ISSN 2090-3359 (Print)
ISSN 2090-3367 (Online)

ΑΔΣ

Advances in Decision Sciences

Volume 26
Issue 1
March 2022

亞洲大學
ASIA UNIVERSITY

Published by Asia University, Taiwan

Extension of Classical TOPSIS Method Using Q-Rung Orthopair Triangular Fuzzy Number

Meltem Yontar Aksoy (corresponding author)

Department of Industrial Engineering, Istanbul Technical University, Faculty of Management, Istanbul, Turkey

yontarm@itu.edu.tr

Ayşe Nur Karabayır

Department of Industrial Engineering, University of Passau, Passau, Germany

ayse.karabayir@uni-passau.de

Zahide Özden Ceylan Güngör

Department of Industrial Engineering, Istanbul Technical University, Faculty of Management, Istanbul, Turkey

ceylanz15@itu.edu.tr

Received: December 19, 2021; First Revision: March 18, 2022;

Last Revision: April 15, 2022; Accepted: April 18, 2022;

Published: April 22, 2022

Abstract

Purpose: As an extension of pythagorean fuzzy sets, the q-rung orthopair fuzzy sets (q-ROFS) is proposed by Yager in 2017. The q-ROFS offers a novel calculation form for the loss function and effectively deals with unclear information of multi-attribute decision-making (MADM) problems. The concept of q-rung orthopair fuzzy number (q-ROFN) is introduced to facilitate the use of q-ROFS in 2018. This study proposes a comprehensive q-rung orthopair triangular fuzzy number (q-ROTFN) which is a special notation of q-ROFN, to cope with supplier selection problems.

Design/methodology/approach: A new method is developed in this paper for supplier selection MADM problems in uncertain situations. The proposed technique utilizes experts' knowledge represented by q-ROFN. It considers the selection of the most proper supplier taking into account flexibility, quality, price, supplier profile, and delivery criteria. Based on the advantages of q-ROFN, this article proposes an extended fuzzy TOPSIS method that does not require aggregation technology.

Findings: To verify the proposed technique, a case study is conducted to evaluate and rank the alternative suppliers for an automotive company. As a result of the outcomes, it is shown that the proposed method is suitable for MADM problems.

Originality/value: The main contributions of this paper are as follows: (i) Traditional TOPSIS method has been extended using the q-ROTFN to solve multi-attribute decision problems, (ii) It is shown that aggregation techniques are not needed for q-ROTFN based TOPSIS method, (iii) A novel expert weight calculation technique is proposed.

Keywords: Q-Rung orthopair fuzzy number, TOPSIS, supplier selection, multiple attribute decision-making

JEL classification: D7 Analysis of Collective Decision-Making, D81 Criteria for Decision-Making under Risk and Uncertainty

Introduction

Multi-attribute decision making (MADM) are widely known methods to make a decision using many conflicting criteria and attitudes. There are many well-known MADM methods like Technique for Order Preference by Similarity to Ideal Solution (TOPSIS), The Analytic Hierarchy Process (AHP), Preference Ranking Organization Method for Enrichment Evaluation (PROMETHEE), etc. (Alibeigi et al., 2021). As observed from the literature, the results can vary when different methods are implemented for the same problem with the same data. While some techniques are related to calculating weights of decision-making criteria and alternatives such as AHP, others as TOPSIS are used to define the differences of alternatives according to their order preference concept. Due to the variety of methods, one of the important questions is to decide which method is suitable for a defined problem. The best way to determine the method is to understand the problem structure.

TOPSIS is one of the most popular methods introduced by Yoon and Hwang (1980) and is based on the distances of the decision points to the ideal solution. There are numerous applications of fuzzy TOPSIS to solve MADM problems (Tang et al., 2020; Jahanshahloo et al., 2006; Chen, 2000; Hwang et al., 1981). Compared to AHP, TOPSIS gives better results in a short calculation procedure. One of the other advantages of the TOPSIS method is calculation ease. However, the nature of decision-making problems has uncertainty which can be concluded with wrong results to solve using the traditional MADM methods. The fuzzy set theory also provides a better outcome for handling the vagueness of human thought, the primary input of MADM problems. Therefore, instead of using the classical decision-making methods, it is better to use the fuzzy extension.

In this study, our problem is the selection of appropriate alternative suppliers among a set of alternatives for supplier selection problems. TOPSIS is one of the most suitable traditional MADM methods compared to other methods. However, regarding all the aforementioned explanations and considering our problem structure, the fuzzy extension of TOPSIS can have a better solution.

The q-rung orthopair fuzzy set (q-ROFS), developed by Yager (2017), is a powerful set that effectively deals with uncertainty in real-life problems and covers complicated and hesitant fuzzy information. Therefore, this model has a more robust capability than other fuzzy sets, such as intuitionistic or pythagorean fuzzy sets to manage the uncertainty. Liu and Wang (2018) introduced the concept of q-rung orthopair fuzzy number (q-ROFN) to facilitate the use of q-ROFS.

Numerous researches apply the q-ROFS and q-ROFN methods to solve the MADM problems; these methods' procedure of decision information is too old and limited. Furthermore, the q-ROFS and q-ROFN are generally based on different aggregation operators. No matter what aggregation operators are used, it may distort decision information. Also, some decision-makers might give the criteria evaluation value of a scheme too low or too high owing to individual bias.

Based on the advantages of q-ROFN, this study proposes an extended fuzzy TOPSIS method that does not require aggregation technology. The proposed method attempts to extend the traditional TOPSIS with q-ROTFN which is a special notation of q-ROFN. Another outstanding point of this paper a novel expert's weight calculation technique that can successfully cope with the expert's possible bias and solve experts' circumstances with significantly different opinions has been proposed.

The article's primary contributions are as follows: (i) to demonstrate how the proposed model, which is an extension of the traditional TOPSIS method based on q-ROTFN, can efficiently solve multi-attribute decision problems without the requiring aggregation operators, (ii) to introduce a new expert weight calculation technique, (iii) to conduct a case study in the automotive industry to show the applicability of the proposed method for assessing and ranking suppliers.

The second section presents a literature review about TOPSIS and q-rung orthopair fuzzy environment. The third section explains the theoretical fundamentals of q-ROFS and q-ROTFN. Moreover, the proposed model is shown in the third section. The case study results and sensitivity analysis are presented in the fourth and fifth sections. In the last section, the findings and conclusions are discussed.

Literature Review

Supply chain is the concept that defines all the logistics activities, starting from the required raw material supplied for producing a product to reaching the final consumer. All these processes include people, technologies, and resources involved in these activities. On the other hand, supply chain management defines all the managerial tasks of the material and information flow that occur during these processes. Supply chain management contains four core phases: planning, procurement, production, and distribution. Supplier selection appears in the procurement phase according to the conditions determined from the planning phase, followed by processes in producing and distributing the products from the supplied materials (David, 2016; Liang and Cao, 2019). Defining the proper supplier is a crucial and difficult problem for companies. The reason for this is that numerous criteria must be considered when selecting the most suitable supplier(s) to meet the expectations of the business. It is possible to see that there are abundant studies on supplier selection from the literature. One of the oldest studies was conducted by Youssef et al. (1996), in which existing methods have been grouped under five headings as categorical models, matrix models, cost-based models, multi-criteria selection models and supplier profile evaluation model. These models can be reclassified as multi-attribute decision models, cost-based models, mathematical programming models, and statistical models for supplier selection problems. Researchers widely deal with supplier selection as a MADM problem involving conflicting and non-commensurable criteria, assuming that compromising is acceptable for conflict resolution.

TOPSIS, which was developed by Yoon and Hwang (1980), is one of the widely used techniques among MADM methodologies. It aims to evaluate the relative closeness of the alternatives to the ideal solution with the Euclidean distance approach. The basic layout of this technique can be described as the selected alternative should be geometrically the nearest distance from the positive ideal solution and the furthest from the negative ideal solution. Compared with other MADM methods like ELECTRE or VIKOR, the TOPSIS method has an easy calculation procedure; hence, it has been widely implemented in many different fields. However, exact numerical values for evaluation may be inadequate when real-life cases take into account. This situation comes from the uncertainty of human thoughts and judgments, especially preferences. Therefore, fuzzy TOPSIS proposed by Chen (2000) is a systematic approach that extends classical TOPSIS to solve decision-making problems under a fuzzy environment.

Fuzzy TOPSIS has linguistic expressions to reflect the judgments more realistic way. In this method, decision-makers verbally express their thoughts when evaluating alternatives using specified criteria. Then, these linguistic expressions are converted into fuzzy numbers. After that, the proximity coefficients of each alternative are calculated. Finally, calculated closeness coefficients are sorted according to their values, and the appropriate alternative is selected.

After TOPSIS is extended to a fuzzy environment by Chen (2000), many researchers have attention to different extensions of fuzzy TOPSIS using the interval type-2 (Lee and Chen, 2008; Dymova et al., 2015; Liao, 2015, Pham et al., 2018), intuitionistic (Ervural et al., 2015; Wood, 2016), interval-valued intuitionistic hesitant fuzzy Choquet integral (Joshi and Kumar, 2016), hesitant (Xu and Zhang, 2013; Senvar et al., 2016); spherical (Kutlu and Kahraman, 2020), Pythagorean (Akram et al., 2019; Rani et al., 2020; Yucesan and Gul, 2020), etc.

