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Rational Irrationality:
A Two Stage Decision Making Model*

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Abstract

This paper proposes a mathematical two-stage decision making model based on dual-decision models from behavioral economics that includes, in addition to cognitive and affective systems, an individualistic human factor and a stochastic shock. The model provides a new vision of the decision-making process and the impact of individualism. In the first stage, the agent´s initial willingness to choose is obtained following traditional economic theory but including an individual human factor, which is composed by the learning process, free will, and other human factors. This allows us to explain the reason why sometimes people are inclined to choose options that seem to be irrational decisions from the view of traditional economics logic. In the second stage, the model explains how the cognitive and affective systems and the influence of a stochastic shock affect the initial willingness to choose, obtained in the first stage. The shock might be produced by those negative and/or positive feelings and information not known or considered previously that allows the individual arrive to the final decision. Finally, our model demonstrates that the individual human factor and the stochastic shock are fundamental elements that define the rational irrationality when traditional economic theory fails to explain individuals´ choices.

Keywords: Decision making, Expected utility, Behavioral economics, Cognitive and affective systems, Human factor.

JEL: C44, C90, D03.

1. Introduction

Over time, the gap between Microeconomics and social and cognitive Psychology has been narrowing due to the rapid developments in the field of behavioral economics, game theory, and economic psychology, as pointed out by Carmerer and Loewenstein (2004). According to Alós-Ferrer and Strack (2014), behavioral economics has matured due to the contributions that range from development of experiments to modifications of the assumptions of traditional models, which assume that decisions are made by rational agents, —agents who have well-defined preferences, clearly understand the environment in which they must make decisions, and have an ability to learn rapidly (Brocas and Carrillo, 2014).

However, as has already been demonstrated by models in the fields of behavioral economics (see Kandori, Mailath, and Rob, 1993; Young, 1993) and game theory (see Weibull, 1995), human decisions are not always derived from rational analysis (see Alchian, 1950; Simon, 1959). In fact, these new developments fit best the empirical evidence. This clearly reveals that decision-making does not derive from a single entity, but from a complex system of entities (Brocas and Carrillo, 2014; Cervantes and Dzhafarov, 2018; Lee, Gluck, and Walsh, 2019; Wallin, Swait, and Marley, 2018). This has given relevance to the initial experiments carried out by Schneider and Shiffrin (1977a) and Schneider and Shiffrin (1977b), who proposed what has been called in Psychology dual decision models and proposed the dual decision theory (Brocas and Carrillo, 2014).

Dual decision models have the common assumption that there are two kinds of processes influencing the human mind: controlled, reflective, or rational on one side and automatic, impulsive, reflexive, or experiential on the other side (Alós-Ferrer and Strack, 2014). Given rational or controlled behavior, in the first type, cognitive resources are assumed to be consumed since the individual's behavior patterns are derived from the traditional model of economic rationality. In these models, an agent with predetermined preferences described by her utility function, and given a certain information to construct her conjectures, seeks to maximize the utility derived from the selection over uncertain relevant events (Alós-Ferrer and Strack, 2014; Brocas and Carrillo, 2014).

On the other side, in automatic, impulsive (emotional) or reflective processes, the chosen alternative is a suboptimal choice that could be related to addiction, previous learning patterns, or low-level cognitive processes where decisions are made effortless (Brocas and Carrillo, 2014; Alós-Ferrer and Strack, 2014) and faster than in cognitive processes (Kahneman, 2011).

An important feature that needs to be considered in dual decision models refers to the interaction between these two processes. Some suggest that these two processes work in parallel and at sometimes cooperate with one another but at other times get into conflict. If both processes cooperate in parallel, the automatic or deliberative process becomes a tool to classify the processes rather than a crucial feature of decision conflict, as seen in the seminal works of Sloman (1996) and Epstein (1994). The deliberative or automatic process comes to play a larger role (cooperates) if the controlled or cognitive process is unable to reach a decision (see e.g. Botvinick, Carter, Braver, Barch, and Cohen, 2001).

However, there could be cases when the parallel work between the two processes conflicts. In this situation, time becomes an important variable and, as a result, the automatic or deliberative process, which is much faster, might select the option suppressing the decision from the controlled process. In either situation, it can be seen that the parallel dual model becomes in fact a two-stage model, where both processes work sequentially and the controlled process can influence the result only if the impulse created by the automatic process is suppressed or vice verse.

The literature shows that neo-classical models are not accurate at describing decision-making. Neoclassical economics states that "… economic agents rationally make decisions, then, optimize their utility in a predictable way when they consume, and efficiently produce by combining production factors in the best feasible way" (De Schant, Martin, and Martín Navarro, 2012, p. 2). This argument has been questioned by a series of experiments conducted by expert psychologists, sociologists, and economists in which a supposed "irrationality" seems to take over the decision-making process to such an extent that consumers select the alternative that breaks the boasted rationality in traditional economic theory. From this arises the thesis of irrationality or cognitive biases in the decision-making process.

Per De Schant, Martin, and Martin (2012), human behavior could be understood as the result of the interaction of processes and systems. This implies that human behavior is determined by 1) controlled processes that assume a subjective feeling of effort to reach the objective; 2) automatic processes that are carried out without mental effort and do not imply cognitive activity; and 3) affective systems (related to feelings) and cognitive systems (knowledge). Acevedo (2013) argues that there are several other aspects that influence decision making such as beliefs (religion and political judgement), education, gender, age, and space-time location.

In this paper, we follow Acevedo (2013) and build a mathematical model that explains the cooperation or conflict between the cognitive or controlled process and the deliberative or automatic process that takes into account individual human factors and show that in cases where these two processes cannot come up with a selection, then the choice would depend fundamentally on stochastic or random shocks and other individual human factors. Particularly, our model considers not only the theory and approaches of behavioral economics but also how human action (individual freedom, learning, and other human factors related to the decision being evaluated) exerts the greatest influence in the first stage of decision-making. In the second stage, everything else being equal, the factor with the greatest influence is one that we define as a stochastic shock formed by all non-measurable probabilities associated with feelings and negative and/or positive information about the options to be chosen. These two factors are *proxies* for the "irrational" part of the decision-making process.

Throughout time, scientific studies have allowed progress on this subject and have permitted the inclusion of several factors that the expected utility theory did not consider in the decision-making process. However, in the light of the revised literature, we argue that the inclusion of individual factors is a necessity. This, we think, would be an advancement, based on what Von Mises (1949) defined as the elements of human action, which go beyond a simple computation or generalized mathematization of a process carried out in a framework of means that allow an individual to reach some goals. Nevertheless, the arguments against and for among authors does not get to a halt. Nobel Prize Vernon Smith (1999) says that the human action, which according to Mises is consciously intentional, is not a necessary condition of the system and that underestimates the unconscious processes that are activated at the time of decision making.

Two stage decision models have been used quite frequently in the literature to study or test their implied hypothesis or theories. Here, we provide a short list of several works that have addressed two stage decision models. For instance, (Huang et al., 2020) propose a two-stage decision making method that considers the burden distribution of stability and energy consumption in the iron-making sector. Han, Zhu, Ke, and Lin (2019) developed a two-stage decision framework based on the graph model of conflict resolution to facilitate the resolution of conflicts. Zheng, Su, and Zheng (2019) for designing flexible warranty policies they developed a two-stage decision framework. Andersen, Harrison, Lau, and Rutström (2014) considered multiple criteria when studying the behavior of contestans in the *Deal or No Deal* TV show.

