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Wilson Models and its Applications in Decision Sciences

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Abstract

Purpose: The problems of getting the optimal order quantity and reducing reserve management costs play an abundantly important role in economics and several other disciplines such as construction, business, and finance. Thus, it is tremendously meaningful to study the issues.

Design/methodology/approach: Wilson is a ubiquitous model utilized to get optimize order quantity and reduce reserve management costs. This model was first developed by Harris (1913) and followed up by Wilson (1934) who expanded it to become a Wilson model. Although the Wilson model has been used for a very long time, the theory and applications of this model need to be stated clearly and systematically. To bridge the gap in the literature in this area and provide academics and practitioners with an overview of the Wilson model, in this paper, we explore the issue.

Findings: We first introduce the general concepts and principles of reserve management. We then present the origin and workaround of the Wilson model and exhibit two examples to illustrate the approach. In addition, we also provide some applications of the Wilson model in Economics.

Originality/Value: All of the issues presented and discussed in this paper are unique and new in the field.

Practical implications: This work will help people interested in finance and economics have the best plan to carry out the work in the most cost-effective and efficient way.

Keywords: Wilson model, Management system, Economic.

JEL Classifications: C01, D12, E23

Paper Type: Research Paper

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1 Introduction

The storage of goods or materials in the enterprise's warehouse to meet the business's production needs as well as the demand for customers' products. A business or a company, a manufacturing industry that wants to produce products needs to have materials and goods to meet the demand for making that product. Therefore, a reserve management system is required so that production is not delayed due to a lack of materials or too many materials or products manufactured without storage space leading to broken materials or products. To address this issue, a reserve management model is proposed and widely utilized in the literature.

For the reserve management model, there are several scholars have researched and utilized it in their research. For instance, Poole (1968) introduces commercial bank reserve management in a stochastic model: implications for monetary policy. Howe (1984) provides implications of animal seed dispersal for tropical reserve management. Dooley et al. (2004) present the revived Bretton Woods system: the effects of periphery intervention and reserve management on interest rates & exchange rates in center countries. Dong et al. (2005) introduce the improvement of voltage stability by reactive power reserve management. Borio et al. (2008) provide about FX reserve management: trends and challenges. Alfaro et al. (2009) present optimal reserve management and sovereign debt. Vandoorn et al. (2013) introduce Microgrids: Hierarchical control and an overview of the control and reserve management strategies. Mohan et al. (2015) provide an efficient two-stage stochastic optimal energy and reserve management in a microgrid. Bakar et al. (2018) present about provision for Bad & Doubtful Financing and Contingency Reserve Management: Assessing Resilient and Stable Islamic Banks.

Until today the problem of the optimal order quantity and reserve management costs is still a concern, and thus, it has been researched by several scientists. Readers may refer to Rezaei and Kalantar (2015), Rahman and Chowdhury (2016), Han (2017), Zhao et al. (2017), Zhou et al. (2017), Cagnano et al. (2018), Han, Lai, and Ho (2018), Ferreira et al. (2018), Wright, Cundill and Biggs (2018) and Castillo et al. (2019), etc.

For numerous businesses today, the traditional method of reserve management is still essential and necessary, especially for small and medium enterprises. We are often concerned about the following issues: what basic elements of the reserve management system? Which common model manages adequate reserves? What reserves are products or raw materials? How much to reserve? These issues play exceedingly vital roles in current economics.

The Wilson model is a widespread model utilized to address the problem of the optimal order quantity and reserve management costs. In inventory management, economics order quantity (EOQ) is the order quantity that minimizes the total holding costs and ordering costs. It is one of the oldest classical production scheduling models. The model was developed by Harris (1913), and then Wilson (1934) expanded this model to the Wilson model. Although the Wilson model has been used for a very long time, the theory and applications of this model have yet to be stated clearly and systematically. To bridge the gap in the literature in this area and provide academics and practitioners with an overview of the Wilson model. This is the primary motivation for us to study this article. This first paper presents a detailed and complete system of the origin and applications of the Wilson model.

The rest of the paper is organized as follows. We review the general concepts and principles of reserve management in Section 2. In Section 3, we introduce to Wilson model, a workaround, and some examples of this model. Some applications of the Wilson model are presented in Section 4. Conclusions and discussions will be provided in the last section.