Fuzzy sets can be added to multi-criteria decision problems to get more robust results and cover real-life uncertainty (Ayyildiz, 2021). In this manner, the q-ROFS and q-ROFN are novel and effective tools for dealing with uncertainty in real-life problems and covering more complicated and uncertain fuzzy evaluation information. Moreover, representation of decision information effectively and reducing distortion put forward research for q-ROFS and q-ROFN.

Generally, studies about q-ROFN focus on theoretical research, especially on aggregation operators. Peng et al. (2018) presented a new score function of q-ROFN to solve the failure problems when comparing two q-ROFN. They defined a new exponential operational law about q-ROFN, in which the bases are positive real numbers, and the exponents are q-ROFN. The notion of interval-valued q-rung orthopair fuzzy sets (IVq-ROFS) that allows decision-makers to present their satisfactory and unsatisfactory ratings about a given set of alternatives based on a range value is presented by Joshi et al. (2018). They provided the aggregation of IVq-ROFS based on essential operations of the proposed model, such as negation, union, and intersection. In another study, to quantify the loss function of decision- theoretic rough sets, Liang and Cao (2019) utilized q- ROFSs and constructed a new three- way decisions model by TOPSIS with projection- based distance measures. To aggregate different thoughts based on the power average (PA) operator and power geometric (PG) operator, the authors designed four operators: (i) q- rung orthopair fuzzy power average (q- ROFPA), (ii) q- rung orthopair fuzzy power weighted average (q- ROFPWA), (iii) q- rung orthopair fuzzy power geometric (q- ROFPG), (iv) q- rung orthopair fuzzy power weighted geometric (q- ROFPWG). Wei et al. (2019) introduced four multi-criteria selection model operators to use q-ROFI, including q-ROFMSM, the q-ROFWMSM, q-ROFDMSM and q-ROFWDMSM operators. A new notion for q-rung orthopair regular fuzzy set is developed by Yang et al. (2019). Yang and Pang (2019) introduced some q-rung orthopair fuzzy partitioned Bonferroni mean operators. After that Liu D. et al. (2019) presented two novel q-rung orthopair fuzzy extended Bonferroni mean (q-ROFEBM) operators and their weighted form (q-ROFEWEBM) to select a location.

The main advantage of the proposed operators is that they can aggregate input arguments with heterogeneous relationships more intuitively and effectively. Liu and Wang (2020) proposed two new operators: the q-rung orthopair fuzzy weighted averaging operator (q-ROFWAO) and the q-rung orthopair fuzzy weighted geometric operator (q-ROFWGO). Another study suggested advanced weighted generalized Maclaurin symmetric mean (q-ROFWGMSM) and the weighted generalized geometric Maclaurin symmetric mean (q-ROFWGGMSM) operator (Liu and Wang, 2020).

As a result of current research on q-ROFS and q-ROFN, it can be mentioned that most of the studies focused on the forms of various collecting operators. Associated with deeper research, these aggregation operators become more complex. However, q-ROFN is a comprehensive and effective information expression tool that is able to deal with uncertain knowledge and adjust the representation of information based on different decision-makers and decision scenarios.

Apart from theoretical research, there have been some q-ROFS and q-ROFN empirical studies with real-world applications in recent years, as this method is well suited for complex decision-making situations in a variety of fields, such as stock investment. (Tang et al., 2020), e-commerce (Liang and Cao, 2019), energy source selection (Krishankumar et al., 2021), education (Hussain, 2019), partner selection (Yang et al., 2019), construction project selection (Wang et al., 2019), investment selection (Liu, P. et al., 2019), location selection (Liu, D. et al., 2019), etc. Some investment types, like stock investment, are characterized by high risk and immense profit. Therefore, the selection method should avoid investment risks and obtain increased returns. Tang et al. (2020) combined q-ROFS with decision-theoretic rough sets considering these critical advantages. Thanks to the q-ROFS, a new generalized form of Pythagorean fuzzy sets (PFS) and intuitionistic fuzzy sets (IFS) show uncertain information more extensively and flexibly. Thanks to the reasonableness and effectiveness of the q-ROFS, it has also been implemented for other risky investment selections. Liu et al. (2018) introduced the q-rung orthopair hesitant fuzzy set (q-ROHFS) TOPSIS method to select energy projects. Another study for selecting renewable energy sources from a range of sources based on sustainability characteristics has been conducted using the q-ROF information to minimize vagueness and subjective randomness by providing a flexible and generalized preference style (Krishankumar et al., 2021). Hussain et al. (2019) presented the q-rung orthopair fuzzy TOPSIS (q-ROF-TOPSIS) methodology for the MADM problem in the education field thanks to the core advantage of the q-ROF information, which cope with complexities and uncertainties efficiently. The developed model depends on the cover-based q-rung orthopair fuzzy sets (Cq-ROFRS). Wang et al. (2019) introduce the q-rung orthopair fuzzy MABAC (multi-attributive border approximation area comparison) model on account of the traditional MABAC model to compute the distance between each alternative.

As a result of the aforementioned studies, q-ROFS or q-ROFN are efficient tools to solve a multi-variable problem, and the results are satisfactory. Table 1 presents a summary of these studies, including the using methodologies. Moreover, Table 1 consists of alternatives for each reviewed research, a list of criteria, the number of decision-makers, and the q value used.

The literature review showed that almost all the research on q-ROFN agrees that this method is an efficient and influential instrument for gathering different decision-makers evaluations under uncertain and complicated conditions. In the other word, q-ROFN is capable of encompassing more complex and hesitant fuzzy evaluation information. However, our q-ROFN research is generally based on complex aggregation operators. Even if aggregation operators are a powerful tool for solving decision problems, while it has been applied to these new types of fuzzy sets with defective operational rules, the result has significant information distortion frequently. To solve this problem, Liu P. et al. (2019) proposed a novel decision-maker weight calculation model applying the TOPSIS method with q-ROFN. The outstanding point of this calculation, it hasn't included complex aggregation operators.

In this study, an extension of TOPSIS using q-ROTFN has been used to solve a supplier selection problem in the automotive sector. The reasons why the q-ROTFN based TOPSIS method is used in the state of traditional fuzzy TOPSIS can be listed as follows: (i) the form of decision information used by fuzzy TOPSIS methods is too restricted and an old procedure, (ii) traditional fuzzy TOPSIS is not capable of dealing with existing complicated decision situations, (iii) the aggregation methods used in the traditional fuzzy TOPSIS method distort decision information, (iv) each decision-makers' weight sometimes cannot be determined by fuzzy TOPSIS directly and accurately (v) the defect of relative closeness calculation of the TOPSIS method which is nearest ideal solution to positive ideal solution is not necessarily the farthest from the negative ideal solution, which makes the evaluation result inaccurate. From the application perspective to the best of our knowledge, to solve group decision problems, even supplier selection problems, q-ROTFN-based TOPSIS method has never been used.

Methodology

q-Rung Orthopair Fuzzy Sets (q-ROFS)

Zadeh (1965) introduced the concept of fuzzy sets in 1965 as an extension of classical sets. Fuzzy sets use a similar form of probability to express a person's degree of satisfaction with a particular entity, which is referred to as the membership degree (u). Further, Atanassov (1986) extended the fuzzy sets by adding non-membership degree (v) to express one's degree of dissatisfaction with a particular entity. This novel formula of information has been called intuitionistic fuzzy sets (IFSs) and specified that it must satisfy the constraints of $u \in [0, 1]$, $v \in [0, 1]$, and $0 \leq u + v \leq 1$. IFS also includes a hesitation degree (π), which is an indicator of an entity's level of uncertainty, and must satisfy $\pi = 1- u - v$. Xu (2017) refers to the single element of IFS as an intuitionistic fuzzy number (IFN). However, due to the uncertainty and conflict inherent in people's subjective cognition, expert evaluation values do not always fully satisfy the constraints of $0 \leq u + v \leq 1$. To solve this problem, Yager (2014) proposed pythagorean fuzzy sets (PFSs) that extend $0 \leq u + v \leq 1$ to $0 \leq u^2 + v^2 \leq 1$. A single element of PFS is called pythagorean fuzzy number (PFN). While PFN broadens the representation scope of decision-making information, its capacity to express vague and hesitant information remains limited as people's vague consciousness and degree of hesitation increase. Subsequently, Yager (2017) proposed q-rung orthopair fuzzy sets (q-ROFS), which provide a novel calculation form of the loss function. As shown in Eq. (1), q- ROFSs can be expressed by the sum of q^{th} power of the membership degree and q^{th} power of the non-membership degree. Liu and Wang (2018) introduced the concept of q-rung orthopair fuzzy number (q-ROFN) to facilitate the use of q-ROFS.

$$0 \leq u^q + v^q \leq 1, \text{ where } q \geq 1 \qquad (1)$$

For general set Y, a q-ROFS $\tilde{Q}$ is the form of $\tilde{Q} = \{\langle y, u_{\tilde{Q}}(y), v_{\tilde{Q}}(y) \rangle : y \epsilon Y\}$, where the degree of membership $u_{\tilde{Q}}$: Y → [0, 1] and degree of non-membership $v_{\tilde{Q}}: Y \rightarrow [0,1]$ of element y ∈ Y correspondingly, with constraint $0 \leq u_{\tilde{Q}}(y)^q + v_{\tilde{Q}}(y)^q \leq 1$ (Chen, 2000). When q = 1, the q-ROFN become IFNs, and when q = 2, the q-ROFN become PFNs. $\pi_{\tilde{Q}}(y) = (1 - u_{\tilde{Q}}(y)^q - v_{\tilde{Q}}(y)^q)^{1/q}$ is represented by hesitancy degree or indeterminacy degree $\pi_{\tilde{Q}}: Y \rightarrow [0,1]$ for each member of y ∈ Y.