Jin, Chen, and Lingling (2009) studied decision making in the context of e-commerce in China. Fudenberg and Levine (2006) argued that their dual-selves model explains a broad range of behavioral anomalies and better fit the modular structure of the brain than the quasi-hyperbolic models. Loewenstein and O'Donoghue (2004) develop a model in which the outcome of an agent´s behavior results from the interaction between two processes: deliberative processes that assess options with a broad, goalbased perspective, and affective processes that encompass emotions and motivational drives. Finally, Pohlmeier and Ulrich (1995) estimated a tw stage econometric model regarding where the decision of visiting the doctor for the first time is an individual´s decision but the number of visits will depend on the physician.

The model we propose in this paper rests on individualism. There are human, non-measurable elements, that affect rational decisions driven by utility (profit) maximization or cost minimization at the time of selection. Therefore, the objective of our model is not to predict the behavior or decision of an agent, but rather to analyze the impact of human factors and human random shocks and thus theoretically demonstrate that these elements, that represent what is theoretically defined as "irrational", are the most influential factors in the decision-making process.

Finally, we have organized this paper as follows: the introduction presents a brief state and scope of our proposal. In section two, we conceptualize the proposed model and present some theoretical examples related to perfect substitute goods and pure public goods. In the third section, we report the procedure, results, and analysis of a series of simulations we carried out using Python to test the perfect substitute goods theoretical example. Lastly, section fourth concludes.

2. A Two-Stage Decision Making Model with Human Factors

Our model assumes that an agent makes a final decision after a two-stage process of evaluation of different options. Initially, and considering the contributions of Von Neumann and Morgenstern (1944), the individual carries out the economic analysis of the options based on expected utility and cost, determined by the aversion to risk and loses.

We developed the first stage based on the contributions of Von Neumann and Morgenstern (1944). We state that, in this stage, the individual or agent (for example, a consumer) carries out the economic analysis of the options considering their expected utility and costs. Following Tversky and Khaneman (1991) and Starmer (2000), our model accounts for the agents´ aversion to risk and losses and the human factor (Acevedo, 2013), see Figure 1. In the second stage, choice is given by the interaction between cognitive and affective systems, and a random shock only if agents' economic willingness to choose among different market baskets are the same.

[Figure 1 here]

When an individual must decide on what quantity of what goods and services to choose or to include in her market basket should she choose it to maximize her utility, the decision problem arises. Let $\{x_i\}_{i=1}^{n}$ be a finite-countable set of goods and/or services that represents the consumption bundle and are openly trade at monetary units publicly quoted (principle of universality of markets). Furthermore, the individual does not have any influence on prices (price-taking assumption). For convention, we assume that the individual has enough economic resources to access any of the options she evaluates, then: $p_i x_i \leq b$, where p_i is the price of x_i and b defines the boundary of having a fixed income.

We assume that when the individual evaluates an option, she considers its expected utility and costs by means of her cognitive bias "mental accounting" (Thaler, 1999). Now, the agent's economic willingness to consume x_i would be represented by: $X_{x_i} = U_{x_i} - C_{x_i}$, where U_{x_i} represents the expected utility or reward to be received and C_{x_i} is the total cost of choosing x_i and no another option. We assume that the consumer determines the total cost of choosing x_i by adding the opportunity cost of selecting an option other than x_i and other unknown costs that the she expects to incur. Let $C_{x_i} = U_{x_i'} + \varsigma_{x_i}$. According to what have been defined, $U_{x_i'}$ is the opportunity cost of choosing an option other than x_i and ς_{x_i} are the other unknown costs associated with x_i.

The capacity to measure the unknown costs varies among consumers. It depends on their experience, exogenous information, and risk aversion. Nevertheless, we assume that in case of total absence of information, agents will value $\varsigma_{x_i} = U_{x_i}$. As a result, a consumer's economic willingness to choose x_i would be: $X_{x_i} = -U_{x_i'}$[1]. Likewise, her economic willingness to choose x_i' would be: $X_{x_i'} = -U_{x_i}$.

The traditional theory states that after this economic evaluation, the individual, under a rational decision, will select x_i over any other option x_i', if and only if, $X_{x_i} > X_{x_i'}$.

We consider the Bayesian approach that allows us to analyze results not just in terms of utility but probability. Shifting our analysis to probabilities allowed us to include the *individual human factor* and obtain the probability of initial willingness to choose any option.

Let $P_{U_{x_i}}$ and $P_{\left(C_{x_i} \geq U_{x_i}\right)}$ be, respectively, the probability of incidence of the expected utility and the probability that the cost of choosing x_i is equal or greater than its expected utility. In an automatic and unconscious process, consumers set these probabilities based on exogenous information and/or their aversion to losses and risks. If the agent risk aversion is high, then her probability of expected reward will be low and cost will be high, and vice versa.

Following Acevedo (2013), when consumers evaluate their different options, there is an *individual human factor* that affects their probability of initial willingness to choose. This author defines it as a parameter that represents a dynamic learning process. However, in this paper, we extend the discussion over this *individual human factor* when choosing x_i , or, x_i' at time t. That is, at different points in time $t_1, t_2, \ldots, t_m$, the parameter, ψ_t, can take different values, even when a consumer is deciding among the same options. Thus, ψ_t presents a stochastic behavior. Also, we suppose that this is a dynamic parameter that includes elements of learning, freedom, and other human factors.

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[1] That is $X_{x_i} = U_{x_i} - C_{x_i} = U_{x_i} - \left(U_{x_i'} + \varsigma_{x_i}\right) = U_{x_i} - \left(U_{x_i'} + U_{x_i}\right) = -U_{x_i'}$

Let the following be vectors that contain values of learning (L), freedom (F), and other human factors (OHF) that are accumulated from previous experiences and affect the decision-making process, respectively:

$$\begin{bmatrix} L_1 \\ L_2 \\ L_3 \\ \vdots \\ L_u \end{bmatrix} \quad \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ \vdots \\ F_v \end{bmatrix} \quad \begin{bmatrix} OHF_1 \\ OHF_2 \\ OHF_3 \\ \vdots \\ OHF_z \end{bmatrix} \tag{1}$$

and we assume that the "individual human factor" ψ_t at any point in time $t \in \{t_1, \dots, t_m\}$, is obtained weighing the values of the components (previously and individually weighted) learning, freedom, and other human factors. For this, with $w_{1,t}$, $w_{2,t}$, and $w_{3,t}$ weight parameters, we define ψ_t as follows:

$$\psi_t = \frac{\left[w_{1,t}\left(\frac{\sum_{r=1}^{u} L_r * v_r}{\sum_{r=1}^{u} v_r}\right)\right] + \left[w_{2,t}\left(\frac{\sum_{s=1}^{v} F_s * z_s}{\sum_{s=1}^{v} z_s}\right)\right] + \left[w_{3,t}\left(\frac{\sum_{y=1}^{z} OHF_y * u_y}{\sum_{y=1}^{z} u_y}\right)\right]}{\sum_{k=1}^{3} w_{k,t}} \tag{2}$$

where v_r, z_s, and u_y are values that consumers determine, based on their personality, preferences, or the importance that they give to each one. Next, the human factor is determined by means of $w_{1,t}, w_{2,t}$, and $w_{3,t}$, which are parameters that randomly change over t and allow obtaining the value of ψ_t, but:

i) if $P_{U_{x_i}} \geq P_{\left(C_{x_i} \geq U_{x_i}\right)}$ then $0 \leq \psi_t \leq 1 - \left(P_{U_{x_i}} - P_{\left(C_{x_i} \geq U_{x_i}\right)}\right)$

or

ii) if $P_{U_{x_i}} < P_{\left(C_{x_i} \geq U_{x_i}\right)}$ then $\left|P_{U_{x_i}} - P_{\left(C_{x_i} \geq U_{x_i}\right)}\right| \leq \psi_t \leq 1 + \left|P_{U_{x_i}} - P_{\left(C_{x_i} \geq U_{x_i}\right)}\right|$

Therefore, the probability of initial willingness to choose x_i can be written as follows:

$$P_t^I(x_i) = \left(P_{U_{x_i}} - P_{\left(C_{x_i} \geq U_{x_i}\right)}\right) + \psi_t \tag{3}$$

As we stated before, using the Bayesian approach allowed us to measure the economic willingness to select any option in probabilistic terms. Furthermore, including a probabilistic parameter defined as individual human factor allows us to shift the expected utility assumption described by the linear function $X_{x_i} = U_{x_i} - C_{x_i}$ and use it as a stochastic variable. Now, we have defined the probability of initial willingness to choose x_i as:

$$X_{x_i}(t) := P_t^I(x_i) = \left(P_{U_{x_i}} - P_{\left(C_{x_i} \geq U_{x_i}\right)}\right) + \psi_t,$$

and for each $t \epsilon \{t_j\}_{j=1}^{m}$, under two conditions: a) for each $x_i \epsilon \{x_i\}_{i=1}^{n}$, $0 \leq X_{x_i}(t) \leq 1$; and b) $\sum_{i=1}^{n} X_{x_i}(t) = 1$.

