ISSN 2090-3359 (Print)
ISSN 2090-3367 (Online)

Advances in Decision Sciences

Volume 26
Issue 3
September 2022

亞洲大學
ASIA UNIVERSITY

Published by Asia University, Taiwan

Review of Matrix Theory with Applications in Economics and Finance

Dao Le Kieu Oanh
Ho Chi Minh University of Banking, Ho Chi Minh City, Vietnam.
Email: oanhdlk@buh.edu.vn

Le Dinh Hac
Ho Chi Minh University of Banking, Ho Chi Minh City, Vietnam.
Email: lacld@buh.edu.vn

Do Phuong Thao
Faculty of Accounting - Auditing, Ho Chi Minh City Open University
Email: thao.dp@ou.edu.vn

Shin-Hung Pan**
Department of M-Commerce and Multimedia Applications
Asia University, Taiwan

Received: June 22, 2022; First Revision: July 20, 2022;
Last Revision: August 22, 2022; Accepted: September 2, 2022 ;
Published: September 2, 2022

Acknowledgments: The fourth author would like to thank Robert B. Miller and Howard E. Thompson for their continuous guidance and encouragement.

** Corresponding author: vincentpan@asia.edu.tw

ABSTRACT

Matrix theory plays a vital role in Mathematics and several other sciences due to its numerous applications. The topic of matrix theory has a lot of great content for discussion and research. It offers countless beautiful theorems that are straightforward and yet striking in their formulation, uncomplicated and yet ingenious in their proof, and diverse as well as powerful in their application. In the current period, the countries in the world most concerned are Economics and Finance. To the best of our knowledge, these two sciences have the most significant impact on the development and sustainability of each nation in the world. This paper presents the application of matrix theory to important and ubiquitous problems in Economics and Finance, namely, the problem of determining capital demands and the problem defining the quantity of each type of product. Furthermore, we also introduce applications of matrix theory in Decision Sciences to help readers have an overview of applications of matrix theory in other fields.

Keywords: Matrix theory, Review, Applications, Finance, Economics.

JEL: A09, G12, G22, O25.

1. Introduction

Since ancient times, people have known numbers. For example, we need to count the number of people in a village, a commune, or a family. In the family, we also need to count the number of pets, the number of ornamental plants, and the number of problems or certain phenomena. Along with the development of society, information has gradually become an indispensable field in the community. After we get the data, it is necessary to write it somewhere, such as a matrix.

It can be seen that the data is an essential basis for the heads of state to set out the country's development policy and to have an overview for the managers. It is also a prerequisite for all studies. In short, all areas of life around us use numbers, numbers, or, scientifically speaking, matrices. Matrix theory plays a critical role in Mathematics and several other sciences due to its numerous applications. We can easily find enormous, rich, and diverse applications of matrix theory in all sciences, including critical disciplines such as Applied Mathematics, Economics, Finance, Engineering, Education, and so on.

The applications of matrix theory in Economics and Finance are abundant and diverse. Economics models often use the approach of matrices a lot, for example, the Market equilibrium model, the macroeconomic equilibrium model, the IS-LM model (Investment/Saving - Liquidity preference/Money supply model), and the input-output model of Leontief, and many other economic models. As an illustration, Bruce (1977) presented the IS-LM model of macroeconomic equilibrium and the monetarist controversy. Baldick (2002) introduced the effect of parametrization in the electricity market equilibrium models.

Besides, Draganska and Jain (2004) provided a likelihood approach to estimating market equilibrium models. Barker and Santos (2010) presented measuring the efficacy of inventory with a dynamic input-output model. Dymova, Sevastjanov, and Pilarek (2013) provided a method for solving systems of linear interval equations applied to the Leontief input-output model of economics.

Furthermore, some critical problems in Economics and Finance have also used matrices, including the issue of the relationship between cost and quantity of products produced, with some common questions such as: finding ways to combine the input materials to get the most products or the problem of finding the optimal way profits earned. Some of the other important problems are the problem of determining capital demands and the quantity of each type of product.

For the literature on this, readers may refer to Hailu (2010) presented the demand side factors affecting the inflow of foreign direct investment to African countries. Ovchinnikov (2011) introduced revenue and cost management for remanufactured products. Digiesi, Mossa, and Mummolo (2013) provided a sustainable order quantity model under uncertain product demand. Besides, Asphjell et al. (2014) illustrated the sequentiality versus simultaneity: Interrelated factor demand. Alfares and Ghaithan (2016) presented an inventory and pricing model with price-dependent demand, time-varying holding cost, and quantity discounts. Recently, many typical studies on matrices have appeared in the literature, see, for example, Walker et al. (2018), Guo and Wong (2019), Chi et al. (2019), Chu et al. (2020), Norboevna and Husenovich (2020), Hau et al. (2020), Acevedo et al. (2021), Alghalith (2021), Allen and McAleer (2021), Darsono, et al. (2021), Cox (2021), Hon et al. (2021), Kilic and GÖKSEL (2021), Nguyen et al. (2021), Nhan et al. (2021), Suu et al. (2021), Tang et al. (2021), Brahma et al. (2022), Mahmood et al. (2022), and many others.