For convenience, $\langle u_{\tilde{Q}}(y), v_{\tilde{Q}}(y) \rangle$ is called q-ROFN by Liu and Wang (2018), which can be denoted by $\tilde{Q} = \langle u_{\tilde{Q}}, v_{\tilde{Q}} \rangle$. Assume $\widetilde{Q_1} = \langle u_1, v_1 \rangle$ and $\widetilde{Q_2} = \langle u_2, v_2 \rangle$ are two q-ROFN. Some operations rules between these two q-ROFN are listed below.

$$\tilde{Q}_1 \vee \tilde{Q}_2 = \langle (u_1, u_2), min(v_1, v_2) \rangle \qquad (2)$$

$$\tilde{Q}_1 \wedge \tilde{Q}_2 = \langle (v_1, v_2) \rangle \qquad (3)$$

$$\tilde{Q}_1 \oplus \tilde{Q}_2 = \langle v_1 . v_2 \rangle \qquad (4)$$

$$\tilde{Q}_1 \otimes \tilde{Q}_2 = \langle (v_1^q + v_2^q - v_1^q v_2^q)^{\frac{1}{q}}) \rangle \qquad (5)$$

$$\lambda\tilde{Q}_1 = \langle (1-(1-u_1^q)^{\lambda})^{1/q}, v_1^{\lambda} \rangle, \ \lambda>0 \qquad (6)$$

$$\tilde{Q}_1^{\lambda} = \langle u_1^{\lambda}, (1-(1-v_1^q)^{\lambda})^{1/q} \rangle, \ \lambda>0 \qquad (7)$$

If $u_1 \leq u_2$ and $v_1 \geq v_2$ then $\tilde{Q}_1 \leq \tilde{Q}_2$ (Yager and Alajlan, 2017)

q-Rung Orthopair Triangular Fuzzy Number (q-ROTFN)

The q-rung orthopair triangular fuzzy number (q-ROTFN*)* is a special notation of orthopair fuzzy set. For universal set Y, a q-rung orthopair triangular fuzzy set (q-ROTFS) $\tilde{Q}$ in Y is defined by

$$\widetilde{Q} = \{\langle y, [l_{\tilde{Q}}(y), m_{\tilde{Q}}(y), u_{\tilde{Q}}(y)], \quad [s_{\tilde{Q}}(y), b_{\tilde{Q}}(y), h_{\tilde{Q}}(y)] \rangle : y \epsilon Y\}$$

where $[l_{\tilde{Q}}(y), m_{\tilde{Q}}(y), u_{\tilde{Q}}(y)]$ and $[s_{\tilde{Q}}(y), b_{\tilde{Q}}(y), h_{\tilde{Q}}(y)]$ stand for the membership degree and the non-membership degree, respectively, with constraints $l_{\tilde{Q}}(y) \epsilon [0,1]$, $m_{\tilde{Q}}(y) \epsilon [0,1]$, $u_{\tilde{Q}}(y) \epsilon [0,1]$, $s_{\tilde{Q}}(y) \epsilon [0,1]$, $b_{\tilde{Q}}(y) \epsilon [0,1]$, $h_{\tilde{Q}}(y) \epsilon [0,1]$ and $0 \leq l_{\tilde{Q}}(y)^q + s_{\tilde{Q}}(y)^q \leq 1$, $0 \leq m_{\tilde{Q}}(y)^q + b_{\tilde{Q}}(y)^q \leq 1$, $0 \leq u_{\tilde{Q}}(y)^q + h_{\tilde{Q}}(y)^q \leq 1$. The hesitancy degree is given as follows;

$$\pi1_{\widetilde{Q}}(y) = (1 - l_{\widetilde{Q}}(y)^q - s_{\widetilde{Q}}(y)^q)^{1/q}$$

$$\pi2_{\widetilde{Q}}(y) = (1 - m_{\widetilde{Q}}(y)^q - b_{\widetilde{Q}}(y)^q)^{1/q}$$

$$\pi3_{\widetilde{Q}}(y) = (1 - u_{\widetilde{Q}}(y)^q - h_{\widetilde{Q}}(y)^q)^{1/q}$$

Let $\widetilde{Q_1} = \{\langle [l_1, m_1, u_1], [s_1, b_1, h_1] \rangle\}$ and $\widetilde{Q_2} = \{\langle [l_2, m_2, u_2], [s_2, b_2, h_2] \rangle\}$ be two q-ROTFNs, then the normalized Hamming Distance d $(\widetilde{Q_1}, \widetilde{Q_2})$ between the q-ROTFN $\widetilde{Q_1}$ and $\widetilde{Q_2}$ is calculated using Eq. (8).

$$d(\widetilde{Q_1}, \widetilde{Q_2}) = \frac{\begin{pmatrix} |l_1^q - l_2^q| \\ |m_1^q - m_2^q| \\ |u_1^q - u_2^q| \end{pmatrix} + \begin{pmatrix} |s_1^q - s_2^q| \\ |b_1^q - b_2^q| \\ |h_1^q - h_2^q| \end{pmatrix} + \begin{pmatrix} |\pi1_1^q - \pi1_2^q| \\ |\pi2_1^q - \pi2_2^q| \\ |\pi3_1^q - \pi3_2^q| \end{pmatrix}}{2} \quad (8)$$

where $[\pi1_1, \pi2_1, \pi3_1]$ and $[\pi1_2, \pi2_2, \pi3_2]$ are hesitancy degree.

The Proposed Model

The q-ROFN is a very efficient and influential instrument for expressing information of different decision-makers. Generally, q-ROFN is based on aggregation operators. Nevertheless, aggregation operators reason information distortion easily. To address this issue, Liu P. et al. (2019) proposed a novel decision-maker weight calculation model and applied it to the TOPSIS method based on q-ROFN. This calculation does not include complex aggregation operators. On the basis of the advantages of q-ROFN, this article proposes a new extended fuzzy group TOPSIS method that does not require aggregation technology. The process of extended classical TOPSIS method based on q-ROTFN consists of 13 steps. The specific steps are as follows:

Assume DM = (DM_1, DM_2,...,DM_p) is a pool of decision-makers. A = (A_1, A_2…A_m) is a collection of alternatives. C = (C1, C_2…C_n) is a criteria pool. Assume the assessment value of the criteria C_j given by decision-maker DM_k for alternative A_i is $Q_{ij}^k = (u_{ij}^k, v_{ij}^k)$ where k=1,2,…,p; i= 1,2,…,m; j=1,2,…,n and $u_{ij}^k \epsilon$ [0,1], $v_{ij}^k \epsilon$ [0,1] and $0 \leq ((u_{ij}^k)^q + (v_{ij}^k)^q) \leq 1.$ Assume the decision matrix given by decision-maker DM_k is $Q^k = [Q_{ij}^k]_{mxn,}$ and η= (η_1, η_2, …., η_p) is the degree of decision-maker importance in this field and η_k ϵ [0,1]. w = (w_1, w_2, …, w_n) is criteria weight vectors and satisfies the constraints w_j ϵ [0,1], $\sum_{j=1}^n w_j = 1$. With this information, the goal of the MADM problem is to choose the best alternative from a set of alternatives.

Step 1: Construct the normalized decision matrix. There are two types of criteria in a decision-making environment: benefit type and cost type. While the former implies that the bigger is the better, the latter means that the smaller is the better. To eliminate the impact of different criteria types, criteria should be standardized.

Table 1. Studies of q-ROFS and q-ROFS applied in various fields

Author(s) and Year	Application Fields	Proposed Methodology	Alternatives	Criteria	Number of Decision Makers	q Value
Krishankumar et al. (2021)	Renewable energy sources selection	q-rung orthopair fuzzy set (q-ROFS)	Solar energy, wind energy, small hydro energy, biomass energy, tidal energy	C1: air pollutant emissions, C2: need for waste disposal, C3: water pollution, C4: land disruption, C5: land requirement, C6: economic risk, C7: security, C8: sustainable energy, C9: durability, C10: adaptability to energy policy, C11: cost, C12: feasibility	3	3
Tang et al. (2020)	Stock investment selection	q-rung orthopair fuzzy decision-theoretic rough sets (q-ROFDTRS)	Six stocks	C1: loss Function, C2: expected loses	5	4
Hussain et al. (2019)	Personal selection for new faculty positions in universities	q-rung orthopair fuzzy TOPSIS (q-ROF-TOPSIS) methodology depends on the cover-based q-rung orthopair fuzzy sets (Cq-ROFRS) model	Five applicants who achieve the necessities for the senior faculty position in a university	C1: research productivity, C2: managerial skill, C3: impact on the research community, C4: the ability to work under pressure, C5: academic leadership qualities, C6: contribution to Y University	2	3
Liang and Cao (2019)	Rural e-commerce selection	q- rung orthopair fuzzy power average (q- ROFPA), q- rung orthopair fuzzy power weighted average (q- ROFPWA), q- rung orthopair fuzzy power geometric (q-ROFPG), q- rung orthopair fuzzy power weighted geometric (q- ROFPWG) operators	Five regions	C1: e-commerce sell channel, C2: traditional sell channel	4	3
Yang et al. (2019)	Enterprise partner selection	q-rung orthopair normal fuzzy set (q-RONFS)	Five alternative enterprises	C1: Rand D capability, C2: business operation level, C3: international cooperation level, C4: credit level	NA	2