The second stage, see Figure 1, following the argument of Loewenstein and O'Donoghe (2004), is characterized by two systems, affective and cognitive, that affect the final decision. However, our model includes a stochastic shock, which is considered as all negative and/or positive feelings and information not known at the moment of making the decision but affects the final decision. We understand that including such a term could not be a novel concept in behavioral economics (i.e. Laibson, 2001), nevertheless our contribution goes beyond the use of a stochastic shock but the definitions, approach, and assumptions we have considered.

Individuals have three different options at the end of this process, see figure 1. The first (Yes) is if their final decision is to choose that specific product or service (and what quantity in the market basket), then, the process ends. The second option is “Do not know”, which means that the individual will repeat the second stage. The third option is “No”, in this the individual will value or consider another option.

While the probability of initial willingness is determined, the affective and cognitive systems will activate and begin to send unconscious, unfavorable, or favorable, impulses in relation to the evaluated option, let us say x_i. Within this frame of reference, the contributions of Tversky and Kahneman (1991) as well as the findings of Ariely (2010a, 2010b), indicate that all the negative impulses of the affective and cognitive system will be confronted by positive impulses from both systems.

Let the following be vectors of negative feeling probabilities and negative information probabilities respectively, for each one of the options, x_i, that the individual must select:

$$\begin{bmatrix} \alpha_1^N \\ \alpha_2^N \\ \alpha_3^N \\ \vdots \\ \alpha_p^N \end{bmatrix} \text{ and } \begin{bmatrix} \lambda_1^N \\ \lambda_2^N \\ \lambda_3^N \\ \vdots \\ \lambda_q^N \end{bmatrix} \tag{4}$$

Individuals will determine the total probabilities of affective (α^N) and cognitive rejection (λ^N) of the evaluated option as follows: $\alpha^N = \frac{1}{p}\left(\sum_{l=1}^{p} \alpha_l^N\right)$ and $\lambda^N = \frac{1}{q}\left(\sum_{l=1}^{q} \lambda_l^N\right)$. Then, the joint probability of final rejection towards any option x_i at any point in time t_j can be defined by:

$$\beta_{t_j} = 0.5(\alpha^N + \lambda^N).$$

Let us α^P and λ^P be the total probabilities of affective and cognitive acceptance of the evaluated option. Then: $\alpha^P = 1 - \alpha^N$ and $\lambda^P = 1 - \lambda^N$ allow us to define the joint probability of final acceptance towards any option x_i at any point in time t_j as follows:

$$z_{t_j} = 0.5(\alpha^P + \lambda^P).$$

Considering all above assumptions, we can define the final probability of choosing any option x_i at time t_j as follows:

$$Y_{x_i}(t_j) := \mathrm{P}_{t_j}^{\boldsymbol{F}}(x_i) = X_{x_i}(t_j)\left(z_{t_j} - \beta_{t_j}\right) + \varepsilon_{t_j} \tag{5}$$

Considering that ε_{t_j} is our stochastic shock formed by all the probabilities associated with negative and/or positive feelings and information (previously defined as α and λ) not considered in β and z at time t_j, but that have an influence on the final choice of the individual subject to these restrictions:

i) $0 \leq \varepsilon_{t_j} \leq 1 - X_{x_i}(t_j)\left(z_{t_j} - \beta_{t_j}\right)$ for the case $z_{t_j} \geq \beta_{t_j}$,

or

ii) $\left|X_{x_i}(t_j)\left(z_{t_j} - \beta_{t_j}\right)\right| \leq \varepsilon_{t_j} \leq 1 + \left|X_{x_i}(t_j)\left(z_{t_j} - \beta_{t_j}\right)\right|$ for the case $z_{t_j} < \beta_{t_j}$.

Finally, our model should be analyzed considering that:

- This evaluation can be carried out for a single option or more than one, following the same procedure and in the end selecting the one or, in some cases, those with greater $Y_{x_i}(t_j)$.
- The unlikely and even extreme scenario, not denied in this model, that $X_{x_i}(t_j) = 0$, or $z_{t_j} = \beta_{t_j}$ in equation (5). In this case, we will have that $Y_{x_i}(t_j) = \varepsilon_{t_j}$, which means that the final choice x_i in the time t_j will depend solely on the stochastic shock ε_{t_j}.
- The first decision making process of an agent, is at time t_1. Our model considers that $z_{t_j} = \beta_{t_j} = 0$, because the agent does not have any previous experience with the product. In this case the final decision will be made considering other positive and negative impulses that interact in ε_{t_j}, as in the previous consideration.
- The final decision is determined by the individual probability of the initial willingness, the affective system, the cognitive system, and the shock defined above. Finally, the individual's final probability of choosing, defined in equation (5), will determine which of the options considered will be selected.
- Equation (5) allows us to prove the preference-based approach. For example: i) if $Y_{x_i}(t) > Y_{x_i'}(t)$, then we can state that exists a strict preference in time t and $x_i \succ x_i'$; ii) if $Y_{x_i}(t) \cong Y_{x_i'}(t)$, then we can state that exists an indifference preference in time t and $x_i \sim x_i'$. It also allows us to keep the completeness and transitivity properties, i.e: iii) for all $x_i, x_i' \in \{x_i\}_{i=1}^{n}$, we have that $Y_{x_i}(t) \geq Y_{x_i'}(t)$ or $Y_{x_i'}(t) \geq Y_{x_i}(t)$; and iv) if $Y_{x_i}(t) > Y_{x_i'}(t)$ and $Y_{x_i'}(t) > Y_{x_i''}(t)$, then we can state that in time t preferences are transitive: $x_i \succ x_i'$, $x_i' \succ x_i''$, and $x_i \succ x_i''$.

2.1 Theoretical Examples

In order to facilitate the understanding of our model, we present two theoretical cases: perfect substitutes goods and pure public goods.

