ISSN 2090-3359 (Print)
ISSN 2090-3367 (Online)

ΑΔΣ

Advances in Decision Sciences

Volume 22(A)
22nd Anniversary Special Issue
December 2018

Michael McAleer

Editor-in-Chief
University Chair Professor
Asia University, Taiwan

Published by Asia University, Taiwan

ADS@ASIAUNIVERSITY

The Noise Trader Effect in a Walrasian Financial Market *

Jukka Ilomäki **

Faculty of Management
University of Tampere, Finland

Hannu Laurila

Faculty of Management
University of Tampere, Finland

December 2018

* The authors would like to thank Michael McAleer and a reviewer for invaluable comments and suggestions.

** Corresponding author: jukka.ilomaki@uta.fi

Abstract

We assume rational risk averse informed investors who observe noisy information about the true value of a risky asset, rational risk averse uninformed investors who infer the true value from the price, and noise traders without any inferences. We have a static two period model where all trading happens in the first period. We show that, due to a negative shock caused by a random sentiment of noise traders, uninformed investors follow the noise because their risk increases. If there is a positive sentiment shock, uninformed investors bet against the noise. However, the equilibrium price stays at the fundamental value as long as the aggregate effect of informed investors is larger than that of noise traders. Thus, the risk premium adjusts perfectly in the market. This is consistent with the common finding of dynamic adjustment of the fundamental value with a time-varying risk premium.

Keywords: Risk aversion, informed investors, uninformed investors, sentiment shock, fundamental value.

JEL: G1, G11, G14, G4

1. Introduction

Can noise traders drive the equilibrium price of a risky asset away from the fundamental value? Many studies answer *yes* in the literature of financial economics (e.g. Shiller 1984; DeLong et al. 1990; Campbell and Kyle 1993; Wang 1993; Mendel and Shleifer 2012; Ilomäki 2016; and Cespa and Vives 2015). More importantly, why rational risk averse investors may end up to follow the noise even in the last trading day?

By Friedman (1953), noise traders who buy (sell) when risky asset is over (under) valued are quickly out of the market. Samuelson (1973) states that the fundamental value is the present value of expected dividends discounted with risk-free rate when rational investors are risk neutral. Merton (1973) demonstrates that rational risk averse investors require a risk premium that varies in time. Grossman and Stiglitz (1976) show that the equilibrium price equals the fundamental value, if there are informed investors who get a signal about the true value of the risky asset, and rational uninformed investors who can interpret the true value from the price.

Grossman and Stiglitz exclude noise traders, which implies that they should exist to make uninformed investors unable to infer the true value from the price. Grossman and Stiglitz (1980) demonstrates that with asymmetrically informed rational investors, there has to be enough gain from trading so that the equilibrium should reflect the cost of fundamental information: there has to be just enough gains from trading to cover the cost of information. Tirole (1982) proves that there are no gains to be made by trading, if homogenous rational investors have similar information.

Shiller (1984) shows that if risk averse informed and ordinary investors trade an infinitely lived risky asset, its fundamental value and the equilibrium price can drift apart for long periods, if ordinary investors herd with some common story. This can be explained by the price effect of market psychology. DeLong et al. (1990) and Campbell and Kyle (1993) demonstrate with dynamic models that the risk aversion of informed investors prevents them to take large positions against correlated noise traders with due effects on prices. Wang (1993) shows in an infinite horizon dynamic model, where risk averse rational informed and uninformed investors trade with noise traders, that uninformed investor can rationally follow noise traders, because they make estimation errors. We show that this can happen also in a simple one period static trading model.

Allen et al. (2006) address that the law of iterated first order expectations does not hold. Higher-order expectations arise before the liquidation date, if rational investors have short investment horizon (one period). This indicates that public information prevails over private information and the equilibrium price drifts away from the fundamental value. Furthermore, Bacchetta and VanWincoop (2008) show that rational higher-order expectations drive the equilibrium price from the first order expectations even in infinite horizon markets.