2 General concepts and principles of reserve management

2.1 Reserve management system

There are two ubiquitous problems that inventory managers have to address: when do they need to order, and how many orders need to be placed? To solve the first issue, one must determine when one starts ordering. Two main systems are used: A fixed order quantity system - K model that requires managers to reorder inventories when they are below the minimum level that is called the reorder point. A fixed time period system (or issuing a production order) - P model: a week, a month, or any specific time, regardless of the inventory reserve. Next, to reply to the second question, this answer is based on the previous answers. If reserve managers order in fixed days and numbers, it can be challenging to adapt to the demand fluctuations.

Therefore, it can be seen that there is a change in either times or quantities. This leads to the principles of these two inventory management systems: If managers utilize a fixed order quantity system, they only have one number at a time. Meanwhile, suppose managers use the fixed time period system. In that case, they will order with different quantities according to each time which the inventory manager appreciates retaining the target inventory levels.

It can be observed that managers can choose between two inventory management systems: Looking at the first system could x the number of order goods(K), and when the reserve drops gradually to reorder point. Whereas the order cycles could be changed and depend on the demand fluctuations of goods. If the demand increases (decreases), the reorder point will occur sooner (later). The second system could x the order cycles (monthly, quarterly,...) while the change in the order quantity. They check the remaining and calculate the quantities of goods to achieve the target inventory level. The order quantity is equal to the previous period's sold quantity.

2.2 Research ordering system/ fixed order quantity model

The principle of the K model is that when the reserve drops to reorder point (the alarming stock), managers need to calculate precisely the expected demand during the time span to guarantee that their company can respond to the requirements of customers as also for manufacturing during the time waiting for goods from suppliers, which leads a company to prevent the stock-out situation.

The expected demand and lead time vary around the average value. Thus, the reorder point is the sum of the average expected demand and safety stock during the average time span. These systems require managers to conduct ordering or producing goods immediately when reaching a reorder point. This causes some problems below: Initially, many kinds of goods are made by one supplier, but their reorder points happen at different times. Hence, managers need to separate orders differently.

The manufactured organizations (in the enterprise as well as in the supplier) conduct an order that is often influenced by themselves because of before and after manufactured programs so that time-spans and cycle criteria usually last longer, and these systems do not follow fixed order quantity. Therefore, companies need to have flexible structural manufacturing or have certain reserves in suppliers to operate effectively. The carrying cost is large because managers must always check the inventory level to notify when the reserves reach the reorder point immediately. Nevertheless, this problem can be addressed through computer systems; it records changes in reserve at all times and reports as soon as the reserve touches its lowest point.

In conclusion, the fixed order quantity system is appropriate when the following factors are satisfied: The request is highly volatile, the high-value products need to limit because it will cause great damage, the production systems are flexible, and there are reserves at the suppliers.

2.3 Periodic regeneration system/ fixed time period system

This system checks cyclical inventory levels and orders quantities of goods sold out in the previous period. The quantity of goods for each order is determined by target inventory minus inventory position (need to ensure safety stock). When the target inventory is fixed at a high level, the average reserve will be high, and the carrying costs will increase drastically. On the other hand, if the target level is too low, which leads to a low average reserve and higher risk.

The advantages of a fixed time period system are possible to connect the requirements at the same suppliers to deduct the costs of management, order, and delivery. On the contrary, this system will be unsuitable when the demands change suddenly. So as to limit them, companies must accept large inventory levels.

It can be observed that a fixed time period system is effective when the following conditions are satisfied: The demands and lead time have fewer changes, the manager cannot request or regularly order from suppliers or production processes, a large number of low-value products cannot cause the increase of carrying cost. We next turn on to discuss the Wilson model in the next section.

3 Wilson model

In this section, we will present a detailed and complete system of the origin and applications of the Wilson model and two examples to illustrate the approach. The Wilson model is a ubiquitous model used to determine the optimal order quantity and reserve management costs. In inventory management, economics order quantity (EOQ) is the order quantity that minimizes the total holding costs and ordering costs. It is one of the oldest classical production scheduling models. This model was first developed by Harris (1913) and extended by Wilson (1934) to become a Wilson model. We first describe the problem.

3.1 Problem

We use an example in which there is only one kind of good for demand in the T period $(T = 1\, year)$ and the good has H units. Consumption of this good is regular and the ordering cost is L, unit price is P, carrying cost per one unit is I, ordering time is T_0. Assume that one needs to end the number of orders and the quantity of each order so that the total cost is the smallest.