As we know, Economics and Finance are the two sciences that have the most significant impact on the development and sustainability of each nation in the world. Therefore, it would be interesting to have a detailed and specific study on applying the matrix for Economics and Finance. Motivated by this, it is studied in this work. Our paper is constructed as follows. We review the basic knowledge of matrices in Section 2. The problem of determining capital demands and the quantity of each type of product are presented in Sections 3 and 4, respectively. Applications of matrix theory in Decision Sciences are offered in Section 5. The final section is destined for the conclusion of the paper.

2. Literature Review

The theory and properties of matrices are diverse and plentiful. In this section, we only recall some basic knowledge of matrix theory, including the determinant of matrices and inverse matrices, which apply to the following sections. However, in the era of rapid and robust technology development today, just only need to input data for computers, and the results can be found quickly.

2.1. The determinant of matrices

Let $M = \begin{bmatrix} m_{11} & m_{12} & \dots & m_{1n} \\ m_{21} & m_{22} & \dots & m_{2n} \\ \dots & \dots & \dots & \dots \\ m_{n1} & m_{n2} & \dots & m_{nn} \end{bmatrix}$ be a square matrix with dimension n (n=j+1).

The determinant of M is a real number, denoted by det M and it is defined as below:

$$\det M = \begin{vmatrix} m_{11} & m_{12} & \dots & m_{1n} \\ m_{21} & m_{22} & \dots & m_{2n} \\ \dots & \dots & \dots & \dots \\ m_{n1} & m_{n2} & \dots & m_{nn} \end{vmatrix} = m_{n1}M_{n1} + m_{n2}M_{n2} + \dots + m_{nn}M_{nn},$$

where M_{nk} is the product of $(-1)^{n+k}$ and the determinant with dimension j of the matrix got from M by deleting the nth row and the kth column; k = 1, 2, …, n.

2.2. Special cases

Example 1: Two-dimensional case

Let $M = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix}$ be a square matrix with dimension 2. The determinant of M is a real number, denoted by det M and it is determined as follows:

$$\det M = \begin{vmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{vmatrix} = m_{21}M_{21} + m_{22}M_{22} = m_{21}(-m_{12}) + m_{22}m_{11} = m_{11}m_{22} - m_{12}m_{21}.$$

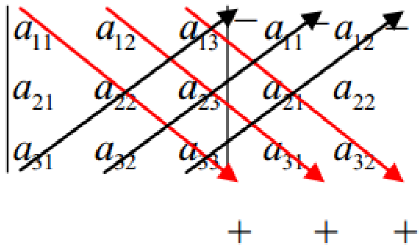
Example 2: Three-dimensional case

Let $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ be a square matrix with dimension 3. The determinant of A is a real number, denoted by detA as given below:

$$\text{detA} = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{31}(-1)^{3+1}\begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix} + a_{32}(-1)^{3+2}\begin{vmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{vmatrix} + a_{33}(-1)^{3+3}\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}.$$

$$= a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - a_{31}a_{22}a_{13} - a_{32}a_{23}a_{11} - a_{33}a_{21}a_{12}$$

To simply remember the above formula, the Sirus formula can be used as the following diagram:



2.3. Inverse matrix

The square matrix M is called an invertible matrix if have a square matrix N such that MN=NM=I (where I is a unit matrix, provided that the same dimension as M and N). Then N is called an invertible matrix of M, denoted by M^{-1}.

It should be noted that, if M is a square matrix and $\det M \neq 0$ then M is an invertible matrix, and we always have M M^{-1}=M^{-1}M=I.

The paper aims to introduce two great applications of matrix theory, one of which is the application in Economics and Finance through two significant problems: the problem of determining capital demands and the quantity of each type of product.

3. The problem of determining capital demands

Determining capital demands is a ubiquitous and crucial problem in Economics and Finance. This is also the problem that every factory, company, or enterprise must solve when planning production. Many types of capital need to be prepared for a plan, such as capital for materials, labor, taxes, insurance, benefits and other expenses, and so on. Although the content or approach may differ, the way of preparing and solving problems to determine capital demands is analogous. We now present the problem of determining the capital demands of materials for production in this work.

3.1. General problem

The factory has plans to produce n types of products for the year, namely P_1, P_2, ..., P_n from m materials M_1, M_2, ..., M_m. To produce a unit of product P_j, it is necessary to use simultaneously b_{ij} unit of material M_i (i = 1, 2, ..., m). It is supposed that the factory has production demand in the year for each product category P_j is D_j product units. Assume that we need to find the capital required for each type of material.