However, Cespa and Vives (2015) find two stable equilibriums in terms of liquidity, volatility and informational efficiency. The authors assume short term rational informed investors and a continuum of noise traders that raise the higher-order expectations. They address that if volatility is high and liquidity is low, higher order expectations drive the equilibrium prices away from the fundamental value. Thus, if volatility is low and liquidity is high, equilibrium price equals fundamental value.

In this paper, we have simple one period trading model with rational risk averse informed and uninformed investors, and noise traders. The rational risk averse informed investors observe a noisy signal about the true value of the risky asset. The rational risk averse uninformed investors try to estimate the true value of the risky asset from the current price. The noise traders trade strictly on noise, being incapable to infer the true value from the price. The model omits short term trading, because we have only one trading period before the payoffs are distributed.

We find that if there emerges a negative noise trader shock, rational uninformed investors follow noise traders, and the informed investors act just oppositely. The uninformed investors follow noise simply because they are risk averse and they have to infer the true value from market price. If there is a positive noise trader shock, uninformed investors bet against noise and sell, while informed investors start to buy since their risk reduces. However, the equilibrium price is not driven away from the fundamental value, if the effect of informed investors is greater than that of noise traders.

The results are consistent with the findings of Mendel and Shleifer (2012), which shows that a rational uninformed investor can regard noise as information, thus chasing the noise. In Mendel and Shleifer, this happens when a part of the fundamental information about the true value is available in the first period, and the rest of the information becomes available when payoffs are distributed. We show that the same can happen also when all information is available in the first period. However, we also find that the equilibrium price in the final trading period equals the fundamental value, when the rational risk averse (informed and uninformed) investors and noise traders trade before the liquidation date, and if the effect of informed investors overcomes that of noise traders. Otherwise, the final trading price differs from the fundamental value.

2. Model

The model is based on Grossmann and Stiglitz (1980), Kreps (1977), Milgrom and Stokey (1982), Admati (1985), Shiller (1984), and Mendel and Shleifer (2012). There is a set $\left[0,1\right]$ of rational constant absolute risk-averse (CARA) investors in a Walrasian market. The investors live for two periods, trading in the first period, and consuming in the second period. That is, all trades occur in period 1, and the payoffs take place in period 2.

The investors allocate their investments between risk-free and risky assets. The risk-free asset pays one unit of consumption in period 2, and the risky asset pays $\tilde{D} \sim N(\tilde{D}, \sigma_D^2)$ in terms of consumption in period 2. The price of risky asset is P per share, calculated in terms of the risk-free asset. The investors have asymmetric information. There are $0 < \mu < 1$ informed investors, who observe a noisy signal $\tilde{s} = \tilde{D} + \varepsilon$ with $\varepsilon \sim N(0, \sigma_\varepsilon^2)$, and $1 - \mu$ uninformed investors, who observe P, and form rational expectations of $\tilde{D}$ based on their observation.

In addition, there is a measure 1 of correlated noise traders, who exchange risk-free assets to $\tilde{N} \sim N(0, \sigma_N^2)$ units of the risky asset. The existence of noise traders is important, because otherwise the uninformed investors could infer the true value of $\tilde{D}$ from P. Assume also that $\sigma_D^2 < 1$, $\sigma_\varepsilon^2 < 1$ and $\sigma_N^2 < 1$.

Both informed and uninformed investors have the same utility function $u(c) = -e^{-c}$, so that their CARA coefficient is one. The initial asset allocation is the following: informed investors hold a_I, uninformed investors hold a_U, and noise traders hold a_N of the risky asset, and all have a_O of the risk-free asset. The informed investors base their expectation of $\tilde{D}$ on their private signal $\tilde{s}$,

$$E\left[\tilde{D} \mid \tilde{s}\right] = \tilde{D} + \beta\left[\tilde{s} - \tilde{D}\right], \tag{1}$$