Before setting up the model and addressing this problem, one needs to have the following concepts: The reserve time T (often T is 1 year or 365 days), the lead time is the period from the beginning of the order to the time that the order is to be reserved and consumed, denoted by T_0. The review period: is the part of the time between two warehousings (or ex-warehousing) denoted by

$$t_i,\ i = 1,...,n. \tag{1}$$

The reorder point: place a new order whenever the inventory level drops to themselves, denoted by B. We address this problem in the next sub-section.

3.2 Modeling the problem

We divide T into n periods of reserve and consumption, the corresponding quantity of goods in the ith period that is $K_i, i = 1,..,n$. Thus, in t_i cycle, the average inventory level is $K_i / 2$, carrying cost is $\frac{K_i t_i IP}{2}$; ordering cost L is a fixed cost for an order, including the cost of transaction and other business. The total cost of purchase is \$PH\$. Hence, the total cost function is given by

$$G(T,n) = nL + \sum_{i=1}^{n} \frac{IPt_i K_i}{2} + PH \ . \tag{2}$$

Because consumption is regular, it can be seen that $t_i = \frac{K_i}{H}, \forall i$. Thus, one has t_i is defined in (1). Thereafter, we get

$$\begin{cases} G(T,n) = nL + \sum_{i=1}^{n} \frac{IPK_i^2}{2H} + PH \\ \sum_{i=1}^{n} K_i = H \end{cases}, \tag{3}$$

for n, it can be observed that $G(T,n)$ reaches a minimum value when $K_i = K = H / n, \forall i$, it mean that the quantity of goods is K in each time. Thus, it can be seen that to find the minimum of $G(T,n)$ one needs to find the minimum of:

$$G(K) = LH / K + IPK / 2 + PH \ .$$

Because PH is a constant, one needs to figure out K to minimize the following function

$$F(K)=\frac{LH}{K}+\frac{IPK}{2},$$

where LH/K is ordering cost, $IPK/2$ is carrying cost and $F(K)$ is the function of variable K. We now turn on discuss to how to address this problem in the following subsection.

3.3 Solving the problem

It is well known that $F(K)$ is a function of variable K, while L, H, I, P are constants. To find the minimum of this function, one needs to find its derivative with respect to K and then let it equal to 0.

$$\frac{dF(K)}{dK}=-\frac{LH}{K^2}+\frac{IP}{2}=0\Leftrightarrow\frac{K^2}{LH}=\frac{2}{IP}.$$

Thereafter, we get

$$K^2=\frac{2LH}{IP}\Rightarrow K^*=\sqrt{\frac{2LH}{IP}},$$

and

$$\frac{d^2F(K)}{dK^2}=\frac{2LK}{K^3},\frac{d^2F\left(K^*\right)}{dK^2}=\frac{2LH}{\left(K^*\right)^3}>0.$$

Hence, K^* is the minimum point, or the optimal order quantity.

The total cost minimum can be expressed as follows

$$F\left(K^*\right)=\frac{LH}{K^*}=\frac{IPK^*}{2}=\frac{LH}{\sqrt{\frac{2LH}{IP}}}+\frac{IP}{2}\sqrt{\frac{2LH}{IP}}=\sqrt{2LHIP}. \quad (4)$$

Thus, the total cost is provided by $G\left(K^*\right)=L\frac{H}{K^*}+\frac{IPK^*}{2}+PH$ or $G\left(K^*\right)=F\left(K^*\right)+PH$.

Thereafter, we get the optimal order times : $n^*=\frac{H}{K^*}$,

and the optimal of review period: $t^*=\frac{1}{n^*}=\frac{K^*}{H}$.

Thus, the optimal reorder point is given by

$$B^* = H\left(T_0 - t^*\left\lfloor \frac{T_0}{t^*} \right\rfloor\right), \tag{5}$$

where, $\left\lfloor \frac{T_0}{t^*} \right\rfloor$ is a floor of $\frac{T_0}{t^*}$. The process work of the Wilson model is illustrated in Figure 1.

To help readers understand the model and know how to apply the model, we offer two examples in the following subsection.