This problem is summarized as follows:

Product / Material	Material cost / unit of product			
	P_1	P_2	…	P_n
M_1	c_{11}	c_{12}	…	c_{1n}
M_2	c_{21}	c_{22}	…	c_{2n}
…	…	…	…	…
M_m	c_{m1}	c_{m2}	…	c_{mn}
Demand (unit)	d_1	d_2	…	d_n

3.2. Description through the matrix

Let $U = \begin{pmatrix} u_1 \\ u_2 \\ \dots \\ u_m \end{pmatrix}$ be the vector representing the number of units of material in each type.

Let $c_j = \begin{pmatrix} c_{1j} \\ c_{2j} \\ \dots \\ c_{mj} \end{pmatrix}$ be the vector denoting the norm of material to produce a unit product P_j.

Then $d_j c_j = d_j \begin{pmatrix} c_{1j} \\ c_{2j} \\ \dots \\ c_{mj} \end{pmatrix}$ be the vector representing the number of material units each need to produce d_j unit of product P_j.

Then the total amount of materials needed to produce n items as required is:

$$U = d_1c_1 + d_2c_2 + \dots + d_nc_n = \begin{pmatrix} c_1 & c_2 & \dots & c_n \end{pmatrix} \begin{pmatrix} d_1 \\ d_2 \\ \dots \\ d_n \end{pmatrix} = \begin{pmatrix} c_{11} & c_{12} & \dots & c_{1n} \\ c_{21} & c_{22} & \dots & c_{2n} \\ \dots & \dots & \dots & \dots \\ c_{m1} & c_{m2} & \dots & c_{mn} \end{pmatrix} \begin{pmatrix} d_1 \\ d_2 \\ \dots \\ d_n \end{pmatrix}.$$

Let $C = \begin{pmatrix} c_1 & c_2 & \dots & c_n \end{pmatrix} = \begin{pmatrix} c_{11} & c_{12} & \dots & c_{1n} \\ c_{21} & c_{22} & \dots & c_{2n} \\ \dots & \dots & \dots & \dots \\ c_{m1} & c_{m2} & \dots & c_{mn} \end{pmatrix}$ is the cost matrix, and $D = \begin{pmatrix} d_1 \\ d_2 \\ \dots \\ d_n \end{pmatrix}$ be a vector denoting production demands, then we have: $U = CD$.

Thus we have the relationship: [capital] = [cost]. [production demands].

It should be remarked that the cost matrix C is often stable over the long term. Meanwhile, the production demand vector (vector D) changes over a short period (such as day, week, month, quarter, year, etc.).

3.3. The process of solving the problem

Step 1: Making cost matrix C. In which the i^{th} column of the C matrix is the i^{th} vector column, indicating the quantity of each type of material to produce the i^{th} product of the company or enterprise.

Step 2: Set vector D to indicate the company's or business's production demands.

Step 3: Then, the required investment is the result of product of matrices C and D.

3.4. Example 3

Suppose a sugar company produces two types of products including white sugar and refined sugar. To produce these two types of products, the company has to use two types of materials A and B know the cost of materials is shown below:

Product / Material (tons)	Material cost / unit of product	
	White sugar	Refined sugar
A	4	6
B	5	7
Production demands (tons)	2000	3000

Suppose the company wants to produce d_1 tons of white sugar and d_2 tons of refined sugar. How many tons of each kind of material should be prepared?

Applies to d_1 = 2000 tons and d_2 = 3000 tons

Solution

Let $c_1 = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$ be the vector denoting the quantity of materials A and B it takes to produce one ton of white sugar, then the materials needed to produce d_1 tons of white sugar will be: $d_1 c_1 = d_1 \begin{pmatrix} 4 \\ 5 \end{pmatrix}$.

Likewise, let $c_2 = \begin{pmatrix} 6 \\ 7 \end{pmatrix}$ be the vector denoting the quantity of materials A and B it takes to produce one ton of refined sugar, then the materials needed to produce d_2 tons of white sugar will be: $d_2 c_2 = d_2 \begin{pmatrix} 6 \\ 7 \end{pmatrix}$.

The total material required would be:

$$U = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = d_1 c_1 + d_2 c_2 = d_1 \begin{pmatrix} 4 \\ 5 \end{pmatrix} + d_2 \begin{pmatrix} 6 \\ 7 \end{pmatrix} = \begin{pmatrix} 4 & 6 \\ 5 & 7 \end{pmatrix} \begin{pmatrix} d_1 \\ d_2 \end{pmatrix}.$$

Let $C = \begin{pmatrix} c_1 & c_2 \end{pmatrix} = \begin{pmatrix} 4 & 6 \\ 5 & 7 \end{pmatrix}$, then C is called the cost matrix, and $D = \begin{pmatrix} d_1 \\ d_2 \end{pmatrix}$ is a vector denoting the production demands of the company. Thus, to produce d_1 tons of white sugar and d_2 tons of refined sugar, a number of materials must be prepared: $U = CD$

Because d_1 = 2000 and d_2 = 3000, and thus,

$$U = \begin{pmatrix} 4 & 6 \\ 5 & 7 \end{pmatrix} \begin{pmatrix} 2000 \\ 3000 \end{pmatrix} = \begin{pmatrix} 4*2000 + 6*3000 \\ 5*2000 + 7*3000 \end{pmatrix} = \begin{pmatrix} 26000 \\ 31000 \end{pmatrix}$$

So, to produce 2,000 tons of white sugar and 3,000 tons of refined sugar, one needs to have 26,000 tons of material A and 31,000 tons of material B.

