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Mathematical Modelling of Decision-Making: Application to Investment *

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Abstract

In this paper, the mathematical modelling of decision-making problems based on probabilistic graphical models is presented. The models, in addition to random variables, also include decision and profit variables. The proposed decision models contain one or more decisions and their purpose is to assist the decision maker in choosing the best decision under conditions of uncertainty. The techniques used include modelling and evaluating decision trees, as well as evaluating, modelling and presenting the algorithm of influence diagrams (as the expansion of the Bayesian networks) to show the communication structure of the problem and thus provide a coherent presentation with an effective evaluation. The limited memory influence diagram and dynamic decision networks (as a dynamic Bayesian network) have also been developed to avoid the limitations of the influence diagram limitation. An application of the exposed models is presented in investment decision-making.

Keywords: Decision models, Decision tree, Influence diagram, Bayesian network, Investment decision-making.

JEL: B16, C52, D81

1. Introduction

Decision theory was originally proposed in economics and operations research (cf. von Neumann and Morgenstern (1944)), and then attracted the attention of artificial intelligence researchers (AIs) who are interested in understanding and creating intelligent agents (cf. Jensen (2001)). These intelligent agents, such as robots, financial advisors and intelligent instructors, should deal with similar problems such as those found in economics and operations research.

At the same time, graphical models may be used to solve decision problems. Specifically, a decision model that includes one or more decisions to help decision makers to choose the best decision in terms of uncertainty include these problems. In fact, the best decision is the decision that maximizes the expected utility of an agent, called rational agent, based on current knowledge (evidence) and its goals and within the framework of decision theory.

Some developments have been made to prevent the limitations of the influence diagrams (cf. Section 5), including the limited memory influence diagrams (cf. Section 5.1) and the dynamic decision networks (cf. Section 5.2). An application of the proposed models and the decision graph (cf. Section 6) in decision-making on investment has been proposed.

After introducing the summary of the decision theory and its principles in Section 2, two types of modelling techniques are presented for problems with one or more decision including decision trees (cf. Section 3), Modelling of decision trees (cf. Section 3.1) and their evaluation (cf. Section 3.2) as well as influence diagrams (cf. Section 4), Modelling (cf. Section 4.1) and their evaluation and presentation of the influence diagram algorithm (cf. Section 4.2).

These techniques illustrate the communication structure of the problem and, as a result, provide a coherent display with an operational evaluation.

2. Decision Theory

The decision theory provides a framework for decision-making in uncertainty and is based on the concept of rationality, that is, an agent tries to maximize its utility or minimize its costs. This assumes that at least there is a way to allocate the utility to the outcome of each alternative action. The best decision is to make the highest utility. In general, an agent is uncertain about the outcome of each of its possible decisions, so it should pay attention to it when it evaluates the value of each alternative.

In decision theory, the expected utility is the average of all possible results of the decision that is calculated with their probability. Therefore, a rational agent must make a decision that maximizes the expected utility.

Principles of decision theory were originally developed in the theory of games and economic behavior (cf. von Neumann and Morgenstern (1944)), and a set of intuitive constraints was defined as the principles of the utility theory. These constraints should guide the preferences of a rational factor. Before listing these principles, some of the notations should be defined. In the decision scenario, there are four elements including alternatives, events, outcomes, and preferences. In terms of utility, various scenarios are called lotteries.

In a lottery, each possible outcome or state A has a certain probability p, and an associated preference to the agent which is denoted by a real number U. For example, a lottery L with two possible outcomes A (with probability p) and B (with probability $1-p$) is denoted by $L = [p, A; 1-p, B]$. If an agent prefers A rather than B, it is written as $A \succ B$, and if there is no difference between the two outcomes, it is represented as $A \sim B$.

The axioms of utility theory consist of order, transitivity, continuity, substitutability, monotonicity, and decomposability. A utility function follows the principles of utility which means that there is a real-valued utility function U such that $U(A) \succ U(B)$ if and only if the agent A prefers A over B, and $U\ (A)\ = U\ (B)$ and only if the agent is indifferent between A and B. Also, the maximal expected utility principle states that the utility of a lottery is equal to:

$$U[P_1, S_1; P_2, S_2; P_3, S_3; \dots] = \sum_j P_j U_j.$$

Thus, the expected utility (EU) of a particular decision D taken by an agent given by N possible results of this decision with the probability P such as $EU(D) = \sum_{j=1}^{N} P\left(result_j(D)\right) U(result_j(D))$. The principles of maximum expected utility states that a rational agent should choose an action that maximizes its expected utility.

2. Decision Trees

In more complex decision-making problems, applying the principle of maximum expected utility to determine the best decision is not easy, and a systematic approach is needed to model and solve such problems. One of the oldest modeling tools used to solve decision problems is decision tree (cf. Cole and Rowley, 1995).