2.1.1 Perfect Substitutes

Let us assume an agent must choose between two perfect substitute goods x_i and x_i' with the same choice costs. Then, at time t_j, the agent´s probability of initial willingness to pay for either product is the same. Therefore:

$$X_{x_i}(t_j) = \left(P_{U_{x_i}} - P_{\left(C_{x_i} \geq U_{x_i}\right)}\right) + \psi_{t_j} = \left(P_{U_{x_i'}} - P_{\left(C_{x_i'} \geq U_{x_i'}\right)}\right) + \psi_{t_j} = X_{x_i'}(t_j)$$

Case A:

If we assume that x_i has a higher level of acceptance (lower level of rejection) than x_i', then:

$$\mathbb{E}\left(z_{t_j} \middle| x_i\right) > \mathbb{E}\left(z_{t_j} \middle| x_i'\right)$$
$$\mathbb{E}\left(\beta_{t_j} \middle| x_i\right) < \mathbb{E}\left(\beta_{t_j} \middle| x_i'\right)$$

and

$$\mathrm{P}_{t_j}(x_i) = X_{x_i'}(t_j)\left[\left(z_{t_j} \middle| x_i\right) - \left(\beta_{t_j} \middle| x_i\right)\right] + \left(\varepsilon_{tj} \middle| x_i\right)$$
$$\mathrm{P}_{t_j}(x_i') = X_{x_i}(t_j)\left[\left(z_{t_j} \middle| x_i'\right) - \left(\beta_{t_j} \middle| x_i'\right)\right] + \left(\varepsilon_{tj} \middle| x_{i'}\right)$$

as a result:

$$x_i \succ x_i' \;\forall\; \left\{\varepsilon_{tj} : Y_{x_i}(t_j) > Y_{x_i'}(t_j)\right\}$$

This result shows that in this case the agent will select x_i at t_j always and for every value of the stochastic shock, conditioned on the final probability of choosing x_i being greater than that of x_i'. This is a clear indication of the importance of the human factor shock in our model.

Case B:

Once again, let us consider that an agent is indifferent between two goods and that x_i and x_i' generate the same levels of acceptance and rejection. Then, the conditioned expected values of z_{t_j} and β_{t_j} are equal; this implies that:

$$\mathbb{E}\left(z_{t_j} \middle| x_i\right) = \mathbb{E}\left(z_{t_j} \middle| x_i'\right)$$
$$\mathbb{E}\left(\beta_{t_j} \middle| x_i\right) = \mathbb{E}\left(\beta_{t_j} \middle| x_i'\right).$$

Then:

$$Y_{x_i}(t_j) = X_{x_i'}(t_j)\left[z_{t_j} - \beta_{t_j}\right] + (\varepsilon_{tj} | x_i)$$
$$Y_{x_i'}(t_j) = X_{x_i}(t_j)\left[z_{t_j} - \beta_{t_j}\right] + (\varepsilon_{tj} | x_i')$$

as a result:

$$x_i \succ x_i' \ \forall \ \{\varepsilon_{tj} : (\varepsilon_{tj} | x_i) > (\varepsilon_{tj} | x_i')\}$$

Thus, the decision will be determined by the value of the stochastic shock. This means that if the agent picks x_i at time t_j, it is not strictly necessary that she would do the same in $t_{j+1}, t_{j+2}, \dots, t_{j+m}$ given that the shock changes for each t.

2.1.2 Pure Public Goods

Let x_i and x_i' be two pure, mutually exclusive, public goods. Suppose an agent expects the same level of utility from each. Then:

$$U_{x_i} = U_{x_i'}$$

Since these are pure public goods, there are no costs associated with their use[2], then:

[2] Pure public goods have no costs to an agent, except other costs that she must pay to have free access to the good. However, these costs are compensated when the agent determines the expected utility she will get from such an option. A good example is given when a person wants to go to a public park, each located at extremes borders of the city (north and south). If she has to pay for transportation (gas, parking, and so on), then she considers such costs at the time of estimating her expected utility. Now, suppose that going to either park has the same

$$C_{x_i} = U_{x_i} = U_{x_i'}$$
$$C_{x_i'} = U_{x_i'} = U_{x_i}$$

Thus, the probability of initial willingness to select option x_i would be given by equation (3), while for x_i' would be[3]:

$$X_{x_i'}(t_j) = \left(\left(1 - P_{U_{x_i}}\right) - \left(1 - P_{\left(C_{x_i} \geq U_{x_i}\right)}\right)\right) + \psi_{t_j}$$

Lastly, the second stage of the model determines the final decision.

3. Simulations and Analysis of Results

In this section, we carried out an extensive simulation of example 2.1.1 using Python with the purpose of analyzing the decision-making process within the context described above. First, we determined the expected utility and costs associated with each option and, second, considered three individuals: investor A was supposed to be an optimistic person (we assign the value of 35% for risk aversion), investor B, an indifferent individual (45% risk aversion), and investor C, a pessimist individual (65% risk aversion).

Let us assume that three investors are interested in two corporations and need to decide where to invest. Suppose x_i and x_i' are each firm´s shares and that these are perfect substitutes of one another, in the sense that their returns and expected utility are the same. Following the considerations and assumptions defined in the first stage, Table 1 shows that all investors have the same economic willingness to choose each option, which is -15 monetary units (m.u.) because the opportunity cost of x_i (15 m.u.) is the expected utility of x_i', and vice versa.

[Table 1 here]

transportation cost. Then, the unknown cost of going to either park would be given by a stochastic shock: raining in the north park. As a result, the expected utility of going to the north park becomes zero.

[3] Given these are mutually exclusive options.

We are assuming that there is total absence of information to estimate unknown costs. Since there is no exogenous information, the probabilities of incidence of the expected utility and costs of choice are determined by the investors' risk aversion. Considering the three agents previously mentioned, Table 2 below shows the probabilities of incidence of the expected utility and costs of choice. We assume the same values for each share.

[Table 2 here]

According to the weights assigned in equation (2), each agent or individual has a finite number of values in the vectors of learning, freedom, and other human factors, so each time the decision-making process ends, the quantity of values in these vectors rises. To simulate these scenarios, we need to establish an array of finite and random values for any t. For simplicity, we imposed the restriction that these random values should be between 0 and 100. Table 3 shows the values with which we proceed to the simulation.

[Table 3 here]

Given the values in Table 3, we used the values presented in Table 4 (for both x_i and x_i' and for different levels of utility and cost), as a basis to find through the different simulation procedures the different values of w_k and, consequently, those of the human factor from equation (2) for each of the routines.

[Table 4 here]

Theoretically, we know that a stochastic process with space of states and discrete parameter indexes describes our model. To tests this, we designed a computational experiment as follows: for a given generic time t_j, mathematically fixed, we carried out, for each agent, a total of 10 simulation routines, where each value associated with the probability of initial willingness and the final probability of choosing an option, see equation (5), defines the behavioral set of X_{t_j} and Y_{t_j} as random variables.

[Table 5 here]

After carrying out the simulations, we found very interesting and suggestive results. In tables 5, 6, and 7 we report these results. To begin with, let us consider individual A, whose results are presented in Table 5. Her initial state of optimism (weighting her individual human factor, see equation 2, in the same way), does not necessarily mean that the probability of initial willingness value is the same. This depends on the interaction between the expected utility generated by the choice of that good and its corresponding opportunity cost, where the magnitude of the human factor does not mark a strictly increasing behavior on those probabilities.

On the other hand, if the individual were to increase her risk aversion by 30 percentage points, considering herself as an economically "pessimistic" person, then regardless of whether her weight towards the human factor remains constant, increases or decreases, the initial willingness probability values will now decrease considerably. This is an expected result because in this case such a drastic paradigm shift towards the preference of such a good prevails over its human factor.

Table 6 below shows the results for individual B. We observed strictly random behavior for equal weighting magnitudes and between different weighting magnitudes for the individual human factor considered in the initial state of neutrality. However, a risk aversion increase of 10 percentage points generates a decrease in the probability of initial willingness of such an individual, but not as drastic as in the situation simulated for agent A.

[Table 6 here]

Finally, when considering results for individual C (pessimist-optimist), the probability of initial willingness shown in Table 7 are probabilistically different from the optimist-pessimist values shown in Table 5. This is another interesting finding from our computational experiment. Before increments or reductions of equal magnitude with respect to the percentage of risk aversion of individuals, in general, which ultimately exerts the greatest influence on the differences between the probabilities of initial willingness for both scenarios, is the value of the individual human factor (ψ_{t_j}).