Regarding economics significance: The problem of determining capital demands is of great importance and plays a critical role in ensuring the smooth and sustainable production and business activities of every business or company. Businesses or companies need to address this problem before producing or selling products. Suppose the business or company does not solve this problem first. In that case, it could lead to a shortage of capital, failure to implement the plan as intended, and may lead to bankruptcy in the business or company.

4. The problem of determining the quantity of each type of material to use

This problem appears in many manufacturing industries such as pharmaceuticals, cosmetics, food processing and apparel, and so on. The main content of the problem is that we intend to produce many types of products with several specific products. Then we need to find the quantity of each kind of material required to meet the requirements. It should be noted that the total amount of the product and the total amount of material required should be equal. To illustrate the general problem, we consider the problem of production for the apparel industry as follows:

4.1. General problem

Garment company is expected to produce m types of product P_1, P_2, ..., P_m from n types of materials are M_1, M_2, ..., M_n. Each material unit M_j contains b_{ij} the unit of product P_i. Set up a formula to find out the number of each type of material to produce a unit of product with all m types of materials and meet the demand for d_i unit of material P_i. The above problem can be summarized as follows:

Material / Products	The quantity of product in a unit of material				The total of products
	M_1	M_2	…	M_n	
P_1	c_{11}	c_{12}	…	c_{1n}	d_1
P_2	c_{21}	c_{22}	…	c_{2n}	d_2
…	…	…	…	…	…
P_m	c_{m1}	c_{m2}	…	c_{mn}	d_m

4.2. Description through the matrix

Let $U = \begin{pmatrix} u_1 \\ u_2 \\ \dots \\ u_m \end{pmatrix}$ be the vector representing the quantity of each type of material.

Let $c_j = \begin{pmatrix} c_{1j} \\ c_{2j} \\ \dots \\ c_{mj} \end{pmatrix}$ be the vector denoting the quantity of m product in a unit of material j.

Then $u_j c_j = u_j \begin{pmatrix} c_{1j} \\ c_{2j} \\ \dots \\ c_{mj} \end{pmatrix}$ be the vector denoting the quantity of m product in x_j unit of material j.

Let $D = \begin{pmatrix} d_1 \\ d_2 \\ \dots \\ d_n \end{pmatrix}$ be a vector denoting the total of products, we have:

$$\sum_{j=1}^{n} u_j c_j = D \Leftrightarrow u_1 c_1 + u_2 c_2 + \dots + u_n c_n = D$$

$$\Leftrightarrow u_1 \begin{pmatrix} c_{11} \\ c_{12} \\ \dots \\ c_{m1} \end{pmatrix} + u_2 \begin{pmatrix} c_{12} \\ c_{22} \\ \dots \\ c_{m2} \end{pmatrix} + \dots + u_n \begin{pmatrix} c_{1n} \\ c_{2n} \\ \dots \\ c_{mn} \end{pmatrix} = \begin{pmatrix} d_1 \\ d_2 \\ \dots \\ d_n \end{pmatrix}$$

$$\Leftrightarrow \begin{pmatrix} c_{11} & c_{12} & \dots & c_{1n} \\ c_{21} & c_{22} & \dots & c_{2n} \\ \dots & \dots & \dots & \dots \\ c_{m1} & c_{m2} & \dots & c_{mn} \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \\ \dots \\ u_n \end{pmatrix} = \begin{pmatrix} d_1 \\ d_2 \\ \dots \\ d_n \end{pmatrix}$$

We can rewrite the above expression as a matrix as follows: $CU = D$. If C is an invertible matrix, then: $U = C^{-1}D$.

Where $C = \begin{pmatrix} c_{11} & c_{12} & \dots & c_{1n} \\ c_{21} & c_{22} & \dots & c_{2n} \\ \dots & \dots & \dots & \dots \\ c_{m1} & c_{m2} & \dots & c_{mn} \end{pmatrix}$ be a matrix denoting "the quantity of product" in each of materials. We have the relationship:

[the quantity of product].[the quantity of material]= [demands].

The formula for $U = C^{-1}D$ is very convenient to calculate and it is very easy to program for computers. The same as the problem of determining capital demands are described in the previous section, using the formula written in matrix form helps us quickly calculate the results when matrix D changes.