A decision tree is a graphical representation of a decision problem that has three types of elements or nodes that represent the three main components of a decision: decision, uncertain events, and results. A decision node is displayed as a multi-branch rectangle, each branch shows one of the possible alternatives at this decision point. At the end of each branch, there may be another decision point (an event or a result). An event node is displayed as a circle with multiple branches, each branch shows one of the possible results of this uncertain event that they can be mutually exclusive and exhaustive.

A probability value is assigned to each branch so that the sum of probabilities for all branches is equal to one. At the end of each branch, another event node (a decision node or a result) may be placed. The results are interpreted based on the utility presented for each agent and are usually located at the end of each branch of the tree (the leaves).

2.1 Modelling of Decision Trees

Decision trees are tools for modeling and solving sequential decision problems. These trees are usually drawn from left to right, the root of the tree (a decision node) at the extreme left and the tree leaves to the right.

2.2 Evaluation of Decision Trees

To determine the best decision for each decision point (according to the maximum expected utility principle), the decision tree must be evaluated. Decision tree evaluation involves determining the value of both types of nodes, the event node and the decision node. This is done from right to left and can start from the node that has only results for all its branches. The value of a node of decision D is the maximum value of the branch from which that emanate from it. The value of each node E is the expected value of all its output branches, that is:

$$V(E) = \sum_j P\left(result_j(E)\right) U(result_j(E)).$$

3. Influence Diagrams

The size of the tree (number of branches), exponentially increasing, increases with the increase in the number of event nodes or decision. Therefore, this type of display only applies to small issues. Impact diagrams are a tool for solving decision problems as a substitute for decision trees to simplify modeling and analysis (cf. Howard and Matheson, 1984). From another angle, they can be seen as an extension of the network of business that integrates the nodes of utility and decision.

3.1 Modelling of Influence Diagrams

The influence diagram is a directed acyclic graph whose nodes represent utility variables (with a diamond representation), decision variables (with a rectangle representation), and random variables (with elliptical views). There are three types of arcs in the influence diagram: probabilistic, informational, and functional.

In an influence diagram, there should be a directed path in the underlying directed graph that includes all the decision nodes and refers to the order in which the decisions are made. In this way, this order induces a partition on the random variables in the influence diagram so that if there are n decision variables, the random variables are divided into $n+1$ subsets. Each subset, R_i, contains all random variables that are known prior to the decision D_i and are unknown to previous decisions. Some of the influence assessment algorithms use the opportunities that create these features to make the evaluation function more efficient.

Influence diagrams are used to help the decision maker in maximizing the expected utility. Therefore, the purpose of the decision analysis is to find the optimal policy $\pi = \{d_1, d_2, \dots, d_n\}$, which selects the best decisions for each decision node to maximize the expected utility of $E_\pi(U)$. If there are several utility nodes, we must maximize the sum of these individual utilities:

$$E_\pi(U) = \sum_{u_i \in U} E_\pi(u_i).$$

3.2 Evaluation of Influence Diagrams

Evaluating an influence diagram means finding the sequence of best decisions or optimal policies. For a simple effect diagram, which has a single decision node and a simple utility node, we have:

Simple Impact Diagram Algorithm:

1) For each $d_i \in D$:
1-1) Put $D = d_i$.
1-2) Enter all known random variables.
1-3) Broadcast the probabilities according to the Bayesian network.

1-4) Gain the expected value of the utility node, U.

2) Select the decision, d_k, that maximizes U.

In the complex decision-making problems in which there are several decision nodes, the above algorithm will be ineffective. In general, there are three main approaches to solving the influence diagrams:

i) Transformation the influence diagram to a decision tree and apply standard solution techniques for decision trees.

ii) Expose a direct solution to the influence diagram by applying a series of transforms to the graph.

iii) Transform the influence diagram to a Bayesian network and use Bayesian network deduction techniques.

For more details about the first approach refer to Cole and Rowley (1995) and the second approach to Shachter (1986). A diagram of the effect of limited memory can be found in Lauritzen and Nilsson (2001) that its upgrading to dynamic models is presented in van Gerven et al. (2007).

4. Extensions

4.1 Limit Memory Influence Diagrams

In order to avoid the previous influence diagram constraints, the limited memory influence diagram is presented as an extension of the influence diagrams. The limited memory is the reflection of this feature, which is the variable when deciding, it is clear that it will not necessarily remain in memory when it decides (cf. Lauritzen and Nilsson (2001)). Removing some variables reduces the complexity of the model so that it can be solved with a computer. Of course, this ends at the expense of obtaining a sub-optimal policy (not a completely optimal policy).