[Table 7 here]

Following our theoretical example 3.1.1 and in accord with results presented in Tables 5, 6, and 7, we know that the economic, and the probability of initial, willingness of choosing either option is the same for both stocks x_i and $x_i{}'$ (under different settings). At this time, the second stage of our model is activated, and it is here where the individual decision is affected. The differentiation process (preference and rejection) begins with the weighing of probabilities of feelings related to the affective and cognitive system, thus determining the joint probabilities of rejection and final acceptance.

Table 8 presents two vectors of probabilities: one for feelings and the other for information for each investor. Within this reference frame and in order to make it as similar as possible to the second stage of our choice model, we have established a matrix of random real numbers within the interval $[0,1]$. Based on the theoretical restrictions imposed on the stochastic shock, as in the first stage, here we carried out 10 simulation routines for each investor. We used all our priori information as well as the one calculated in accord with our definition of the probability of initial willingness and focusing on emulating the final probability of choosing any option, equation (5), for the three investors under the different scenarios and disturbances in terms of their risk aversion.

[Table 8 here]

For the economically optimistic individual, we observe, in Figure 2, that she probabilistically favored stock $x_i{}'$ over x_i (see Table 8) then in most choice trials, her final probability of choosing x_i' over x_i prevails, even when the random shock dominates in magnitude for both options at time t_j.

[Figure 2 here]

Figure 3 reports results when investor A change her optimism to pessimism. Although probabilistically action $x_i{}'$ in general continues to prevail over x_i in the environment of the respective polygonal[4], it is not true that the relationship and co-movements between the two in the face of such a disturbance, regarding their aversion, are exactly the same as the initial state.

[4] We are using the term polygonal to refer to the lines show in the figures. We use this mathematical concept to show the election at a given point in time and therefore do not represent time series.

This occurs because the stochastic shock also has an important influence. Naturally, the average probability of the polygonal that generally prevails over the other in the previous graph is higher than what now prevails in the disturbed system, since now the individual enters an economic state of pessimism and this directly influences the final probability of choosing for both shares.

[Figure 3 here]

For the neutral individual B, in Figures 4 and 5, as the difference between the components for both x_i and x_i' are both negative (see Table 8), being lower for the second one, the share x_i dominates (prevails) throughout the environment of the respective polygonal. However, the transitions of both are more similar and smoother with respect to the graphs presented for investor A. This is a logical finding: given the neutrality in the cost of aversion and the similarity between the individual's final components before the final decision, the co-movements of the final probability of choosing any share, as images of a random variable at time t_j, must be similar for the two possible stocks. In fact, this is the case with a 10 percentage points increase in the aversion cost.

[Figures 4 and 5 here]

Finally, similar to individual A, we observe very erratic co-movements in the polygonal of investor C, where indeed the share x_i prevails probabilistically over share x_i'. Figures 6 an 7 shows the different values of the final probability of choosing x_i and x_i' at the specific value of the stochastic shock. An important remark is that if this initially pessimistic individual reduced her cost of risk aversion by 30 percentage points, this should empirically generate a significant increase in the final probability of choosing independently which one is preferred.

[Figures 6 and 7 here]

4. Conclusions

We present this model with the purpose of not to predict the behavior or decision made by human beings but to analyze the impact of the human factor and the random stochastic shock, commonly defined as the irrational part of decisions, as the most influential factors in the process of individuals' decision making.

Despite the extensive literature in this area, the vast majority has focused on neo-classical analysis or based on expected utility theory without considering any individual human factor. We built a two-stage model that goes more in the direction of the theory and approaches of behavioral economics. It takes in consideration the human action, reflected in the individual human factor and the stochastic shock, which have the greatest role in the first and second stage respectevily. This is supported by individualism at the time of selection. Agents not only select exclusively the option that ensures utility maximization or cost minimization, but also make decisions based on non-measurable human factors. Perhaps, when this happens, researchers tend to classify those decisions as "irrationals".

The decision-making process is not just something that concerns economics. This science provides the fundamental basis for its analysis and interpretation, but other sciences should also be considered, since this process is linked to all areas of daily life. By including other sciences, behavioral economists are trying to find temporary answers and explanations about the decision-making process of individuals. Unquestionably, one cannot think of a single and generalized theory on this subject since it was evidenced that human behavior depends on many elements, factors, and moments that cannot be measured even in controlled experiments. There is still some uncertainty that cannot be explained and that is why equation (5) includes a random stochastic shock and equation (3) the human factor.

Finally, the results we obtained through our simulated experiments suggest that all those human factors and those not observable or measurable factors are the most important explanatory elements in agent´s decision making. Precisely, these elements are what have been called irrational factors at the time of the election. However, further research and the expansion of our model with real experiments could serve to strengthen our findings. Meanwhile, the irrational is the most rational thing in the decision-making process of human beings.