4.3. The process of solving the problem

Step 1: Setting the matrix C, denotes the number of products in each material.

Step 2: Describing the column vector D denotes the total number of products.

Step 3: Then, the quantity of each material required to produce a unit of product is the product of the inverse of the matrix C and matrix D.

It should be noted: that the total quantity of products and the total quantity of materials must be equal; then we will have the same number of equations and unknown variables. In this case, C is a square matrix; then, we can find the inverse of the matrix C if its determinant is different from zero.

4.4. Example 4

The drug manufacturing company needs to produce three types of drugs: antibiotics, cold and cough medicines from 3 kinds of materials: (I), (II), and (III). Figures are given in the following table

Material / Drugs	The quantity of product in a unit of material			The total of products
	(I)	(II)	(III)	
Antibiotics	10	30	20	350
Cold	80	70	90	1430
Cough	40	50	60	880

a) Write matrix equations to find the quantity required for each type of material when processing a unit of three products.

b) Assuming 500 units of three products are needed, how many materials are each required?

Solution

a) Let $U = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}$ be the vector representing the quantity of each type of material.

Let $c_1 = \begin{pmatrix} 10 \\ 80 \\ 40 \end{pmatrix}$ be the vector denoting the quantity of product in a unit of material (I).

Then $u_1 c_1 = u_1 \begin{pmatrix} 10 \\ 80 \\ 40 \end{pmatrix}$ is the vector denoting the quantity of product in u_1 unit of material (I).

Likewise, $u_2 c_2 = u_2 \begin{pmatrix} 30 \\ 70 \\ 50 \end{pmatrix}$ and $u_3 c_3 = u_3 \begin{pmatrix} 20 \\ 90 \\ 60 \end{pmatrix}$ are the vector denoting the quantity of product in u_2 and u_3 unit of material (II) and (III), respectively.

Let $D = \begin{pmatrix} 350 \\ 1430 \\ 880 \end{pmatrix}$ be a vector denoting the total of products, we have:

$$u_1 c_1 + u_2 c_2 + u_3 c_3 = D$$

$$\Leftrightarrow u_1 \begin{pmatrix} 10 \\ 80 \\ 40 \end{pmatrix} + u_2 \begin{pmatrix} 30 \\ 70 \\ 50 \end{pmatrix} + u_3 \begin{pmatrix} 20 \\ 90 \\ 60 \end{pmatrix} = \begin{pmatrix} 350 \\ 1430 \\ 880 \end{pmatrix} \Leftrightarrow \begin{pmatrix} 10 & 30 & 20 \\ 80 & 70 & 90 \\ 40 & 50 & 60 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} = \begin{pmatrix} 350 \\ 1430 \\ 880 \end{pmatrix} \Leftrightarrow CU = D$$

where $C = \begin{pmatrix} 10 & 30 & 20 \\ 80 & 70 & 90 \\ 40 & 50 & 60 \end{pmatrix}$ is a matrix that denotes the quantity of product in each of the materials.

The equation $CU = D$ is a matrix equation used to find the quantity required for each type of material when processing a unit of 3 products.

b) We have: $CU = D \Leftrightarrow U = C^{-1}D = \begin{pmatrix} 10 & 30 & 20 \\ 80 & 70 & 90 \\ 40 & 50 & 60 \end{pmatrix}^{-1} \begin{pmatrix} 350 \\ 1430 \\ 880 \end{pmatrix} = \begin{pmatrix} 7 \\ 6 \\ 5 \end{pmatrix}$.

To produce 500 units of three products then: $500U = 500\begin{pmatrix} 7 \\ 6 \\ 5 \end{pmatrix} = \begin{pmatrix} 3500 \\ 3000 \\ 2500 \end{pmatrix}$.

Thus, to produce 500 units of three products, we need to have 3500 material (I), 3000 material (II), and 2500 material (III).

In terms of economic significance: The problem of determining the quantity of each type of material is very practical and has a critical role for every business or company. When trying to produce new kinds of products, the business or company first needs to determine the specific quantity for new types of products. Thus, using this problem, the particular quantity for each type of material can be found quickly.

Businesses or companies need to address this question before conducting the production or sales of products. If the business or company does not solve this problem first, it can lead to a shortage of raw materials, sometimes leading to a lack of capital. Therefore, it is impossible to carry out the plan as intended and may result in bankruptcy for the business or the company.

5. Applications of matrix theory in Decision Sciences

In addition to the applications of matrices in Economics and Finance, as presented in Sections 3 and 4. It is detailed here to help readers see an overview of the application of matrix theory in other fields.