4.2 Dynamic Decision-Making Networks

Dynamic decision-making networks are another development that is used for sequential decision-making problems and involves several decisions over time. Like the Networks, one can consider decision-making problems in which a series of decisions must be taken at different time intervals; this type of problem is known as the sequential decision-making problem.

A sequential decision problem can be modeled as a dynamic Bayesian network, which is also known as dynamic decision network and can be viewed as an extension of a dynamic Bayesian network, with the additional decision and utility nodes for each time step, Figure 1.

[Figure 1 here]

In principle, one can evaluate a dynamic decision-making network in a manner similar to the influence diagram, given that decisions should be ordered in time. That is, each node of the decision, D_t, has the informational arcs from the previous decision nodes D_{t-1}, D_{t-2}, etc.

In any case, complexity increases with the increasing number of time epochs, and this also affects calculations. In addition, in some applications, the number of decision epochs is not already known and there may be an infinite number of decisions.

5. Application to Investment

An investment decision is considered with three options: stocks, gold and no investment. Assuming that the investment is made for one year, if it is invested in stock, it will earn $1,000 or $3,00 due to the stock market behavior (an uncertain event), even though it is almost equal Happens, if it is invested in gold, another decision must be made; whether the investor has insurance or not.

If an investor has insurance, it can be sure to gain $200. Otherwise, on the basis that the price of gold, high, without changing or lowering, will win or lose, which will be displayed as another event. Each possible output has a certain amount and a probability, Table 1 below. What should the investor decide?

[Table 1 here]

According to Table 1, the best investment decision is gold without insurance. Given the approach outlined in Section 3, the decision tree in Figure 2 can be evaluated.

[Figure 2 here]

6. Conclusion

In this paper, the mathematical modeling of decision-making problems with one or more decisions based on probabilistic graphical models was considered. The proposed decision models also included random variables, decision and profit variables, and to make the best decision in terms of uncertainty.

Modeling and evaluation of decision trees, evaluation, modeling and presentation of the algorithm related to the extended Bayesian network, namely the influence diagram, the limit memory influence diagram and dynamic decision networks were techniques in the model which were presented in order of increasing the functionality and eliminating the limitations of the previous technique. An application of the proposed models for investment decision-making was also explored in detail.

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Table 1

Events and Decisions

Events	**Event 1** Market Price: $V(E_1)$=1000×0.5-300×0.5=350 **Event 2** Gold Price: $V(E_2)$=800×0.7+100×0.2-200×0.1=560
Decisions	**Decision 2** Insurance: $V(D_2)$=max(200,560)=560- No insurance **Decision 1** Investment: $V(D_1)$= max(150,560,0) =560- Invest in Gold

Figure 1

Dynamic decision network with 4-decision epochs

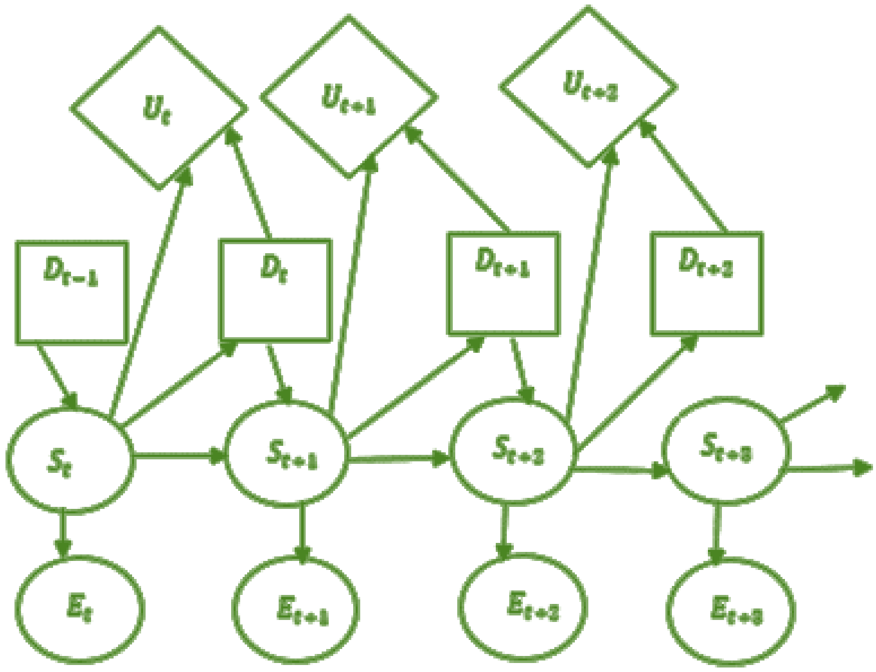

Figure 2

Decision tree for decision-making on investment