Author(s) and Year	Application Fields	Proposed Methodology	Alternatives	Criteria	Number of Decision Makers	q Value
Liu P. et al. (2019)	Investment selection	q-rung orthopair fuzzy number TOPSIS (q-ROFN TOPSIS)	Five small and medium-sized enterprises	C1: company's risk aversion ability, C2: the company's environment, C3: the company's size, C4: the company's growth ability	3	2
Liu D. et al. (2019)	Location selection	q-rung orthopair fuzzy extended Bonferroni mean (q-ROFEBM) operator, q-rung orthopair fuzzy weighted extended Bonferroni mean (q-ROFEWEBM)	Four potential locations in different countries	C1 : market, C2 : investment cost, C3 : labor characteristics, C4 : infrastructure, C5 : possibility for further expansion	NA	3
Wang et al. (2019)	Construction project selection	q-rung orthopair fuzzy MABAC	Five construction projects	C1: human factors; C2: energy cost factors; C3: building materials and equipment factors; C4: environmental factors	3	3
Peng et al. (2018)	Teaching management system selection	a new score function of q-rung orthopair fuzzy number (q-ROFN)	Five system alternatives	C1: operational, C2: functional, C3: security, C4: economic	NA	3
Liu and Wang (2018)	Investment selection	q-rung orthopair fuzzy weighted averaging operator (q-ROFWA), q-rung orthopair fuzzy weighted geometric operator (q-ROFWG)	Three potential companies	C1: the risk analysis, C2: the growth condition, C3: the social-political impact, C4: the environmental impact, C5: the development of the society	NA	3
Liu et al. (2018)	Energy projects selection	q- rung orthopair hesitant fuzzy set (q- ROHFS)	Five energy projects	C1: economic; C2: technological; C3: environmental; C4: sociopolitical	NA	3

Step2: Construct the attribute weighted decision matrix of each decision-maker. This matrix is shown by DM^k = DM_{ij}^k and calculated using Eq. (9).

$$DM_{ij}^k = w_j \tilde{Q}_{ij}^k = \langle (a_{ij}^k, b_{ij}^k, c_{ij}^k), (d_{ij}^k, e_{ij}^k, f_{ij}^k) \rangle = \langle \left(\frac{\left(1-(1-(l_{ij}^k)^q)^{w_j}\right)^{\frac{1}{q}},}{\left(1-(1-(m_{ij}^k)^q)^{w_j}\right)^{\frac{1}{q}}, \left(1-(1-(u_{ij}^k)^q)^{w_j}\right)^{\frac{1}{q}}} \right), \left(\frac{(s_{ij}^k)^{w_j},}{(b_{ij}^k)^{w_j}, (h_{ij}^k)^{w_j}} \right) \rangle \quad (9)$$

where w_j characterizes weight of each j criteria, satisfies $w_j \in [0,1]$, $\sum_{j=1}^{n} w_j = 1$.

Step 3: Compute support degree between attribute decision matrix DM_{ij}^k and attribute decision matrix DM_{ij}^t using Eq. (10) and represent it as Sup (DM_{ij}^k, DM_{ij}^t).

$$\text{Sup}\,(DM_{ij}^k, DM_{ij}^t) = 1 - \text{d}\,(DM_{ij}^k, DM_{ij}^t) \quad (10)$$

where k, t=1,2, ..., p and k ≠ t. d (DM_{ij}^k, DM_{ij}^t) is the distance between DM_{ij}^k *and* DM_{ij}^t.

Step 4: For each DM_{ij}^k calculate total support degree T (DM_{ij}^k) by using Eq. (11).

$$\text{T}\,(DM_{ij}^k) = \sum_{t=1,k\neq 1}^{p} \text{Sup}(DM_{ij}^k, DM_{ij}^t) \quad (11)$$

Step 5: Expert evaluation data frequently appear to be of extreme value due to the influence of personal prejudice. When the expert's weights are equal, the conventional method eliminates the highest and lowest scores, but this approach loses decision information. This study uses expert assessment rationality to preserve complete information and effectively deal with unreasonable excess data. The rationality degree of evaluation means that the assessment information provided by the experts for each alternative is close to the assessment information provided by other experts. Calculate the rationality degree of expert evaluation using Eq. (12).

$$\delta_k = \frac{1}{mn(p-1)} \sum_{i=1}^{m} \sum_{j=1}^{n} T(DM_{ij}^k) \quad (12)$$

where $\delta_k \in [0,1]$.

Step 6: Calculate the comprehensive index of experts by combining the rationality degree δ_k of the evaluation and the importance level η_k of the expert. The formulation of the comprehensive index is shown in Eq. (13).

$$\varsigma_k = \alpha\delta_k + (1-\alpha)\eta_k \quad (13)$$

where $0 \leq \varsigma_k \leq 1$ and α represent adjustment coefficient and α ϵ [0,1].

Step 7: Calculate experts' weight w_k using Eq. (14).

$$w_k = \frac{\varsigma_k}{\sum_{k=1}^{p} \varsigma_k} \quad (14)$$

where $0 \leq w_k \leq 1$ and $\sum_{k=1}^{p} w_k = 1$.

Step 8: Construct the weighted decision matrix $\ddot{DM}_{ij}^k$ of expert DM_{ij}^k by using Eq. (15).

$$D\ddot{M}_{ij}^{k} = w_k DM_{ij}^{k} = \langle (\ell_{ij}^{k}, m_{ij}^{k}, u_{ij}^{k}), (s_{ij}^{k}, b_{ij}^{k}, h_{ij}^{k}) \rangle$$

$$= \left\langle \begin{pmatrix} (1-(1-(a_{ij}^{k})^{q})^{w_k})^{\frac{1}{q}}, \\ (1-(1-(b_{ij}^{k})^{q})^{w_k})^{\frac{1}{q}}, \\ (1-(1-(c_{ij}^{k})^{q})^{w_k})^{\frac{1}{q}} \end{pmatrix}, \begin{pmatrix} (d_{ij}^{k})^{w_k}, \\ (e_{ij}^{k})^{w_k}, \\ (f_{ij}^{k})^{w_k} \end{pmatrix} \right\rangle \quad (15)$$

where w^k signifies the weight of each k expert.

Step 9: Convert each final weighted decision matrix of expert $D\ddot{M}_{ij}^{k}$ into the alternative decision matrix A_{kj}^{i}. A_{kj}^{i} corresponds to the $D\ddot{M}_{ij}^{k}$ in step 8.

Step 10: Using Eq. (16) and Eq. (17), calculate A+ (positive ideal decision matrix) and A- (negative ideal decision matrix). A^{+} should contain the best evaluation data in all alternative decision matrix A_{kj}^{i}.

$$A^{+} = A_{kj}^{+} = [(l_{kj}^{+}, m_{kj}^{+}, u_{kj}^{+}), (s_{kj}^{+}, b_{kj}^{+}, h_{kj}^{+})] \quad (16)$$

where $l_{kj}^{+} = max_{i=1}^{m}\{l_{kj}^{i}\}$, $m_{kj}^{+} = max_{i=1}^{m}\{m_{kj}^{i}\}$, $u_{kj}^{+} = max_{i=1}^{m}\{u_{kj}^{i}\}$,

$s_{kj}^{+} = min_{i=1}^{m}\{s_{kj}^{i}\}$, $b_{kj}^{+} = min_{i=1}^{m}\{b_{kj}^{i}\}$, $h_{kj}^{+} = min_{i=1}^{m}\{h_{kj}^{i}\}$

A^{-} should contain the worst evaluation data in all alternative decision matrix A_{kj}^{i}.

$$A^{-} = A_{kj}^{-} = [(l_{kj}^{-}, m_{kj}^{-}, u_{kj}^{-}), s_{kj}^{-}, b_{kj}^{-}, h_{kj}^{-})], \quad (17)$$

where $l_{kj}^{-} = minx_{i=1}^{m}\{l_{kj}^{i}\}$, $m_{kj}^{-} = min_{i=1}^{m}\{m_{kj}^{i}\}$, $u_{kj}^{-} = min_{i=1}^{m}\{u_{kj}^{i}\}$,

$s_{kj}^{-} = max_{i=1}^{m}\{s_{kj}^{i}\}$, $b_{kj}^{-} = max_{i=1}^{m}\{b_{kj}^{i}\}$, $h_{kj}^{-} = max_{i=1}^{m}\{h_{kj}^{i}\}$

Step 11: Calculate distance S_i^{+} between alternative decision matrix A_{kj}^{i} and the positive ideal decision matrix A^{+}; distance S_i^{-} between alternative decision matrix A_{kj}^{i} and the negative ideal decision matrix A^{-} . Use Eqs. (18) and (19), respectively.