5.1. Applied Mathematics

In Mathematics, in particular, Applied Mathematics, the most widespread application of the Matrix is to address linear equation systems. Then from the original linear equation system, it can be written in the form of matrix equations, and using invertible matrices, afterward, can be found the solutions of this system of linear equations. The general approach for this problem is the Cramer formula. Renowned mathematician Cramer proposed this expression in 1750. It is mainly used to solve systems of linear equations with several unknown variables, provided that the system of equations must be equal to the number of equations included in the design of equations.

Up to the present, there have been many scientists who have applied the Cramer formula so far and proposed many improvements to it. For instance, Ben-Israel (1982) presented a Cramer rule for least-norm roots of consistent linear equations. Wang (1989) provided a Cramer rule to find the basis of a class of singular equations. Besides, Wang and Sun (2004) introduced a Cramer rule for the root of the generally restricted matrix equation. Song and Wang (2011) proposed the condensed Cramer rule for some local quaternion linear equations and so on.

5.2. Applied Statistics

In statistical inference, stochastic matrices are frequently employed to depict sets of probabilities; for example, it is used within the PageRank algorithm that ranks the pages in a Google search. Besides, matrix theory is heavily employed in distribution functions, multi-dimensional density functions, etc. Moreover, the variance of the covariance matrix and the models in multi-dimensional space are also used matrices.

Readers may refer to Rao (1962) presented an approach to finding a general inverse of a matrix with applications to issues in mathematical statistics. Nel (1980) provided the matrix differentiation in statistics. Additionally, Radhakrishna and Bhaskara (1998) developed the Matrix algebra and its applications to Statistics. Lean et al. (2010) apply the theory of matrix to study the relationship between the spot and futures markets in Malaysia. Schott (2016) offered the matrix analysis for statistics. Pho et al. (2019) compared the widespread model selections that include the matrix theory. Magnus and Neudecker (2019) have recently illustrated the matrix differential calculus with applications in Statistics, Econometrics, and many others.

5.3. Numerical Analysis

A primary research direction of numerical analysis whose main aim is to develop efficient algorithms for matrix computation. This is a topic that is thousands of centuries old, and it is considered a very interesting area of research today. Within this topic, there are many areas where there are uses or applications of matrix theory. More specifically, an extremely popular and important matrix is the Jacobi matrix, which contains the first partial derivatives of functions between two vector spaces. This matrix is named after the famous mathematician Carl Gustav Jacobi (Hald, 1976).

Another but no less famous matrix is the Hessian matrix. It is a square matrix of the partial quadratic derivative of some function so that it will represent the curvature of a part of many variables. This matrix was developed in the 19th century by the German mathematician Ludwig

Otto Hesse (Sasson et al., 1973). Newton's formula is an extremely famous formula that uses both Jacobi matrices and Hessian matrices. This formula has enormous applications in Decision Sciences (Truong et al., 2019).

5.4. Education

Education plays a crucial and significant role in the present industrialization era. Higher education plays a vital role in helping students access science and technology and choose a solid future career. Mathematics plays an important role in higher education and is a bridge for all branches of science. Meanwhile, matrix theory is crucial and meaningful in university teaching. As it is well known, matrix theory is taught in most universities in Vietnam and Taiwan to students who are not majoring in Mathematics. Thus, it can be seen that universities have recognized matrix theory for its practicality, importance, and influence.

Besides, as stated in the previous sections on applying Matrix theory to Finance and Economics. These applications are practical applications; they are very close to our lives, following the new trend of the Ministry of Education and Training in Vietnam, as well as in many countries around the world. Teaching needs to be associated with the practice, which is the main content of the Education sector in the current period.

Specifically, the problem of determining the quantity of each type of product appeared in the National Mathematical Olympiad Exam test for students in 2019. It is illustrated as follows:

A factory produces five types of products A, B, C, D, and E. Each of them must go through five stages of cutting, trimming, packing, decorating, and labeling with the time for each step as in the following table:

Product	Cutting (hour)	Trimming (hour)	Packing (hour)	Decorating (hour)	Labeling (hour)
A	1	1	1	1	1
B	4	3	3	2	1
C	8	12	6	3	1
D	12	15	10	4	1
E	20	24	10	5	1

The cutting, trimming, packing, decorating, and labeling parts with the maximum number of working hours in a week are 180, 220, 120, 60, and 20, respectively. In the original design of the plant, there was a plan on the number of each type of product the factory had to produce in a week to use up the capacity of the parts. Calculate the quantity of each type of product made in a week according to that plan.

The solution of this test is straightforward, it is only the application of the problem of determining the quantity of each type of product illustrated in Section 4. So, we do not present solutions here. From the above illustration, it can be seen that matrix has been applied in almost all fields of science, and it has been taught to many university students. It not only helps students do well in exams but also helps students apply the knowledge they learned to everyday life.

This section presents very detailed and complete applications of matrices in many vital areas of life. A matrix is an essential tool that is widely used in the work of professionals in many different disciplines such as medical, psychological, educational, sociological, technical, physical, ... or in other activities of society such as business, industry, and government.