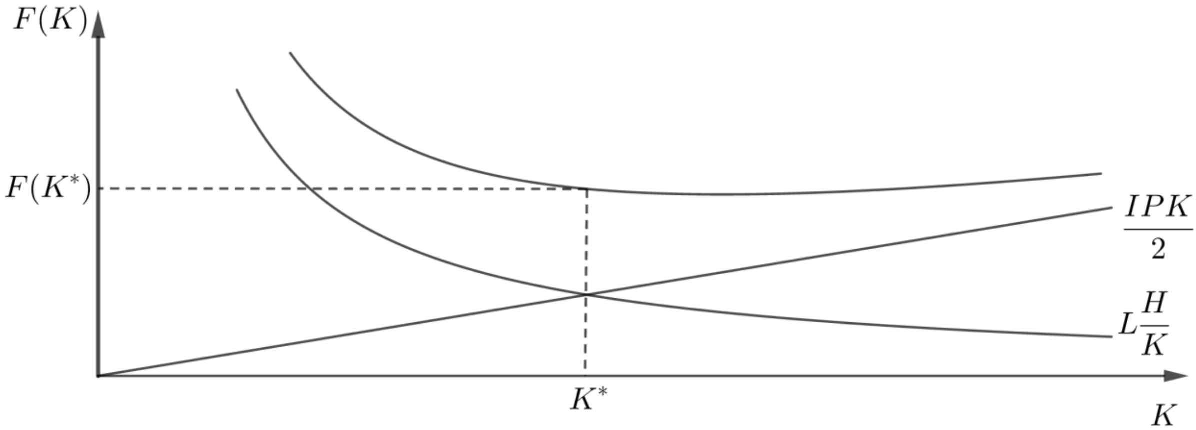

Figure 1: The process work of the Wilson model

3.4 Examples

This section provides two examples to illustrate how to use optimal order quantity and reserve management costs in the analysis. We first discuss the following example to obtain the optimal order quantity and the total cost minimum:

Example 3.1 *Assuming that the demand for store steel is 400,000 tons/year, the ordering cost is \$300 each time, the price is \$120/ton, the carrying cost per one unit is 0.02, and the lead time is two months, what are the numbers of optimal order quantity* K^* *and the total cost minimum* $F(K^*)$, *and* $G(K^*)$?

Solution

It can be seen that $H = 400{,}000$ tons, $L = \$300$, $P = \$120$, and $I = 0.02$. Thus, we get the lead time to be

$$T_0 = \frac{2}{12} = \frac{1}{6},$$

the optimal order quantity K^* to be

$$K^* = \sqrt{\frac{2HL}{IP}} = \sqrt{\frac{2\times 400,000\times 300}{0.02\times 120}} = 10,000 \text{ (tons)},$$

and the total cost minimum $F\left(K^*\right)$ that can be calculated in the following steps:

$$\begin{aligned} F\left(K^*\right) &= \frac{IPK^*}{2} + \frac{LH}{K^*} \\ &= \frac{0.02\times 120}{2} 10,000 + \frac{300\times 400,000}{10,000} \\ &= 24,000 \text{ (dollars)}. \end{aligned}$$

Thereafter, we obtain the optimal order times to be

$$n^* = \frac{H}{K^*} = \frac{400,000}{10,000} = 40 \text{ (times)},$$

and the optimal review period to be

$$t^* = \frac{1}{n^*} = \frac{1}{40}.$$

Hence, the optimal reorder point can be obtained by

$$B^* = 400,000\left(\frac{1}{6} - \frac{1}{40}\left[\frac{\frac{1}{6}}{\frac{1}{40}}\right]\right) = 6,666.7 \text{(tons)}.$$

This is the inventory level when companies need to order for the next period. From this info, we can obtain the total cost that can be calculated by

$$G\left(K^*\right) = 24,000 + 120\times 400,000 = 48,024,000 \text{ (dollars)}.$$

We turn to discuss the second example.

Example 3.2 *Supposing that a construction company has a total demand of 300,000 tons a year, the consumption of materials is regular, the price of one ton is \$2500, and the ordering cost is \$500. In addition, the carrying cost coefficient is 0.05 and the lead time is two months. What are the basic indexes of the company's stock and consumption?*

Solution

From the given conditions, one can easily obtain $H = 300,000$ tons, $L = \$500$, $P = \$2500$, and $I = 0.05$. Hence, the lead time can be calculated by

$$T_0 = \frac{2}{12} = \frac{1}{6},$$

and the optimal order quantity K^* follows:

$$K^* = \sqrt{\frac{2HL}{IP}} = \sqrt{\frac{2 \times 300,000 \times 500}{0.05 \times 2500}} \; ; \; 1549.2 \text{ (tons)}.$$