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Figure 1

Two-Step Decision Making Model with Human Factor

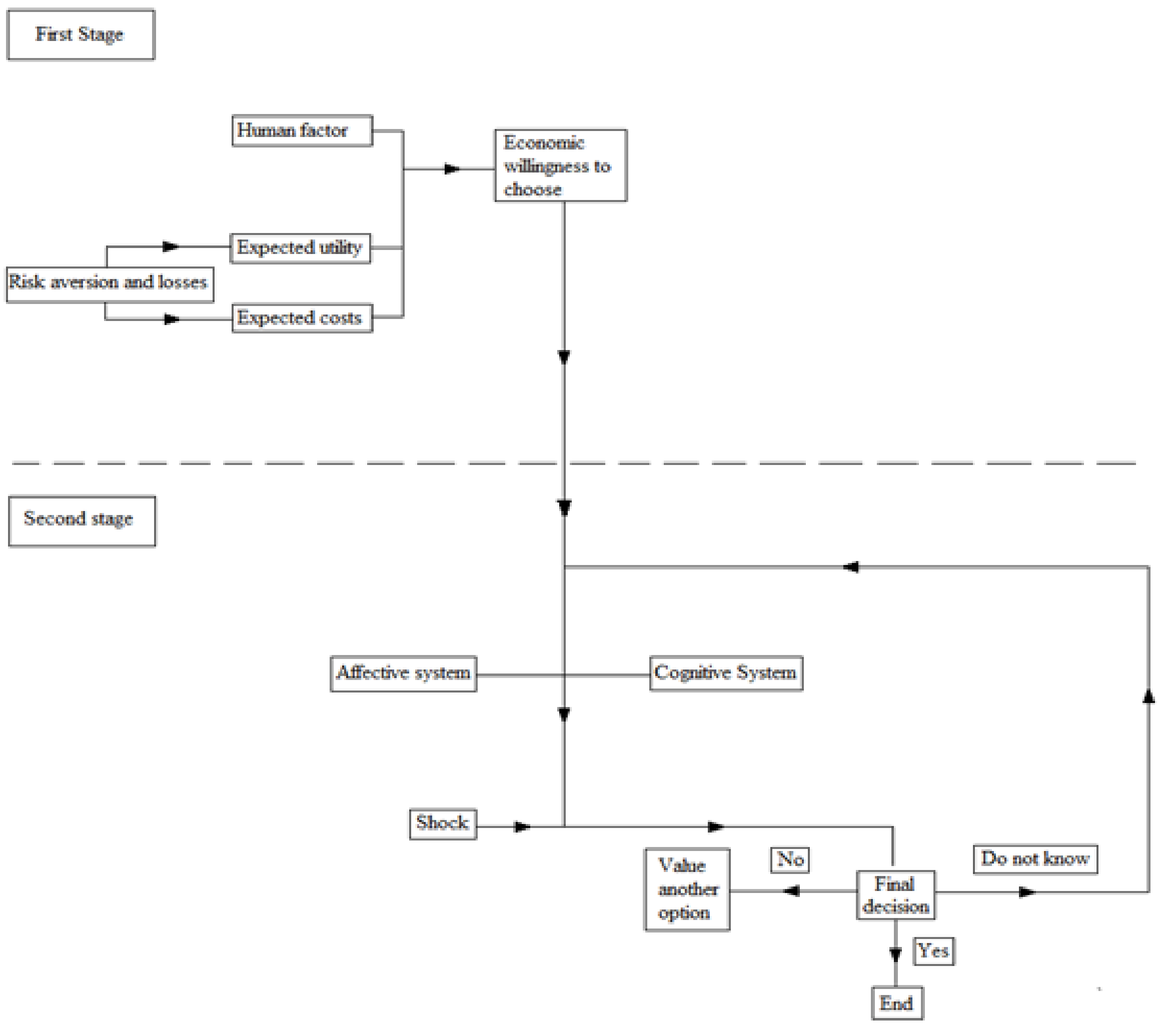

Table 1

Economic willingness to select either option

	Business shares	
	$\boldsymbol{x_i}$	$\boldsymbol{x_i'}$
Expected utility	15	15
Opportunity cost	15	15
Other costs	15	15
Total costs of election	30	30
$X_{x_i} = X_{x_i'}$	-15	-15

Note: Values are expressed in monetary units.

Table 2

Probabilities of Incidence of the Expected Utility and Costs of Choice Under Different Levels

	Extreme cases	
Agent	$\boldsymbol{P_{U_{x_i}} = P_{U_{x_i'}}}$	$\boldsymbol{P_{(C_{x_i} \geq U_{x_i})} = P_{(C_{x_i'} \geq U_{x_i'})}}$
A	0.65	0.35
B	0.55	0.45
C	0.35	0.65

Note: Probability values assigned by the authors.

Table 3

Random Values for Learning, Freedom and other Human Factors at Time t_j

Individual A			Individual B			Individual C		
L	**F**	**OHF**	**L**	**F**	**OHF**	**L**	**F**	**OHF**
90	99	78	66	15	78	37	3	66
71	24	90	80	54	64	94	56	41
70	83	21	98	41	92	54	16	70
59	76	41	33	31	7	66	5	26
22	94	69	100	38	55	41	78	68
90	34	23	99	28	53	54	90	96

Table 4

Values of v_r, z_s and u_t for the Individuals A, B and C

v_r	z_s	u_t
0.78	0.99	0.23
0.43	0.98	0.42
0.97	0.10	0.76
1.00	1.00	0.97
0.33	0.69	0.20
0.81	0.70	1.00

Table 5

Initial Results for the Optimistic Individual

	Individual A			
	U = 0.65; C = 0.35		U = 0.35; C = 0.65	
Routine	ψ_{t_j} for x_i or $x_i{}'$	$X_{x_i}(t_j)$ or $X_{x_i{}'}(t_j)$	ψ_{t_j} for x_i or $x_i{}'$	$X_{x_i}(t_j)$ or $X_{x_i{}'}(t_j)$
1	0,6962	0,9962	0,7133	0,4133
2	0,6962	0,9905	0,6962	0,3962
3	0,6905	0,9996	0,7019	0,4019
4	0,6905	0,9876	0,7047	0,4047
5	0,6962	0,9927	0,7064	0,4064
6	0,6996	0,9986	0,7076	0,4076
7	0,6996	0,9859	0,6905	0,3905
8	0,6876	0,9937	0,6996	0,3996
9	0,6876	0,9981	0,7035	0,4035
10	0,6927	0,9848	0,6876	0,3876

Table 6

Initial Results for the Neutral Individual

	Individual B			
	U = 0.55; C = 0.45		U = 0.45; C = 0.55	
Routine	ψ_{t_j} **for** x_i **or** $x_i{}'$	$X_{x_i}(t_j)$ **or** $X_{x_i{}'}(t_j)$	ψ_{t_j} **for** x_i **or** $x_i{}'$	$X_{x_i}(t_j)$ **or** $X_{x_i{}'}(t_j)$
1	0,7931	0,8931	0,7906	0,6906
2	0,7931	0,8483	0,7471	0,6471
3	0,7931	0,8632	0,7616	0,6616
4	0,7931	0,8707	0,7688	0,6688
5	0,7931	0,8752	0,7732	0,6732
6	0,7931	0,8781	0,7761	0,6761
7	0,7483	0,8334	0,7325	0,6325
8	0,7483	0,8572	0,7558	0,6558
9	0,7632	0,8675	0,7657	0,6657
10	0,7632	0,8259	0,7253	0,6253

Table 7

Initial Results for the Pessimistic Individual

	Individual C			
	U = 0.35; C = 0.65		U = 0.65; C = 0.35	
Routine	ψ_{t_j} **for** x_i **or** x_i'	$X_{x_i}(t_j)$ **or** $X_{x_i'}(t_j)$	ψ_{t_j} **for** x_i **or** x_i'	$X_{x_i}(t_j)$ **or** $X_{x_i'}(t_j)$
1	0,5727	0,2727	0,5727	0,8727
2	0,6040	0,3040	0,5727	0,9040
3	0,5936	0,2936	0,5727	0,8936
4	0,5884	0,2884	0,5727	0,8884
5	0,5852	0,2852	0,5727	0,8852
6	0,5831	0,2831	0,5727	0,8831
7	0,6145	0,3145	0,6040	0,9145
8	0,5978	0,2978	0,6040	0,8978
9	0,5906	0,2906	0,5936	0,8906
10	0,6197	0,3197	0,5936	0,9197

Table 8

Probabilities of Feelings and Negative Information and of Positive Feelings and Information

	Individual A				Individual B				Individual C			
	x_i		x'_i		x_i		x'_i		x_i		x'_i	
	α	λ	α	λ	α	λ	α	λ	α	λ	α	λ
Random values	0.404	0.197	0.234	0.522	0.856	0.898	0.643	0.661	0.438	0.096	0.947	0.481
	0.809	0.790	0.022	0.864	0.736	0.781	0.288	0.628	0.349	0.425	0.939	0.817
	0.569	0.802	0.535	0.580	0.490	0.742	0.790	0.973	0.937	0.093	0.837	0.717
	0.241	0.589	0.521	0.623	0.897	0.417	0.637	0.396	0.352	0.375	0.068	0.728
	0.814	0.703	0.249	0.224	0.090	0.586	0.609	0.836	0.702	0.523	0.870	0.866
	0.793	0.574	0.629	0.006	0.123	0.488	0.569	0.956	0.954	0.412	0.465	0.181
$\alpha^N \mid \lambda^N$	0.605	0.609	0.365	0.470	0.532	0.652	0.589	0.742	0.622	0.321	0.688	0.632
β_{t_j}	0.607		0.417		0.592		0.666		0.471		0.660	
$\alpha^P \mid \lambda^P$	0.395	0.391	0.635	0.530	0.468	0.348	0.411	0.258	0.378	0.679	0.312	0.368
z_{t_j}	0.393		0.583		0.408		0.335		0.529		0.340	
$z_{t_j} - \beta_{t_j}$	-0.214		0.166		-0.184		-0.331		0.058		-0.32	