6. Conclusion

We can see that now the matrix plays a critical role, an indispensable role in matrix works, issues or problems with millions of data will become lifeless, and meaningless. In addition, it can be added that the matrix is a part of providing basic knowledge to serve the learning process of specialized subjects. We can manipulate matrices to solve some practical data analysis and presentation problems. It will help us gain more important and necessary knowledge and recognize the role of many other sciences in many areas of science and life. From there, we can form a love of science, want to explore, learn and have a profound and positive attitude toward the learning process.

Therefore, to help readers learn more about the matrix and gain more knowledge about this part, the article has taken advantage of all the knowledge we know and the reference from various sources to make this article. This article can be viewed as a primary and comprehensive picture of the game. It is intended as a complete and detailed reference to matrices.

Matrix theory is widely applied in science such as Mathematics, Statistics, Finance, Economics, Engineering and Education, and so on. In the current period, the countries in the world most concerned are Economics and Finance. These two sciences have the most significant impact on the development and sustainability of each nation in the world. In this paper, we have presented the application of Matrix Theory to essential problems in Economics and Finance, namely two problems: the problem of determining capital demands and the problem of determining the quantity of each type of product. This work is very detailed and complete to give readers an overview of the matrix application in Economics and Finance. Besides, we presented applications of matrix theory in Decision Sciences to help readers have an overview of applications of matrix theory in other fields.

References

Acevedo, R. A., Aponte, E., Harmath, P., and Mora Mora, J. (2021). Rational irrationality: A two stage decision making model. *Advances in Decision Science*, *25*(1). 1-39.

Alghalith, M., Swanson, N., Vasnev, A., and Wong, W. K. (2021). Editorial Statement In Honor Of Professor Michael Mcaleer. *Annals of Financial Economics*, *16*(03), 2101002.

Alfares, H. K., and Ghaithan, A. M. (2016), Inventory and pricing model with price-dependent demand, time-varying holding cost, and quantity discounts, *Computers and Industrial Engineering*, 94, 170-177.

Allen, D. E., and McAleer, M. (2021). Drawbacks in the 3-Factor Approach of Fama and French (2018). *Annals of Financial Economics*, 2240001.

Asphjell, M. K., Letterie, W., Nilsen, Ø. A., and Pfann, G. A. (2014), Sequentiality versus simultaneity: Interrelated factor demand, *Review of Economics and Statistics*, 96(5), 986-998.

Baldick, R. (2002), Electricity market equilibrium models: The effect of parametrization, *IEEE Transactions on Power Systems*, 17(4), 1170-1176.

Barker, K., and Santos, J. R. (2010), Measuring the efficacy of inventory with a dynamic input–output model, *International Journal of Production Economics*, 126(1), 130-143.

Ben-Israel, A. (1982), A Cramer rule for least-norm solutions of consistent linear equations, *Linear Algebra and its Applications*, 43, 223-226.

Brahma, S., Brandenberger, R., & Laliberte, S. (2022). Emergent cosmology from matrix theory. Journal of High Energy Physics, 2022(3), 1-22.

Bruce, N. (1977), The IS-LM Model of Macroeconomic Equilibrium and the Monetarist Controversy, *Journal of Political Economy*, 85(5), 1049-1062.

Chi, Y., Lu, Y. M., & Chen, Y. (2019). Nonconvex optimization meets low-rank matrix factorization: An overview. IEEE Transactions on Signal Processing, 67(20), 5239-5269.

Chu, A. M., Lv, Z., Wagner, N. F., and Wong, W. K. (2020). Linear and nonlinear growth determinants: The case of Mongolia and its connection to China. *Emerging Markets Review*, *43*, 100693.

Cox, T. R. (2021). The matrix in cancer. Nature Reviews Cancer, 21(4), 217-238.

Cramer, G. (1750). Introduction a l'analyse des lignes courbes algebriques par Gabriel Cramer.. chez les freres.

Darsono, S. N. A. C., Wong, W. K., Thai, N. T., Jati, H. F., and Dewanti, D. S. (2021). Cultural dimensions and sustainable stock exchanges returns in the Asian region. *Journal of Accounting and Investment*, *22*(1).

Digiesi, S., Mossa, G., and Mummolo, G. (2013), A sustainable order quantity model under uncertain product demand, *IFAC Proceedings Volumes*, 46(9), 664-669.

Draganska, M., & Jain, D. (2004), A likelihood approach to estimating market equilibrium models, *Management Science*, 50(5), 605-616.

Dymova, L., Sevastjanov, P., and Pilarek, M. (2013), A method for solving systems of linear interval equations applied to the leontief input–output model of economics, *Expert Systems with Applications*, 40(1), 222-230.

Guo, X., and Wong, W. K. (2019). Comparison of the production behavior of regret-averse and purely risk-averse firms. *Estudios de Economía*, *46*(2), 157-171.