where $\beta = \operatorname{cov}\left(\tilde{D}, \tilde{s}\right) / \operatorname{var}\left(\tilde{s}\right) = \sigma_D^2 / \sigma_s^2$, and $\sigma_s^2 = \sigma_D^2 + \sigma_\varepsilon^2$. The variance of the informed investors' prediction error thus is $\sigma_I^2 = \sigma_D^2 \sigma_\varepsilon^2 / \sigma_s^2$. Equation (1) indicates that the dividend yield $\tilde{D}$ influences the price P through informed investors' private signals. Thus, P depends on $\tilde{s}$, and on $\tilde{N}$, the demand of noise traders. In the OLS fashion, write:

$$P = z + b\tilde{s} + c\tilde{N}, \tag{2}$$

where z, b and c are unknown parameters to be determined. The uninformed investors base their expectations on the observed P. Note that since $\tilde{s} - \tilde{D} = \varepsilon$, $E\left[\tilde{s}\right] = \tilde{D}$, and $E\left[\tilde{N}\right] = 0$, Equation (2) means for uninformed investors that $E[P - z - b\tilde{D}] = 0$. Because they are risk averse, they must take into account the possibility that $P - z - b\tilde{D} \neq 0$. Thus, the unobserved error ε in informed investors' signal $\tilde{s}$ and the effect of noise traders $c^2\sigma_N^2$ add components in the coefficient β of Equation (1), yielding:

$$\gamma = \frac{b\sigma_D^2}{b^2\sigma_s^2 + c^2\sigma_N^2}. \tag{3}$$

The expectation of an uninformed investor reads:

$$E\left[\tilde{D} \mid P\right] = \tilde{D} + \gamma\left[P - z - b\tilde{D}\right], \tag{4}$$

and, using Equation (2):

$$E\left[\tilde{D} \mid P\right] = \tilde{D} + \gamma\left[b(\tilde{s} - \tilde{D}) + c\tilde{N}\right]. \tag{5}$$

The variance of the uninformed investors' prediction error is:

$$\sigma_U^2 = \frac{\left(b^2\sigma_\varepsilon^2 + c^2\sigma_N^2\right)\sigma_D^2}{b^2\sigma_s^2 + c^2\sigma_N^2}. \tag{6}$$

3. Financial Market Equilibrium

The expected utility of an informed investor reads:

$$u(c) = -e^{-x_I E\left[\tilde{D}|\tilde{s}\right] - a_0 + [x_I - a_I]P + x_I^2 \sigma_I^2/2}, \tag{7}$$

where x_I is the total demand of the risky asset by the informed investor. Solve x_I from the first order maximum condition, and have:

$$x_I = \frac{E\left[\tilde{D} \mid \tilde{s}\right] - P}{\sigma_I^2}, \tag{8}$$

where σ_I^2 is the risk-premium of an informed investor and the numerator is their gain for trading. The expected utility of an uninformed investor reads:

$$u(c) = -e^{-x_U E\left[\tilde{D}|P\right] - a_0 + [x_U - a_U]P + x_U^2 \sigma_U^2/2}, \tag{9}$$

which yields:

$$x_U = \frac{E\left[\tilde{D} \mid P\right] - P}{\sigma_U^2} \tag{10}$$

for the uninformed investor's demand of the risky asset, with σ_U^2 describing the risk premium of an uninformed investor and the numerator is their belief on the gain for trading. The market clearing condition for the risky asset reads:

$$\mu x_I + (1-\mu)x_U + \tilde{N} = \mu a_I + (1-\mu)a_U + a_N . \tag{11}$$

Using Equations (8) and (10), and recalling Equations (1), (2) and (5), Equation (11) turns to:

$$\mu\left[\frac{\tilde{D}+\beta(\tilde{s}-\tilde{D})-z-b\tilde{s}-c\tilde{N}}{\sigma_I^2}\right]+(1-\mu)\left[\frac{\tilde{D}+\gamma\left[b(\tilde{s}-\tilde{D})+c\tilde{N}\right]-z-b\tilde{s}-c\tilde{N}}{\sigma_U^2}\right] \tag{12}$$
$$+\tilde{N}=\mu a_I+(1-\mu)a_U+a_N.$$

