

Letter to the Editor

Comment on "Comparison of Some Tests of Fit for the Inverse Gaussian Distribution"

Albert Vexler,¹ Yang Zhao,¹ and Hadi Alizadeh Noughabi²

¹Department of Biostatistics, The State University of New York at Buffalo, Buffalo, NY 14221, USA ²Department of Statistics, University of Birjand, Birjand, Iran

Correspondence should be addressed to Albert Vexler; avexler@buffalo.edu

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The statistical literature shows that the density-based empirical likelihood (DBEL) concept (e.g., [1, pages 150-151], [2]) can be employed successfully to construct efficient non/semiparametric testing procedures. The DBEL approach implies a standard scheme to develop highly efficient procedures, approximating nonparametrically most powerful Neyman-Pearson test-rules.

The paper [3] displayed several concerns regarding the power and practical applicability of the DBEL ratio test for inverse Gaussian (IG) distributions proposed in [4].

(1) Introducing the DBEL ratio test, the authors of [3] wrote, "Observe that, for small *m*, such as m = 1, the statistic can take an infinite value when there are tied data. Vexler et al. [4] do not appear to note this. For the Poisson alternative in Table 1 and $\delta = 0.5$ the log(*TK_n*) statistic is often infinite." The test statistic $\log(TK_n)$ does not depend on *m*. The structure of the test statistic $log(TK_n)$, which consists of the operator "min" over *m*'s, insures that the value of $log(TK_n)$ should not be infinity if just one observed value of the statistics under the "min"-operator is not infinity. The DBEL decision rule says to reject the null hypothesis for large values of the test statistic. If observed values of the statistics involved in $log(TK_n)$ under the "min"-operator are infinity, for all m, and then $\log(TK_n) = \infty$, this implies rejecting the null hypothesis. In these cases, we observe that the data consists of too many equal observations and it is clear that the data cannot follow a continuous IG distribution. In a similar manner to the note mentioned above, we can consider data with many zero

values, for example, when the Poisson alternative is evaluated. Formally speaking, even when we observe one X = 0 we cannot assume our sample is IG-distributed. Taking into account practical issues related to measuring errors, we can impute small values, when X = 0, but evaluations of such techniques do not belong to the aims of this letter.

(2) Considering the power of the test statistics, the authors of [3] evaluated just samples with the size of n = 20. This and the comments above lead us to provide a limited Monte Carlo (MC) study based on 10,000 generations of samples that followed the U[0, 1] and U[1, 2] distributions. Using MC simulations, we compared the powers of the tests, controlling the TIE rate to be 5%. To tabulate the corresponding percentiles of the null distributions of the test statistics, we drew 75,000 replicate samples of the test statistics based on IG(1,1)-distributed data points at each sample size *n*. Table 1 depicts obtained results that can be compared with the outputs of Table 1(b) in [3].

We cannot provide here results corresponding to \widehat{U} , \widetilde{U} type test statistics considered in [3], due to explanation problems in [3] that we will point out in comments below. We just can remark that in the scenarios $\{n = 15, X \sim U[0, 1]\}$ and $\{n = 25, X \sim U[1, 2]\}$ the \widehat{U}_4^2 test statistic gives the power of 0.47 and 0.06, respectively, but we suppose there is a problem in the tests presentations in [3].

Our results are different from those demonstrated in [3] and may change the conclusions shown in [3] with respect to the MC power comparisons. Also these results as

п	\widehat{V}_2^2	\widehat{V}_3^2	\widehat{R}_3	V_0	$\log(TK_n)$	A^2
15	0.69 (0.05)	0.53 (0.08)	0.74 (0.03)	0.63 (0)	0.74 (0.36)	0.75 (0.11)
20	0.79 (0.04)	0.68 (0.19)	0.84 (0.03)	0.74 (0)	0.89 (0.50)	0.87 (0.16)
25	0.87 (0.04)	0.76 (0.31)	0.92 (0.05)	0.82(0)	0.95 (0.62)	0.93 (0.21)

TABLE 1: The Monte Carlo powers of the test-statistics under the alternative hypotheses: $X \sim U[0, 1]$ ($X \sim U[1, 2]$).

well as several MC outputs of [3] raise questions about the consistency of some tests for the IG distribution.

(3) Equations (2.1) in [3] are employed from [5]. However, these equations are different from those used in [5]. The authors of [3] used right formal notations shown in [5] to calculate the tests but provided wrong definitions.

In (2.2) in [3], the authors of [3] used "1+" in {}, whereas in the original paper [6] "1-" is proposed.

In page 5 of [3], line 5 from the bottom, perhaps $\hat{\lambda}$ should be used instead of $\hat{\lambda}$ defined in (1.2).

In the "Jug Bridge Example," the value of V_0 should be 0.0033.

We cannot confirm the MC results of [3] regarding $\widehat{U}_3^2, \widehat{U}_4^2, \widetilde{U}_3^2, \widetilde{U}_4^2$.

(4) The paper [3] considered a very interesting issue to compare the tests for the IG distribution. Perhaps, a more systematic approach to define corresponding alternatives (see, e.g., [7]) can be considered in future studies and various different sample sizes in the relevant MC evaluations can be applied.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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