$$S_i^{+} = \frac{1}{2pn} \sum_{k=1}^{p} \sum_{j=1}^{n} (\left| [(\ell_{kj}^{i})^{q} - (\ell_{kj}^{+})^{q}] + [(m_{kj}^{i})^{q} - (m_{kj}^{+})^{q}] + [(u_{kj}^{i})^{q} - (u_{kj}^{+})^{q}] \right| + (\left| [(s_{kj}^{i})^{q} - (s_{kj}^{+})^{q}] + [(b_{kj}^{i})^{q} - (b_{kj}^{+})^{q}] + [(h_{kj}^{i})^{q} - (h_{kj}^{+})^{q}] \right| + [(\pi 1_{kj}^{i})^{q} - (\pi 1_{kj}^{+})^{q}] + [(\pi 2_{kj}^{i})^{q} - (\pi 2_{kj}^{+})^{q}] + [(\pi 3_{kj}^{i})^{q} - (\pi 3_{kj}^{+})^{q}] \quad (18)$$

$$S_i^{-} = \frac{1}{2pn} \sum_{k=1}^{p} \sum_{j=1}^{n} (\left| [(\ell_{kj}^{i})^{q} - (\ell_{kj}^{-})^{q}] + [(m_{kj}^{i})^{q} - (m_{kj}^{-})^{q}] + [(u_{kj}^{i})^{q} - (u_{kj}^{-})^{q}] \right| + (\left| [(s_{kj}^{i})^{q} - (s_{kj}^{-})^{q}] + [(b_{kj}^{i})^{q} - (b_{kj}^{-})^{q}] + [(h_{kj}^{i})^{q} - (h_{kj}^{-})^{q}] \right| + [(\pi 1_{kj}^{i})^{q} - (\pi 1_{kj}^{-})^{q}] + [(\pi 2_{kj}^{i})^{q} - (\pi 2_{kj}^{-})^{q}] + [(\pi 3_{kj}^{i})^{q} - (\pi 3_{kj}^{-})^{q}] \quad (19)$$

Step 12: Compute the relative closeness of the alternative decision matrix A_{kj}^{i} to the ideal decision matrix. Relative closeness RC_i is calculated by Eq. (20).

$$RC_i = \frac{S_i^-}{max_{i=1}^m S_i^-} - \frac{S_i^+}{min_{i=1}^m S_i^+} \quad (20)$$

Step 13: Determine the priority of the alternatives. The optimal alternative is determined using the ranking rule that the larger the RC_i, the better the alternative.

Application Case in the Automotive Industry

One of the foremost manufacturers of spare parts for automobiles needs to determine a supplier of joint components used in various spare parts. A1, A2, A3 express three potential suppliers (i=1,2,3). The alternatives are evaluated against five decision criteria (j=1,2,3,4,5), the most important ones for the automotive industry. The criteria are defined by the decision-makers using a survey among Turkey's leading automotive companies as flexibility (C1), quality (C2), price (C3), supplier profile (C4), and delivery (C5). The assessment of the potential suppliers in each criteria is based on linguistic decisions provided by the decision-makers D1, D2, D3 (k=1,2,3), who are responsible for purchasing at a different company level. According to these three decision makers' professional backgrounds and specialists, the importance index vector is determined as $\eta = (0.8, 0.65, 0.95)^T$. The assessment data provided by each decision-maker for the criteria of each alternative is symbolized by;

$$Q_{ij}^k = \langle (l_{ij}^k, m_{ij}^k, u_{ij}^k), \qquad (s_{ij}^k, b_{ij}^k, h_{ij}^k) \rangle$$

where i=1,2,3; j=1,2,3,4,5; k=1,2,3.

The q-ROTFN method is applied to this case to show the applicability of q-ROTFN for supplier selection problems. One of the most critical parameters for q-ROFN TOPSIS is determining of q value. It is observed that the most applied value for this parameter is 3 in the literature (see Table 1). The main reason for using this parameter as 3 in our study is that it has developed better results with good performance. Moreover, the other parameter values have been implemented to see the differences in our problem solution.

In this study, three experts who are responsible for the company's purchasing evaluate the criteria's weight and alternatives' rating. It is worth to say the experience years of these experts in this field changes between 3-10 years. This long-term experience not only gives a chance for a proper evaluation of alternatives but also provides a different perspective assessment. Based on Chen's (2000) triangular fuzzy numbers (TFN), the evaluations are presented in Tables 2 and 3, respectively.

Table 2. Linguistic variables for the criteria's weight

Very low (VL)	(0, 0, 0.1)
Low (L)	(0, 0.1, 0.3)
Medium low (ML)	(0.1, 0.3, 0.5)
Medium (M)	(0.3, 0.5, 0.7)
Medium high (MH)	(0.5, 0.7, 0.9)
High (H)	(0.7, 0.9, 1.0)
Very high (VH)	(0.9, 1.0, 1,0)

Table 3. Linguistic variables for the alternatives' ratings

Very poor (VP)	(0, 0, 1)
Poor (P)	(0, 1, 3)
Medium poor (MP)	(1, 3, 5)
Fair (F)	(3, 5, 7)
Medium good (MG)	(5, 7, 9)
Good (G)	(7, 9, 10)
Very good (VG)	(9, 10, 10)

Tables 4, 5, and 6 present the linguistic judgments of the weights of the criteria and the ratings of the alternatives for each three decision-makers.

Table 4. Linguistic ratings of DM^1

		A1	**A2**	**A3**	**WoC**
C1	Membership	G	G	G	MH
	Non-membership	MP	MP	MP	
C2	Membership	G	VG	G	VH
	Non-membership	MP	F	MP	
C3	Membership	F	MG	MP	VH
	Non-membership	VG	F	G	
C4	Membership	VG	VG	VG	MH
	Non-membership	F	F	F	
C5	Membership	G	MP	P	H
	Non-membership	P	G	MG	

Table 5. Linguistic ratings of DM^2

		A1	**A2**	**A3**	**WoC**
C1	Membership	G	MG	G	H
	Non-membership	MP	P	MP	
C2	Membership	MG	VG	G	VH
	Non-membership	P	F	MP	
C3	Membership	F	MG	MP	H
	Non-membership	VG	F	G	
C4	Membership	VG	VG	VG	MH
	Non-membership	F	F	F	
C5	Membership	G	MP	P	VH
	Non-membership	P	VG	MG	

Table 6. Linguistic ratings of DM^3

		A1	A2	A3	WoC
C1	Membership	MG	VG	F	V.H.
	Non-membership	P	F	MG	
C2	Membership	F	VG	G	VH
	Non-membership	MG	F	MP	
C3	Membership	F	MG	G	VH
	Non-membership	VG	F	P	
C4	Membership	P	VG	VG	MH
	Non-membership	G	F	F	
C5	Membership	VG	F	MG	VH
	Non-membership	VP	VG	MP	

The decision-making procedure for the extended fuzzy TOPSIS method based on q-ROTFNs is detailed below.

The initial step is the creation of a normalized decision matrix. For analysis, the linguistic variables must be converted to triangular fuzzy numbers. After the linguistic terms have been converted to fuzzy numbers, a procedure for normalization can be performed. In the second step, the weighted decision matrix for each decision-maker is calculated. As an example, the normalized and weighted decision matrices for DM^1 are presented in Tables 7 and 8, respectively.

Table 7. Normalized decision matrix of DM^1

		A1	A2	A3
C1	**(l,m,u)**	(0.33,0.33,0.33)	(0.33,0.43,0.48)	(0.33,0.43,0.48)
	(s,b,h)	(0.33,0.33,0.33)	(0.05,0.14,0.24)	(0.05,0.14,0.24)
C2	**(l,m,u)**	(0.3,0.32,0.33)	(0.43,0.48,0.48)	(0.33,0.43,0.48)
	(s,b,h)	(0.2,0.27,0.29)	(0.14,0.24,0.33)	(0.05,0.14,0.24)
C3	**(l,m,u)**	(0.33,0.33,0.33)	(0.24,0.33,0.43)	(0.05,0.14,0.24)
	(s,b,h)	(0.47,0.42,0.37)	(0.14,0.24,0.33)	(0.33,0.43,0.48)
C4	**(l,m,u)**	(0.33,0.33,0.33)	(0.43,0.48,0.48)	(0.43,0.48,0.48)
	(s,b,h)	(0.33,0.33,0.33)	(0.14,0.24,0.33)	(0.14,0.24,0.33)
C5	**(l,m,u)**	(0.88,0.69,0.56)	(0.05,0.14,0.24)	(0,0.05,0.14)
	(s,b,h)	(0,0.06,0.14)	(0.33,0.43,0.48)	(0.24,0.33,0.43)

Table 8. Weighted decision matrix of DM^1

		A1	A2	A3
C1	**(l,m,u)**	(0.2,0.2,0.2)	(0.2,0.26,0.29)	(0.2,0.26,0.29)
	(s,b,h)	(0.79,0.79,0.79)	(0.52,0.66,0.73)	(0.52,0.66,0.73)
C2	**(l,m,u)**	(0.18,0.19,0.2)	(0.26,0.29,0.29)	(0.2,0.26,0.29)
	(s,b,h)	(0.7,0.75,0.76)	(0.65,0.73,0.79)	(0.51,0.65,0.73)
C3	**(l,m,u)**	(0.2,0.2,0.2)	(0.14,0.2,0.26)	(0.03,0.09,0.14)
	(s,b,h)	(0.85,0.83,0.81)	(0.65,0.73,0.79)	(0.79,0.83,0.85)
C4	**(l,m,u)**	(0.17,0.17,0.17)	(0.22,0.24,0.24)	(0.22,0.24,0.24)
	(s,b,h)	(0.87,0.87,0.87)	(0.78,0.83,0.87)	(0.78,0.83,0.87)
C5	**(l,m,u)**	(0.6,0.44,0.34)	(0.03,0.09,0.14)	(0,0.03,0.09)
	(s,b,h)	(0,0.54,0.65)	(0.79,0.83,0.85)	(0.73,0.79,0.83)

The following step concerns the determination of the support degree. The support degrees calculated by Eq. (10) are listed in Tables 9, 10, and 11.