Figure 2

Stochastic Shock and Final Choice Probabilities for Individual A

U = 0.65 and C =0.35

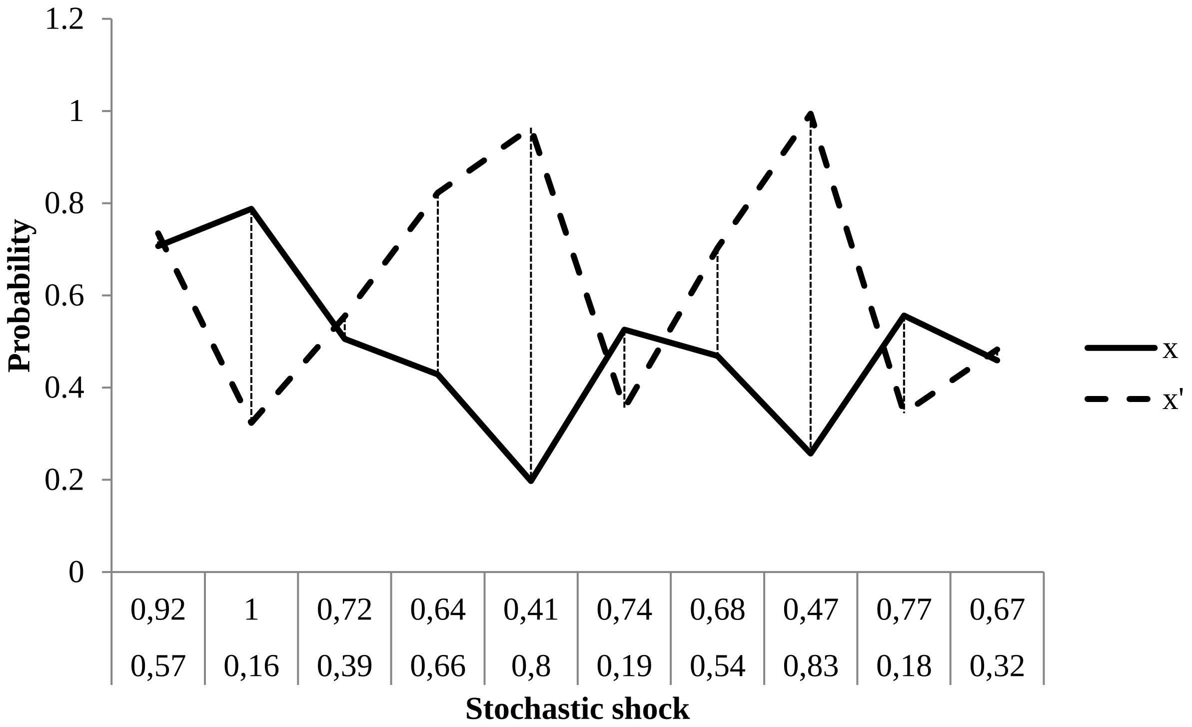

Figure 3

Stochastic Shock and Probability of Final Choice for Individual A

U = 0.35 and C = 0.65

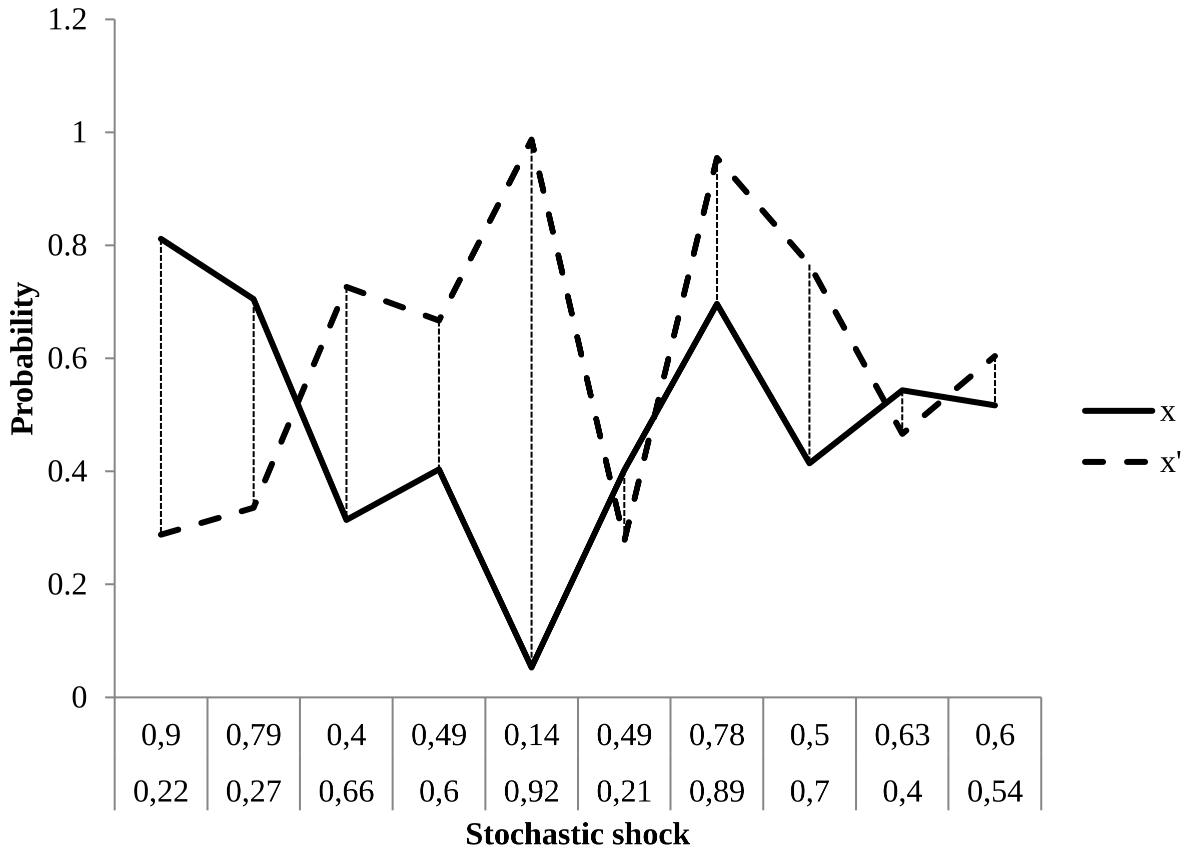

Figure 4

Stochastic Shock and Probability of Final Choice for Individual B

U = 0.55 And C = 0.45

