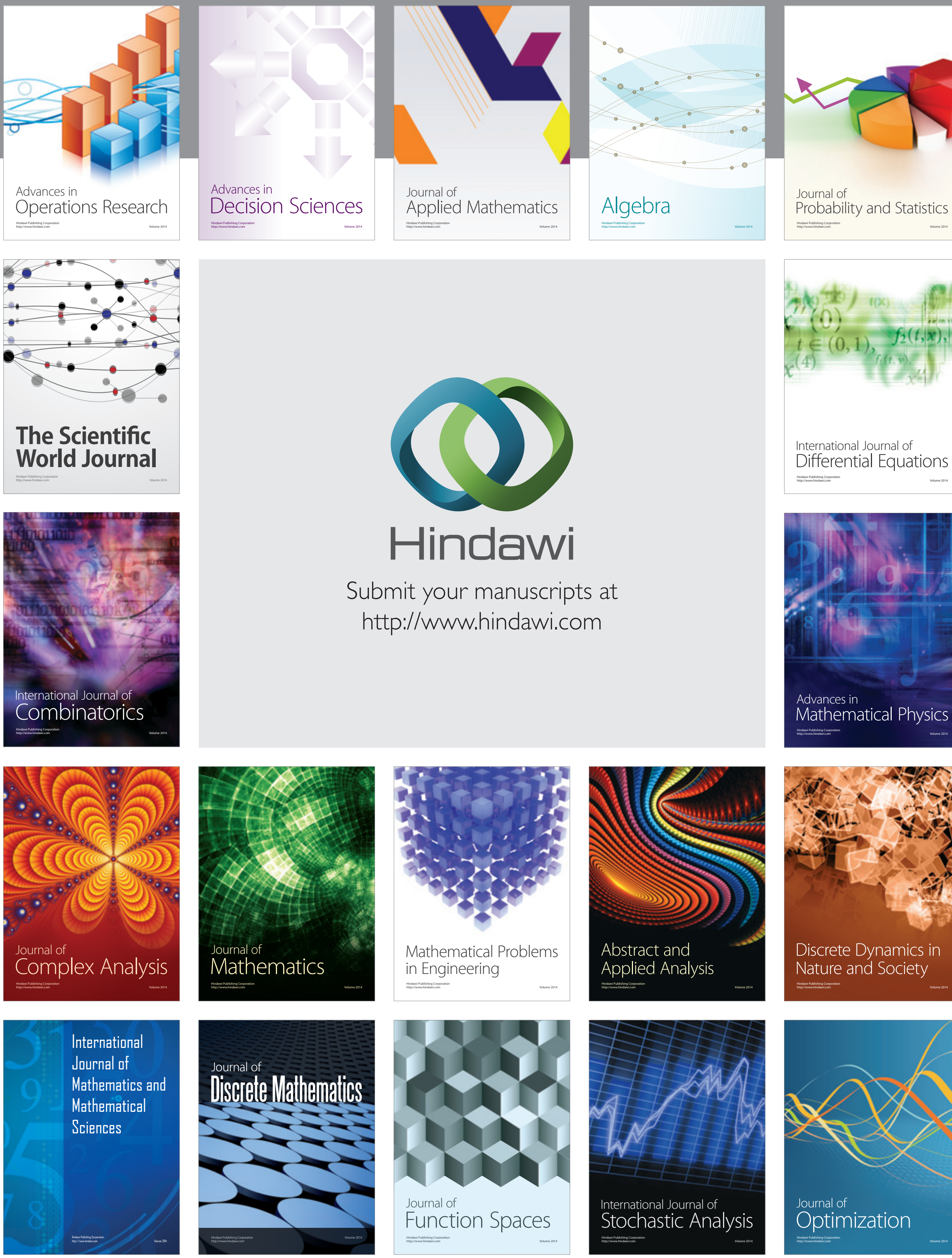
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