Note that the left-hand side of Equation (12) includes a constant, and terms involving $\tilde{s}$ and $\tilde{N}$, whereas the right-hand side expresses the constant supply of the risky asset. In the equilibrium, the terms including the constant z on the left-hand side must equal the right-hand side, and the terms including the coefficients b and c of $\tilde{s}$ and $\tilde{N}$ must be zero. Thus, the unknown parameters of Equation (2) can be solved. They read:

$$z=-\frac{\sigma_I^2\sigma_U^2}{\mu\sigma_U^2+(1-\mu)\sigma_I^2}\left[\mu a_I+(1-\mu)a_U+a_N\right], \tag{13}$$

$$b=\mu\beta\frac{\sigma_U^2}{\mu\sigma_U^2+(1-\mu)(1-\gamma)\sigma_I^2}, \tag{14}$$

$$c=\frac{\sigma_I^2\sigma_U^2}{\mu\sigma_U^2+(1-\mu)(1-\gamma)\sigma_I^2}. \tag{15}$$

Follow Milgrom and Stokey (1982) and set the supply of the risky asset to zero in Equation (13). This means that, in the equilibrium, no trade happens because all investors are satisfied with their current holdings. Using Equations (14) and (15), the equilibrium price is:

$$P=\mu\beta\frac{\sigma_U^2}{\mu\sigma_U^2+(1-\mu)(1-\gamma)\sigma_I^2}\tilde{s}+\frac{\sigma_I^2\sigma_U^2}{\mu\sigma_U^2+(1-\mu)(1-\gamma)\sigma_I^2}\tilde{N}. \tag{16}$$

Examine the coefficient of $\tilde{s}$ on the right-hand side of Equation (16), namely Equation (14). Using Equations (3) and (6), and manipulating, it reduces to:

$$b=\left\{\beta\left[\sigma_\varepsilon^2\left(\mu\sigma_D^2-c^2\sigma_N^2-\sigma_\varepsilon^2\right)\right]\right\}^{\frac{1}{3}}.$$

Now, b is positive, if $\mu\sigma_D^2-\sigma_\varepsilon^2>c^2\sigma_N^2$. This says that the stable equilibrium is reached, if the total effect of noise traders is smaller than the effect of informed investors.

In the equilibrium, noise traders can behave unexpectedly thus affecting the risk of rational investors, both informed and uninformed.

***Proposition 1*:** Increase in noise traders' effect reduces the risk for informed investors.

***Proof* :** Use Equation (14) to solve for σ_I^2, manipulate, and obtain:

$$\sigma_I^2 = \frac{\mu\left(\beta - b\right)\sigma_U^2(b^2\sigma_s^2 + c^2\sigma_N^2)}{(1-\mu)b(b^2\sigma_s^2 + c^2\sigma_N^2) + (\mu - 1)b^2\sigma_D^2}\,. \tag{17}$$

Taking partial derivate from Equation (17) with respect to σ_N^2 yields:

$$\frac{\partial\sigma_I^2}{\partial\sigma_N^2} = \frac{\mu(\beta - b)\sigma_U^2 c^2[(1-\mu)b(b^2\sigma_s^2 + c^2\sigma_N^2) + (\mu-1)b^2\sigma_D^2] - (1-\mu)bc^2\mu\left(\beta - b\right)\sigma_U^2(b^2\sigma_s^2 + c^2\sigma_N^2)}{[(1-\mu)b(b^2\sigma_s^2 + c^2\sigma_N^2) + (\mu-1)b^2\sigma_D^2]^2},$$

which reduces to:

$$\frac{\partial\sigma_I^2}{\partial\sigma_N^2} = \frac{(\mu-1)b^2\sigma_D^2}{[(1-\mu)b(b^2\sigma_s^2 + c^2\sigma_N^2) + (\mu-1)b^2\sigma_D^2]^2} < 0.$$