Hailu, Z. A. (2010), Demand side factors affecting the inflow of foreign direct investment to African countries: does capital market matter?, *International Journal of Business and Management*, 5(5), 104.

Hald, O. H. (1976), Inverse eigenvalue problems for Jacobi matrices, *Linear Algebra and Its Applications*, 14(1), 63-85.

Hau, N. H., Tinh, T. T., Tuong, H. A., and Wong, W. K. (2020). Review of matrix theory with applications in education and decision sciences. *Advances in Decision Sciences*, *24*(1), 1-41.

Hon, T. Y., Moslehpour, M., and Woo, K. Y. (2021). Review on behavioral finance with empirical evidence. *Advances in Decision Sciences*, *25*(4), 1-30.

Kilic, E., and GÖksel, O. (2021). Arbitrageur Behavior In Sentiment-Driven Asset-Pricing. *Annals of Financial Economics*, *16*(03), 2150015.

Lean, H. H., Lien, D. H. D., and Wong, W. K. (2010). Futures versus stocks: a stochastic dominance study in Malaysian markets. *Advances in Investment Analysis and Portfolio Management*, (4), 49-80.

Magnus, J. R., and Neudecker, H. (2019), Matrix differential calculus with applications in statistics and econometrics. *John Wiley and Sons*.

Nel, D. G. (1980), On matrix differentiation in statistics, *South African Statistical Journal*, 14(2), 137-193.

MAHMOOD, F., SHAHZAD, U., NAZAKAT, A., AHMED, Z., RJOUB, H., and WONG, W. K. (2022). THE NEXUS BETWEEN CASH CONVERSION CYCLE, WORKING CAPITAL FINANCE, AND FIRM PERFORMANCE: EVIDENCE FROM NOVEL MACHINE LEARNING APPROACHES. *Annals of Financial Economics*, 2250014.

Nhan, D. T. T., Pho, K. H., ANH, D. T. V., and McAleer, M. (2021). Evaluating the efficiency of Vietnam banks using data envelopment analysis. *Annals of Financial Economics*, *16*(02), 2150010.

Nguyen, T. T., Huynh, T. L. D., and Wong, W. K. (2021). Factors driving openness in China trade: corruption, exchange rate volatility, and macro determinants. *Review of Pacific Basin Financial Markets and Policies*, *24*(02), 2150016.

Norboevna, B. M., & Husenovich, R. T. (2020). The method of using problematic education in teaching theory of matrix to students. Academy, (4 (55)), 68-71.

Ovchinnikov, A. (2011), Revenue and cost management for remanufactured products, *Production and Operations Management*, 20(6), 824-840.

Pho, K. H., Ly, S., Ly, S., and Lukusa, T. M. (2019), Comparison among Akaike Information Criterion, Bayesian information criterion and Vuong's test in model selection: a case study of violated speed regulation in Taiwan, *Journal of Advanced Engineering and Computation*, 3(1), 293-303.

Radhakrishna, R. C., and Bhaskara, R. M. (1998), Matrix algebra and its applications to statistics and econometrics, *World Scientific*.

Rao, C. R. (1962), A note on a generalized inverse of a matrix with applications to problems in mathematical statistics, *Journal of the Royal Statistical Society: Series B (Methodological)*, 24(1), 152-158.

Sasson, A. M., Viloria, F., & Aboytes, F. (1973). Optimal load flow solution using the Hessian matrix. *IEEE Transactions on Power Apparatus and Systems*, (1), 31-41.

Schott, J. R. (2016), Matrix analysis for statistics. *John Wiley and Sons*.

Song, G. J., and Wang, Q. W. (2011), Condensed Cramer rule for some restricted quaternion linear equations, *Applied Mathematics and Computation*, 218(7), 3110-3121.

Suu, N. D., Tien, H. T., and Wong, W. K. (2021). The impact of capital structure and ownership on the performance of state enterprises after equitization: Evidence from Vietnam. *Annals of Financial Economics*, *16*(02), 2150007.

Tang, L., Zhang, J., Tang, Y., Kong, J., Liu, T., & Gu, J. (2021). Polymer matrix wave-transparent composites: A review. Journal of Materials Science & Technology, 75, 225-251.

Truong, B. C., Van Thuan, N., Hau, N. H., & McAleer, M. (2019). Applications of the Newton-Raphson Method in Decision Sciences and Education. *Advances in Decision Sciences,* 23(4), 1-28.

Walker, C., Mojares, E., & del Río Hernández, A. (2018). Role of extracellular matrix in development and cancer progression. International journal of molecular sciences, 19(10), 3028.

Wang, G. R. (1989), A Cramer rule for finding the solution of a class of singular equations, *Linear Algebra and its Applications*, 116, 27-34.

Wang, G. R., and Sun, J. (2004), A Cramer rule for solution of the general restricted matrix equation, *Applied mathematics and computation*, 154(2), 415-422.