In addition, the total cost minimum $F\left(K^*\right)$ can be obtained by

$$\begin{aligned} F\left(K^*\right) &= \frac{IPK^*}{2} + \frac{LH}{K^*} \\ &= \frac{0.05 \times 2500}{2} 1549.2 + \frac{500 \times 300,000}{1549.2} \\ &= 193649.2 \text{ (dollars)}. \end{aligned}$$

From the above, we can get the total cost as follows:

$$\begin{aligned} G\left(K^*\right) &= F\left(K^*\right) + PH \\ &= 193649.2 + 2500 \times 300,000 \\ &= 750193649 \text{ (dollars)}, \end{aligned}$$

and obtain the optimal order times to be

$$n^* = \frac{H}{K^*} = \frac{300,000}{1549.2} = 193.6483 \text{ times.}$$

Thereafter, we can obtain the optimal review period that follows:

$$t^* = \frac{1}{n^*} = \frac{1}{193.6483} \text{ year } ; \; 1.4 \text{ days}.$$

Hence, the optimal reorder point can be calculated by using the following steps:

$$\begin{aligned} B^* &= 300,000 \left(\frac{1}{6} - \frac{1}{193.6483} \left[\frac{\frac{1}{6}}{\frac{1}{193.6483}} \right] \right) \\ &= 300,000 \left(\frac{1}{6} - \frac{32}{193.6483} \right) \\ &= 425.5911 \text{ (tons)}. \end{aligned}$$

This is the inventory level that the company requires to order in the next period. We will discuss some applications using the Wilson model in the next section.

4 Applications of the Wilson model in Computational Sciences

In this section, we introduce the two important applications of the Wilson model in Computational Sciences: recreating continuous reserves and reduced selling prices for large orders. We first present the example of applying the Wilson model in recreating continuous reserves.

4.1 Recreate continuous reserves

In the model of recreating continuous reserves, we assume the reserve process is provided by a production procedure that has N products made per unit of time to reproduce K^* once. During this time, the number of ex-stocking (sales) are d goods per one unit of time, the number of products sold is: $d\frac{K}{N}$. The inventory level is not K, that is: $K-\frac{d}{N}K$ or $\left(1-\frac{d}{N}\right)K$. The average inventory level is not $K/2$, that is: $\frac{\left(1-\frac{d}{N}\right)K}{2}$.

We have the total cost of the order and the reserve is given by

$$F(K)=L\frac{H}{K}+\frac{\left(1-\frac{d}{N}\right)KIP}{2},$$

and the optimal order quantity can be expressed by

$$K^*=\sqrt{\frac{2LH}{IP\left(1-\frac{d}{N}\right)}}.$$

Thereafter, we obtain the total cost minimum is

$$F\left(K^*\right)=\sqrt{2LHIP\left(1-\frac{d}{N}\right)}.$$

It can be observed that the optimal order time is $n^*=\frac{H}{K^*}$, and the optimal review period will be equal to $t^* = \frac{1}{n^*} = \frac{K^*}{H}$.

After that, we get total cost is calculated by

$$G\left(K^*\right) = F\left(K^*\right) + PH = \sqrt{2LHIP\left(1 - \frac{d}{N}\right)} + PH .$$

We next discuss reducing selling prices for large orders in the next section.

4.2 Reduced selling prices for large orders

The suppliers often reduce goods prices if the customer purchases goods that large to increase sell quantities and reduce carrying costs. The purchase quantity is higher, and the discount is higher. The discounts have been identified and noticed by the supplier. The problem is that the buyer must answer the question: Whether the total cost of purchase and reserve of large quantities of goods decreases or not when they accept the supplier's request.

4.2.1 Modeling

The demand for only one kind of good in the T period is H units. Ordering cost is L each time; the carrying cost coefficient is I, the goods price is a function of K (the quantity purchased each time) variable, as follows:

$$\begin{cases} K < S_1 & \text{price is } c_1 \\ S_2 \le K \le S_3 & \text{price is } c_3 \\ \text{..................} & \\ S_{k-1} \le K \le S_k & \text{price is } c_k \\ \text{..................} & \\ S_{n-1} \le K & \text{price is } c_n \end{cases}$$

where $S_1 < S_2 < ... < S_{n-1}$ and $c_1 > c_2 > ... > c_n$, $S_0 = 0$ and $S_n = +\infty$.