Table 9. Support degree between DM^1 and DM^2

	A1	A2	A3
C1	(0.78,0.87,0.93)	(0.78,0.98,0.96)	(0.25,0.56,0.78)
C2	(0.48,0.71,0.85)	(0.17,0.56,0.81)	(0.6,0.68,0.85)
C3	(1,1,1)	(0.92,0.94,0.89)	(0.95,0.93,0.86)
C4	(1,1,1)	(0.72,0.88,0.99)	(0.72,0.88,0.99)
C5	(1,0.99,1)	(0.61,0.84,0.97)	(0.82,0.92,0.97)

Table 10. Support degree between DM^1 and DM^3

	A1	A2	A3
C1	(0.26,0.55,0.72)	(0.37,0.61,0.8)	(0.1,0.41,0.66)
C2	(0.5,0.73,0.81)	(0.68,0.85,0.98)	(0.84,0.89,0.98)
C3	(0.67,0.74,0.83)	(0.81,0.89,0.97)	(0.26,0.37,0.51)
C4	(0.8,0.86,0.91)	(0.85,0.97,0.95)	(0.85,0.97,0.95)
C5	(0.73,0.67,0.8)	(0.33,0.6,0.82)	(0.74,0.84,0.83)

Table 11. Support degree between DM^2 and DM^3

	A1	A2	A3
C1	(0.03,0.41,0.65)	(0.16,0.59,0.79)	(0.85,0.86,0.88)
C2	(0.02,0.43,0.65)	(0.29,0.65,0.8)	(0.82,0.84,0.87)
C3	(0.67,0.74,0.83)	(0.83,0.83,0.86)	(0.21,0.45,0.65)
C4	(0.8,0.86,0.91)	(0.86,0.91,0.95)	(0.86,0.91,0.95)
C5	(0.73,0.68,0.8)	(0.72,0.76,0.79)	(0.56,0.76,0.86)

Following the computation of the support degree, the total support degree is calculated using Eq. (11) and is presented in Tables 12, 13, and 14. For convenience, this study replaced T(DM_{ij}^k) with matrix T^k .

Table 12. Total support degree T^1

	A1	A2	A3
C1	(1.03,1.41,1.64)	(1.16,1.59,1.76)	(0.35,0.97,1.44)
C2	(0.97,1.44,1.66)	(0.85,1.41,1.79)	(1.45,1.58,1.83)
C3	(1.67,1.74,1.83)	(1.72,1.83,1.86)	(1.21,1.3,1.37)
C4	(1.8,1.86,1.91)	(1.57,1.84,1.93)	(1.57,1.84,1.93)
C5	(1.73,1.66,1.8)	(0.94,1.43,1.79)	(1.56,1.76,1.81)

Table 13. Total support degree T^2

	A1	A2	A3
C1	(0.81,1.28,1.57)	(0.94,1.57,1.75)	(1.09,1.41,1.66)
C2	(0.5,1.14,1.5)	(0.47,1.2,1.61)	(1.42,1.52,1.72)
C3	(1.67,1.74,1.83)	(1.74,1.77,1.75)	(1.16,1.37,1.51)
C4	(1.8,1.86,1.91)	(1.59,1.79,1.93)	(1.59,1.79,1.93)
C5	(1.73,1.67,1.8)	(1.33,1.59,1.76)	(1.39,1.68,1.83)

Table 14. Total support degree T^3

	A1	A2	A3
C1	(0.29,0.96,1.36)	(0.53,1.21,1.59)	(0.95,1.27,1.54)
C2	(0.52,1.15,1.46)	(0.97,1.5,1.78)	(1.66,1.73,1.85)
C3	(1.35,1.48,1.66)	(1.63,1.72,1.83)	(0.47,0.82,1.15)
C4	(1.6,1.71,1.82)	(1.71,1.88,1.89)	(1.71,1.88,1.89)
C5	(1.46,1.35,1.59)	(1.05,1.35,1.61)	(1.31,1.6,1.7)

The next step is calculating the rationality degree (Step 5). Using Eq. (12); δ_1, δ_2, and δ_3 are calculated as 0.7732, 0.7631, and 0.7059, respectively.

Experts' comprehensive indexes and weights are calculated in the following two steps. Consistent with the professional background and specialist of the three decision-makers, the importance level of experts is determined as $(0.8, 0.65, 0.95)^T$, and α parameter is set to 0.5. The comprehensive index of each expert is found as 0.78569, 0.70653, and 0.82795. The weight of each expert is found 0.33889, 0.304398, and 0.356712, respectively.

Then final weighted decision matrix of each decision-maker is computed by multiplying the experts' weight and support degree. The final weighted decision matrix for each decision-maker is presented in Tables 15, 16, and 17.

Table 15. Final weighted decision matrix of DM^1

		A1	A2	A3
C1	**(l,m,u)**	(0.14,0.14,0.14)	(0.14,0.18,0.2)	(0.14,0.18,0.2)
	(s,b,h)	(0.92,0.92,0.92)	(0.8,0.87,0.9)	(0.8,0.87,0.9)
C2	**(l,m,u)**	(0.13,0.14,0.14)	(0.18,0.2,0.2)	(0.14,0.18,0.2)
	(s,b,h)	(0.89,0.91,0.91)	(0.86,0.9,0.92)	(0.8,0.86,0.9)
C3	**(l,m,u)**	(0.14,0.14,0.14)	(0.1,0.14,0.18)	(0.02,0.06,0.1)
	(s,b,h)	(0.95,0.94,0.93)	(0.87,0.9,0.92)	(0.92,0.94,0.95)
C4	**(l,m,u)**	(0.12,0.12,0.12)	(0.15,0.17,0.17)	(0.15,0.17,0.17)
	(s,b,h)	(0.95,0.95,0.95)	(0.92,0.94,0.95)	(0.92,0.94,0.95)
C5	**(l,m,u)**	(0.43,0.31,0.24)	(0.02,0.06,0.1)	(0,0.02,0.06)
	(s,b,h)	(0,0.81,0.86)	(0.92,0.94,0.95)	(0.9,0.92,0.94)

Table 16. Final weighted decision matrix of DM^2

		A1	A2	A3
C1	(l,m,u)	(0.15,0.15,0.14)	(0.11,0.11,0.13)	(0.15,0.15,0.14)
	(s,b,h)	(0.96,0.95,0.94)	(0,0.88,0.91)	(0.96,0.95,0.94)
C2	(l,m,u)	(0.1,0.11,0.13)	(0.18,0.16,0.14)	(0.14,0.14,0.14)
	(s,b,h)	(0,0.86,0.9)	(0.98,0.96,0.95)	(0.91,0.93,0.93)
C3	(l,m,u)	(0.14,0.14,0.14)	(0.23,0.19,0.18)	(0.05,0.08,0.1)
	(s,b,h)	(0.95,0.94,0.94)	(0.88,0.9,0.91)	(0.94,0.94,0.94)
C4	(l,m,u)	(0.11,0.11,0.11)	(0.11,0.11,0.11)	(0.11,0.11,0.11)
	(s,b,h)	(0.96,0.96,0.96)	(0.96,0.96,0.96)	(0.96,0.96,0.96)
C5	(l,m,u)	(0.41,0.3,0.23)	(0.05,0.09,0.11)	(0,0.03,0.07)
	(s,b,h)	(0,0.83,0.88)	(0.97,0.96,0.95)	(0.93,0.94,0.94)

Table 17. Final weighted decision matrix of DM^3

		A1	A2	A3
C1	**(l,m,u)**	(0.13,0.14,0.15)	(0.23,0.2,0.16)	(0.08,0.1,0.11)
	(s,b,h)	(0,0.82,0.87)	(0.93,0.93,0.93)	(0.96,0.95,0.94)
C2	**(l,m,u)**	(0.07,0.09,0.11)	(0.21,0.18,0.16)	(0.16,0.16,0.16)
	(s,b,h)	(0.95,0.94,0.94)	(0.92,0.92,0.92)	(0.84,0.88,0.89)
C3	**(l,m,u)**	(0.09,0.1,0.12)	(0.14,0.14,0.15)	(0.2,0.19,0.17)
	(s,b,h)	(0.98,0.96,0.95)	(0.9,0.91,0.92)	(0,0.81,0.86)
C4	**(l,m,u)**	(0,0.02,0.05)	(0.18,0.17,0.16)	(0.18,0.17,0.16)
	(s,b,h)	(0.97,0.97,0.96)	(0.93,0.94,0.95)	(0.93,0.94,0.95)
C5	**(l,m,u)**	(0.23,0.2,0.17)	(0.08,0.1,0.12)	(0.13,0.14,0.15)
	(s,b,h)	(0,0,0.81)	(0.99,0.98,0.96)	(0.84,0.89,0.91)

In step 9, each decision maker's final weighted decision matrix DM^k is transformed into the alternative decision matrix A^i (k = 1, 2, 3; i = 1, 2, 3) and the results shown in Tables 18, 19 and 20.