This says that as noise traders' variance increases, the variance and thus risk of the informed investors reduces. Quite obviously, use of Equation (15) would produce the same result. **QED**

***Proposition 2*:** Increase in noise traders' effect increases the risk for uninformed investors.

Proof: Use Equation (14) to solve for σ_U^2, manipulate, and obtain:

$$\sigma_U^2 = \frac{(b^2\sigma_s^2 + c^2\sigma_N^2)b(1-\mu)\sigma_I^2 + b^2(\mu-1)\sigma_I^2\sigma_D^2}{\mu(\beta - b)(b^2\sigma_s^2 + c^2\sigma_N^2)} \tag{18}$$

Take a partial derivate from Equation (18) with respect to σ_N^2:

$$\frac{\partial\sigma_U^2}{\partial\sigma_N^2}=\frac{c^2b(1-\mu)\sigma_I^2\mu(\beta-b)(b^2\sigma_s^2+c^2\sigma_N^2)-\mu(\beta-b)c^2[(b^2\sigma_s^2+c^2\sigma_N^2)b(1-\mu)\sigma_I^2+b^2(\mu-1)\sigma_I^2\sigma_D^2]}{\left[\mu(\beta-b)(b^2\sigma_s^2+c^2\sigma_N^2)\right]^2},$$

which reduces to:

$$\frac{\partial\sigma_U^2}{\partial\sigma_N^2}=-\frac{\mu(\beta-b)c^2b^2(\mu-1)\sigma_I^2\sigma_D^2}{\left[\mu(\beta-b)(b^2\sigma_s^2+c^2\sigma_N^2)\right]^2}>0.$$

The sign is positive, because $\beta>b$, which can be seen by using Equation (3) in Equation (14). This says that as noise traders' variance increases, the variance and thus risk of the uninformed investors increases. Quite obviously, use of Equation (15) would produce the same result. **QED**

Corollary 1: Propositions 1 and 2 indicate that if noise traders sell more (a negative sentiment shock), uninformed (informed) investors also sell (buy) the risky asset, because their risk increases (decreases). This is to say that uninformed investors follow noise traders, but informed investors have private information on $\tilde{D}$.

Corollary 2: Propositions 1 and 2 indicate that if the noise traders buy more (a positive sentiment shock), uninformed (informed) investors sell (buy) the risky asset, because their risk increases (decreases). This implies that uninformed investors bet against the noise, but informed investors buy more risky assets based on their private information on $\tilde{D}$.

In order to clarify Corollary 2, recall investors' rationality and the initial allocation a_I, a_U and a_N of the risky asset. Recall also Equations (8) and (10), which say that the total demand of informed investors is x_I+a_I and that of the uninformed is x_U+a_U. If $\partial\sigma_N^2>0$, a positive noise shock emerges and noise traders' demand of increases. By Equations (17) and (18), σ_I^2 reduces and σ_U^2 increases. Thus, the denominator diminishes in Equation (8) and grows in Equation (10), and x_I+a_I increases and $x_U + a_U$ decreases. The equilibrium price P adjusts as market clearing occurs. Obviously, the same reasoning applies also to Corollary 1.

Corollary 3: The aggregate market risk premium changes when the risk premiums of rational investors change, since the market clearing condition (11) implies that the aggregate market risk premium is $\sigma_M^2 = \mu\sigma_I^2 + (1-\mu)\sigma_U^2$ (the weighted average risk premiums of rational investors). This indicates that the fundamental value equals the equilibrium price, if $\mu\sigma_D^2 - \sigma_\varepsilon^2 > c^2\sigma_N^2$ that is if the effect of informed investors prevails the effect of noise traders. If not, the model does not have an equilibrium suggesting that the market price drifts away from the fundamental value.

Hence, Corollary 3 implies that if the equilibrium price is found in the economy, it also equals the fundamental value. This can be interpreted as static support for the argument of time-varying discount rates (e.g. Cochrane 2011, and Fama 2014), which offers a rational explanation why stock prices vary so much.