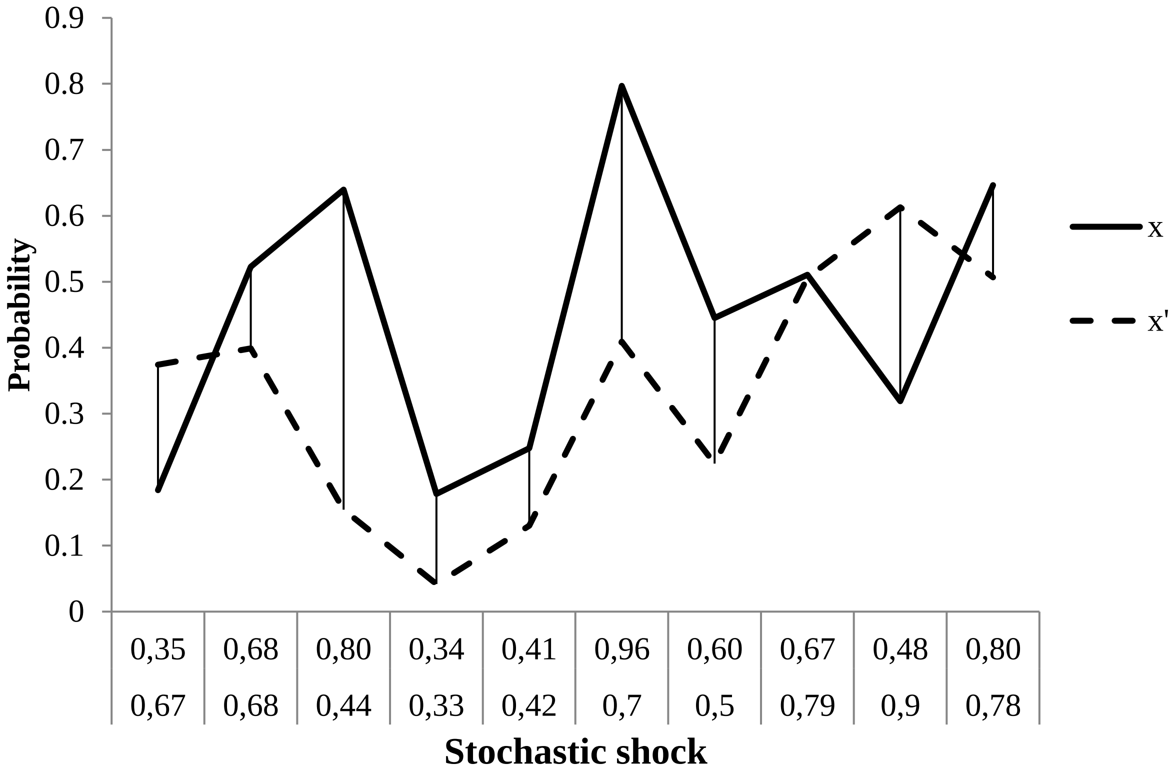

Figure 5

Stochastic Shock and Probability of Final Choice for Individual B

U = 0.45 And C = 0.55

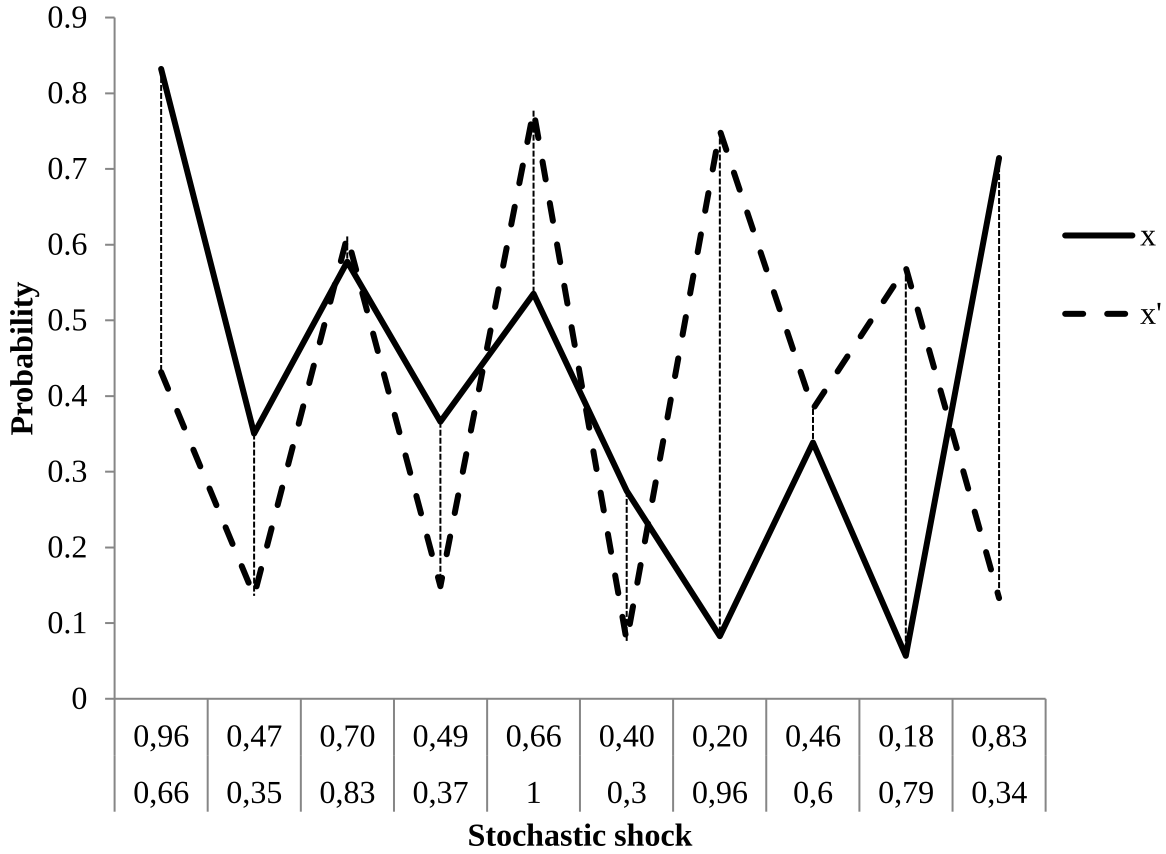

Figure 6

Stochastic Shock and Probability of Final Choice for Individual C

U = 0.35 And C = 0.65

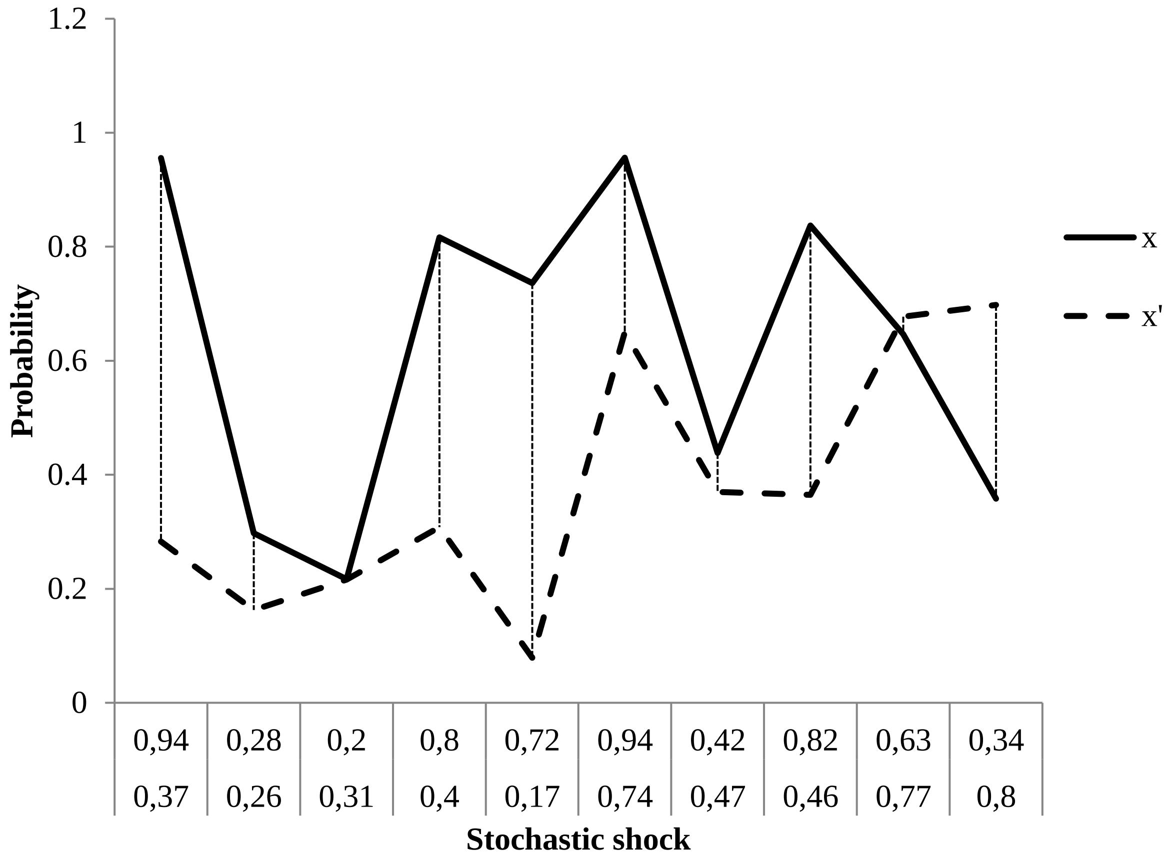

Figure 7

Stochastic Shock and Probability of Final Choice for Individual C

U = 0.65 And C = 0.35