Table 18. Converted matrix of alternative A^1

	DM1	DM2	DM3
C1	(0.59,0.59,0.59)	(0.5,0.53,0.55)	(1,0.76,0.7)
C2	(0.67,0.63,0.62)	(1,0.71,0.65)	(0.51,0.55,0.56)
C3	(0.53,0.56,0.58)	(0.51,0.54,0.56)	(0.4,0.47,0.53)
C4	(0.51,0.51,0.51)	(0.49,0.49,0.49)	(0.43,0.46,0.48)
C5	(0.97,0.76,0.7)	(0.98,0.74,0.68)	(1,1,0.78)

Table 19. Converted matrix of alternative A^2

	DM1	DM2	DM3
C1	(0.78,0.7,0.64)	(1,0.68,0.63)	(0.57,0.57,0.59)
C2	(0.7,0.64,0.59)	(0.37,0.48,0.52)	(0.6,0.61,0.61)
C3	(0.7,0.65,0.59)	(0.67,0.64,0.61)	(0.65,0.62,0.6)
C4	(0.6,0.55,0.51)	(0.49,0.49,0.49)	(0.56,0.55,0.53)
C5	(0.6,0.56,0.53)	(0.44,0.48,0.52)	(0.29,0.39,0.47)

Table 20. Converted matrix of alternative A^3

	DM1	DM2	DM3
C1	(0.78,0.7,0.64)	(0.5,0.53,0.55)	(0.47,0.51,0.54)
C2	(0.79,0.7,0.64)	(0.62,0.58,0.58)	(0.74,0.68,0.66)
C3	(0.6,0.56,0.53)	(0.57,0.56,0.56)	(1,0.78,0.71)
C4	(0.6,0.55,0.51)	(0.49,0.49,0.49)	(0.56,0.55,0.53)
C5	(0.65,0.6,0.56)	(0.57,0.56,0.55)	(0.75,0.66,0.62)

The next step is to calculate A^+ and A^- according to Eq. (16) and (17), respectively. The positive ideal decision matric A^+ and the negative ideal decision matrix A^- are presented in Tables 21 and 22, respectively.

Table 21. Positive ideal decision matrix A^{+}

	DM1	DM2	DM3
C1	(0.78,0.7,0.64)	(1,0.68,0.63)	(0.59,0.58,0.59)
C2	(0.79,0.7,0.64)	(0.62,0.58,0.58)	(0.74,0.68,0.66)
C3	(0.71,0.65,0.6)	(0.68,0.64,0.62)	(1,0.78,0.71)
C4	(0.6,0.55,0.51)	(0.49,0.49,0.49)	(0.56,0.55,0.53)
C5	(0.65,0.6,0.56)	(0.57,0.56,0.55)	(0.75,0.66,0.62)

Table 22. Negative ideal decision matrix A^{-}

	DM1	DM2	DM3
C1	(0.59,0.59,0.59)	(0.5,0.53,0.55)	(0.45,0.5,0.54)
C2	(0.67,0.62,0.59)	(0.37,0.48,0.52)	(0.49,0.54,0.56)
C3	(0.53,0.55,0.53)	(0.5,0.53,0.56)	(0.38,0.46,0.53)
C4	(0.51,0.51,0.51)	(0.49,0.49,0.49)	(0.42,0.45,0.48)
C5	(0.52,0.52,0.52)	(0.24,0.44,0.51)	(0.23,0.37,0.46)

Before computing the relative closeness, the distance between the alternative decision matrix and the positive/negative ideal decision matrix must be calculated by Eq. (18) and (19). The results of this calculation are given in Tables 23 and 24.

Table 23. Distance S_i^{+} between alternative decision matrix A_{kj}^{i} and the positive ideal decision matrix A^{+}

	DM1	DM2	DM3
C1	(0,0.01,0.01)	(0,0,0)	(1.82,0.64,0.38)
C2	(0.01,0.01,0.05)	(1.9,0.5,0.27)	(0.02,0.01,0.01)
C3	(0,0.01,0.1)	(0.02,0.01,0.01)	(0.02,0.01,0.01)
C4	(0,0.01,0.01)	(0,0,0)	(0.01,0.01,0.01)
C5	(1.57,0.59,0.41)	(1.83,0.65,0.36)	(1.95,1.88,0.75)

Table 24. Distance S_i^{-} between alternative decision matrix A_{kj}^{i} and the negative ideal decision matrix A^{-}

	DM1	DM2	DM3
C1	(0.54,0.27,0.11)	(1.75,0.33,0.16)	(1.6,0.5,0.28)
C2	(0.38,0.2,0.07)	(1.52,0.32,0.16)	(0.55,0.3,0.21)
C3	(0.4,0.2,0.04)	(0.34,0.22,0.12)	(1.87,0.75,0.42)
C4	(0.18,0.07,0.01)	(0,0,0)	(0.2,0.14,0.08)
C5	(1.45,0.5,0.37)	(1.63,0.53,0.33)	(1.17,1.42,0.48)

Finally, the relative closeness of the alternative decision matrix A_i to the ideal decision matrix is calculated by Eq. (20). The relative closeness of each alternative (RC_i) is found as -1.089378, -0.714979, and -1.088831. According to the result of step 12, the rank of the three alternatives is RC2>RC1>RC3, so the second supplier alternative is the best.

Sensitivity Analysis

This paper determines different q values to rank the alternatives in order to determine the flexibility and sensitivity of the parameter q. As shown in Table 25 and Figure 1, the experimental results are used to assess the effect of varying the q values on the results. When q is small (q = 3), the RC is small for alternatives A1 and A2; when q is increased, the RC becomes more significant for these two alternatives. By contrast, changes in the q value have a negligible effect on the RC of alternative A3. As shown in Table 25, when the q value increases, the RC of alternatives increases by different ratios. As a result, the ranking results may vary as the q values change. When q is increased from three to five, the ranking of the alternatives changes from A2>A3>A1 to A2>A1>A3, but the best alternatives remain the same. As q increases, the optimal choice changes from A2 to A1. When q=7 or q=9, the ranking results remain constant, but the alternatives' relative closeness values change.

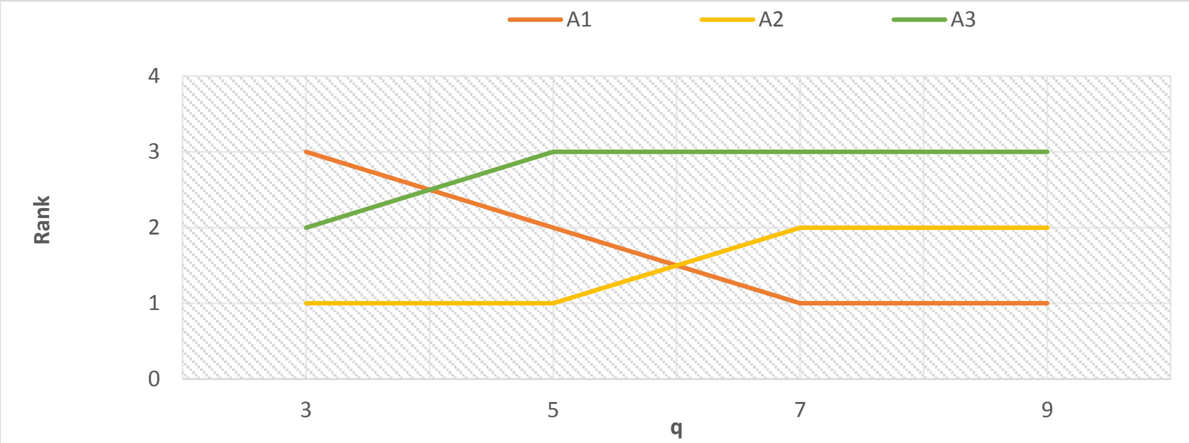

Figure 1. The effect of q-value changes on the ranking result

Table 25. Experimental results of different q values

Scenarios	q	Relative Closeness (RC)	Ranking Alternatives
A	3	RC1 = -1,089378 RC2 = -0,714979 RC3 = -1,088831	A2>A3>A1
B	5	RC1 = -0,777493 RC2 = -0,672265 RC3 = -1,056354	A2>A1>A3
C	7	RC1 = -0,601495 RC2 = -0,639208 RC3 = -1,037384	A1>A2>A3
D	9	RC1 = -0,487982 RC2 = -0,611514 RC3 = -1,024895	A1>A2>A3

Conclusion and Future Studies

This paper develops a new approach as an extension of the traditional TOPSIS method using the q-rung orthopair fuzzy tools to solve MADM problems. The technique utilizes experts' evaluations has been gathered using q-ROTFN which is a special notation of q-ROFN. Additionally, a novel expert weight calculation procedure which is an effective approach to deal with the expert's potential bias and different opinions, is proposed. The outstanding point of this calculation, it hasn't included complex aggregation operators. Last but not least, the result of the automotive industry case study illustrates that the proposed method is suitable for supplier selection problems.

This method is just a new extension of TOPSIS used q-ROTFN, and there are rare studies using this method. Therefore, different MADM problems from various fields can be conducted. Moreover, a sensitivity analysis is set to show the results according to the changing q value, which is the most important parameter of this method.

References

Alibeigi, A., Asemi, A., Munir, A. B., & Baba, M. S. (2021). Evaluating ASEAN E-commerce Laws Using Fuzzy Multi-Criteria Decision Making. Advances in Decision Sciences 25(2), 105-157.