4. Conclusions

The paper presented a simple model with three kinds of investors in a static Walrasian financial market: rational risk averse informed investors who receive noisy private information about true value of the risky asset, rational risk averse uninformed investors who have to infer the fundamental value from market price, and noise traders. The idea was to assess previous findings in a simplified framework.

We found that uninformed risk averse investors follow noise traders' negative sentiment shock by selling their risky assets. This is because of their risk aversion. Moreover, informed investors receive additional gains from trading. The result is consistent with the result of Mendel and Shleifer (2012), in which information about fundamental value is revealed in two parts, step-wise in the first and second periods. Our paper strengthens the intuition by showing it in a simple static one period trading model.

However, in the case of positive sentiment shock, we found that uninformed investors sell, because their risk is increasing again, and thus they do not follow the noise but bet against it. In addition, informed investors buy because their risk is reducing. However, these do not drive the equilibrium price away from the fundamental value, if the effect of informed investors prevails that of noise traders. If not, there is no stable equilibrium. This result is consistent with Cespa and Vives (2015), which finds two equilibriums: one where the equilibrium price equals the fundamental value, and another one where the equilibrium price differs significantly from the fundamental value.

References

Admati, A. (1985). A noisy rational expectations equilibrium for multi-asset securities markets. *Econometrica*, 53, 629-657.

Allen, F., Morris, S., Shin, H. (2006). Beauty contests and iterated expectations in asset markets. *Review of Financial Studies*, 19, 719-752.

Bacchetta, P., Van Wincoop, E. (2008). Higher order expectations in asset pricing. *Journal of Money, Credit and Banking*, 40, 837-865.

Campbell, J., Kyle, A. (1993). Smart money, noise trading and stock price behaviour. *Review of Economic Studies,* 60, 1-34.

Cespa, G., Vives, X. (2015). The beauty contest and short-term trading. *Journal of Finance,* 70, 2099-2153.

Cochrane, J. (2011). Presidential address: Discount rates. *Journal of Finance,* 66, 1047-1108.

DeLong, B., Shleifer, A., Summers, L., Waldmann, R. (1990). Positive feedback investment strategies and destabilizing rational speculation. *Journal of Finance,* 45, 379-395.

Fama, E. (2014). Two pillars of asset pricing. *American Economic Review,* 104, 1467-1485.

Friedman, M. (1953). The case for flexible exchange rates, in Milton Friedman, ed.: *Essays in Positive Economics.* University of Chicago, Chicago.

Grossman, S., Stiglitz, J. (1976). Information and competitive price systems. *American Economic Review,* 66, 246-253.

Grossman, S. Stiglitz, J. (1980). On the impossibility of informationally efficient markets. *American Economic Review*, 70, 393-408.

Ilomäki, J. (2016). Risk-free rates and animal spirits in financial markets. *Annals of Financial Economics,* 11(3), 1-18.

Kreps, D. (1977). A note on fulfilled expectations equilibria. *Journal of Economic Theory,* 14, 32-43.

Mendel, B., Shleifer, A. (2012). Chasing Noise. *Journal of Financial Economics,* 104, 203-320.

Merton, R. (1973). An intertemporal capital asset pricing model. *Econometrica*, 41, 867-887.

Milgrom, P., Stokey, N. (1982). Information, trade and common knowledge. *Journal of Economic Theory,* 26, 17-27.

Samuelson, P. (1973). Proof that properly discounted present values of assets vibrate randomly. *Bell Journal of Economics,* 4, 369-374.

Shiller, R. (1984). Stock prices and social dynamics. *Brookings Papers on Economic Activity,* 2, 457-498.

Tirole, J. (1982). On the possibility of speculation under rational speculations. *Econometrica,* 50, 1163-1181.

Wang, J. (1993). A model of intertemporal asset prices under asymmetric information. *Review of Economic Studies,* 60, 249-282.