Akram, M., Dudek, W. A., & Ilyas, F. (2019). Group decision- making based on pythagorean fuzzy TOPSIS method. International Journal of Intelligent Systems, 34(7), 1455-1475.

Atanassov, K. T. (1986). Intuitionistic fuzzy sets. *Fuzzy Sets System*, 20(1), 87–96. https://doi.org/10.1016/S0165-0114(86)80034-3

Ayyildiz, E. (2021). Interval-valued intuitionistic fuzzy analytic hierarchy process-based green supply chain resilience evaluation methodology in post-COVID-19 era. *Environmental Science and Pollution Research*, 1-19. https://doi.org/10.1007/s11356-021-16972-y

Chen (2000). Extensions of the TOPSIS for group decision-making under fuzzy environment. *Fuzzy Sets and Systems*. https://doi.org/10.1016/S0165-0114(97)00377-1

David A. Wood (2016). Supplier selection for development of petroleum industry facilities, applying multi-criteria decision making techniques including fuzzy and intuitionistic fuzzy TOPSIS with flexible entropy weighting. *Journal of Natural Gas Science and Engineering*, 28, 594-612.

Dymova, L., Sevastjanov, P., and Tikhonenko, A. (2015). An interval type-2 fuzzy extension of the TOPSIS method using alpha cuts. *Knowledge-Based Systems*, 83, 116-127.

Ervural B. Ç., Öner, S. C., Çoban, V., and Kahraman C. (2015). A novel Multiple Attribute Group Decision Making methodology based on Intuitionistic Fuzzy TOPSIS, IEEE International Conference on Fuzzy Systems (FUZZ-IEEE), 1-6.

Hussain A., Irfan A. M., and Mahmood T. (2019). Covering based q-rung orthopair fuzzy rough set model hybrid with TOPSIS for multi-attribute decision making. *Journal of Intelligent and Fuzzy Systems*, 1-13. DOI: 10.3233/JIFS-181832

Hwang C. L. and Yoon, K. (1981). Methods for multiple attribute decision making in Multiple Attribute Decision Making. Berlin, Germany: Springer-Verlag. https://doi.org/10.1007/978-3-642-48318-9_3

Jahanshahloo, G. R., Hosseinzadeh, L.F., and Izadikhah, M. (2006). Extension of the TOPSIS method for decision-making problems with fuzzy data. *Applied Mathematics and Computation*, 181, 1544–1551. https://doi.org/10.1016/j.apm.2010.11.025

Joshi, D., and Kumar, S. (2016). Interval-valued intuitionistic hesitant fuzzy Choquet integral based TOPSIS method for multi-criteria group decision making. *European Journal of Operational Research*, 248-1,183-191.

Joshi, B. P., Singh, A., and P.K. Bhatt (2018). Interval valued q-rung orthopair fuzzy sets and their properties. *Journal of Intelligent and Fuzzy Systems*, 35(5), 5225-5230. DOI: 10.3233/JIFS-169806

Krishankumar, R., Nimmagadda, S.S., Rani, Mishra, P., A. R., Ravichandran, K.S., and Gandomi, A. H. (2021). Solving renewable energy source selection problems using a q-rung orthopair fuzzy-based integrated decision-making approach. *Journal of Cleaner Production*, 279. https://doi.org/10.1016/j.jclepro.2020.123329

Kutlu Gündoğdu, F., and Kahraman, C. (2020). Spherical Fuzzy Sets and Spherical Fuzzy TOPSIS Method. *Journal of Intelligent & Fuzzy Systems*, 36-1, 337-352.

Lee L. W., and Chen, S. M. (2008). Fuzzy multiple attributes group decision-making based on the extension of TOPSIS method and interval type-2 fuzzy sets. *International Conference on Machine Learning and Cybernetics*, 3260-3265.

Liang, D., and Cao, W. (2019). Q-Rung orthopair fuzzy sets- based decision- theoretic rough sets for three- way decisions under group decision making. *International Journal of Intelligent Systems*, 34(12), 3139-3167. DOI: 10.1002/int.22187

Liao, T. W. (2015). Two interval type 2 fuzzy TOPSIS material selection methods. *Materials & Design*, 88, 1088-1099.

Liu P., Liu P., Wang P., and Zhu, B. (2019). An extended multiple attribute group decision making method based on q-Rung orthopair fuzzy numbers. *IEEE Access* 7, 162050-162061. DOI: 10.1109/ACCESS.2019.2951357

Liu, P, Chen, S. M., and Wang, P. (2018). Multiple-attribute group decision-making based on q-rung orthopair fuzzy power maclaurin symmetric mean operators. *IEEE Transactions on Systems, Man, and Cybernetics: Systems,* 50(10), 3741-3756. DOI: 10.1109/TSMC.2018.2852948

Liu, P., and Wang Y. (2020). Multiple attribute decision making based on q-rung orthopair fuzzy generalized Maclaurin symmetric mean operators. *Information Sciences*, 518, 181-210. DOI: 10.1016/j.ins.2020.01.013

Liu, P., and Wang, P. (2018). Some q-rung orthopair fuzzy aggregation operators and their applications to multiple-attribute decision making. *International Journal of Intelligent Systems*, 33(2), 259-280. https://doi.org/10.1002/int.21927

Peng, X., Dai, J., and Garg, H. (2018). Exponential operation and aggregation operator for q-rung orthopair fuzzy set and their decision- making method with a new score function. *International Journal of Intelligent Systems* 33(11), 2255-2282. https://doi.org/10.1002/int.22028

Pham, V. K., Wong, W. K., Moslehpour, M., & Musyoki, D. (2018). Simultaneous Adaptation of AHP and Fuzzy AHP to Evaluate Outsourcing Service in the East and Southeast Asia. *Available at SSRN 3208130*.

Rani, P., Mishra, A.R., Rezaei, G. et al. (2020). Extended Pythagorean Fuzzy TOPSIS Method Based on Similarity Measure for Sustainable Recycling Partner Selection. *Int. J. Fuzzy Syst.* 22, 735–747.

Senvar, O., Otay, I., and Bolturk E. (2016). Hospital Site Selection via Hesitant Fuzzy TOPSIS. *IFAC-PapersOnLine*, 49-12, 1140-1145.

Tang, G., Chiclana, F. and Liu, P. (2020). A decision-theoretic rough set model with q-rung orthopair fuzzy information and its application in stock investment evaluation. *Applied Soft Computing*, 91. https://doi.org/10.1016/j.asoc.2020.106212

Wang, J., Wei, G., Wei, C., and Wei, Y. (2019). MABAC method for multiple attribute group decision making under q-rung orthopair fuzzy environment. *Defence Technology*, 16 (1) (2019), 208-216. https://doi.org/10.1016/j.dt.2019.06.019

Wei G. W., Wei C., Wang J., Gao H., and Wei Y. (2019). Some q-rung orthopair fuzzy maclaurin symmetric mean operators and their applications to potential evaluation of emerging technology commercialization. *Int J Intell Syst*, 34:5081.

Xu, Z. (2007). Intuitionistic fuzzy aggregation operators, *IEEE Trans. Fuzzy Syst.*, 15(6), 1179–1187. DOI: 10.1109/TFUZZ.2006.890678

Xu, Z. and Zhang X. (2013). Hesitant fuzzy multi-attribute decision making based on TOPSIS with incomplete weight information. *Knowledge-Based Systems*, 52, 53-6.

Yager, R. R. (2014). Pythagorean membership grades in multi-criteria decision making. *IEEE Trans. Fuzzy Syst.*, 22 (4), 958–965. DOI: 10.1109/TFUZZ.2013.2278989

Yager, R. R. (2017). Generalized orthopair fuzzy sets. *IEEE Trans. Fuzzy Syst.*, 25(5), 1222–1230. DOI: 10.1109/TFUZZ.2016.2604005

Yager, R. R., and Alajlan, N. (2017). Approximate reasoning with generalized orthopair fuzzy sets. *Information Fusion*, 38, 65–73. https://doi.org/10.1016/j.inffus.2017.02.005

Yang, Z., Li, X., Cao, Z., and Li, J. (2019). Q-rung orthopair normal fuzzy aggregation operators and their application in multi-attribute decision-making. *Mathematics*, 7(12). DOI: 10.3390/math7121142

Yang W., and Pang Y.F. (2019). New q-rung orthopair fuzzy partitioned Bonferroni mean operators and their application in multiple attribute decision making. *Int JIntell Syst*, 34:439-76. DOI: 10.1109/ACCESS.2020.2979780

Yoon K., and Hwang, C. (1980). Multiple attribute decision making: Methods and applications A State-of-the-Art Survey. *Lecture Notes in Economics and Mathematical Systems book series* (LNE, volume 186). SpringerVerlag.

Youssef, M. A., Zairi, M., and Mohanty, B. (1996). Supplier selection in an advanced manufacturing technology environment: an optimization model. *Benchmarking for Quality Management and Technology*, 3 (4), 60-72. https://doi.org/10.1108/EUM0000000004286

Yucesan, M., and Gul, M. (2020). Hospital service quality evaluation: an integrated model based on Pythagorean fuzzy AHP and fuzzy TOPSIS. *Soft Comput*, 24, 3237–3255 (2020).

Zadeh, L. A. (1965). Fuzzy sets. *Inf. Control*, 8(3), 338–35.