4.2.2 Setup the model

For simplicity's sake, one can assume that the lead time is negligible (immediate addition). We now calculate the purchase price.

Let $G_i(K) = \frac{LH}{K} + IP_i\frac{K}{2} + P_iH, i = 1, 2, ..., n.$ one has

$$G(K)=\begin{cases} G_1(K), K\in(0;S_1) \\ G_2(K), K\in[S_1;S_2) \\ \ldots\ldots\ldots\ldots. \\ G_{n-1}(K), K\in[S_{n-2};S_{n-1}) \\ G_n(K), K\in[S_n;+\infty) \end{cases}.$$

4.2.3 Solving

It can be seen that

$$G_1(K) > G_2(K) > \ldots > G_n(K), \forall K > 0 \text{ (for } c_1 > c_2 > \ldots > c_k\text{)}.$$

Assuming that K_i^* is a minimum point of $G_i(K)$, thus, one has

$$K_i^* = \sqrt{\frac{2LH}{IP_i}} = \sqrt{\frac{\beta}{P_i}}; \left(\beta = \frac{2LH}{I}\right).$$

so $G_{i-1}\left(K_{i-1}^*\right) > G_i\left(K_{i-1}^*\right) > G_i\left(K_i^*\right)$.

On the other hand, since c_i decreases, thus, we have $K_{i-1}^* < K_i^*$.

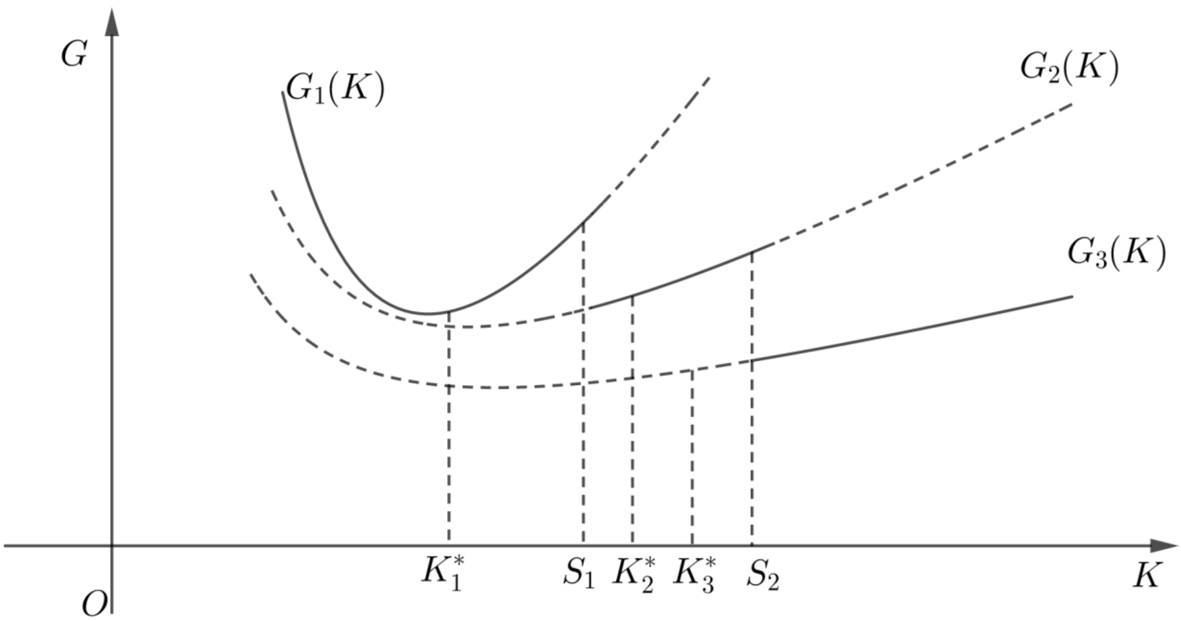

Figure 2: The algorithm to find the K^* optimal value in the case of three prices

The algorithm to find the K^* optimal value in the case of three prices is illustrated in Figure 2.

In general, the algorithm to find the K^* optimal value that can be expressed as follows:

We first consider the case of two prices: Calculate $K_2^* = \sqrt{\dfrac{2LH}{IP_2}}$

If $K_2^* \geq S_1$, the optimum order quantity:

$$K^* = K_2^*$$

This case is described in Figure 3.

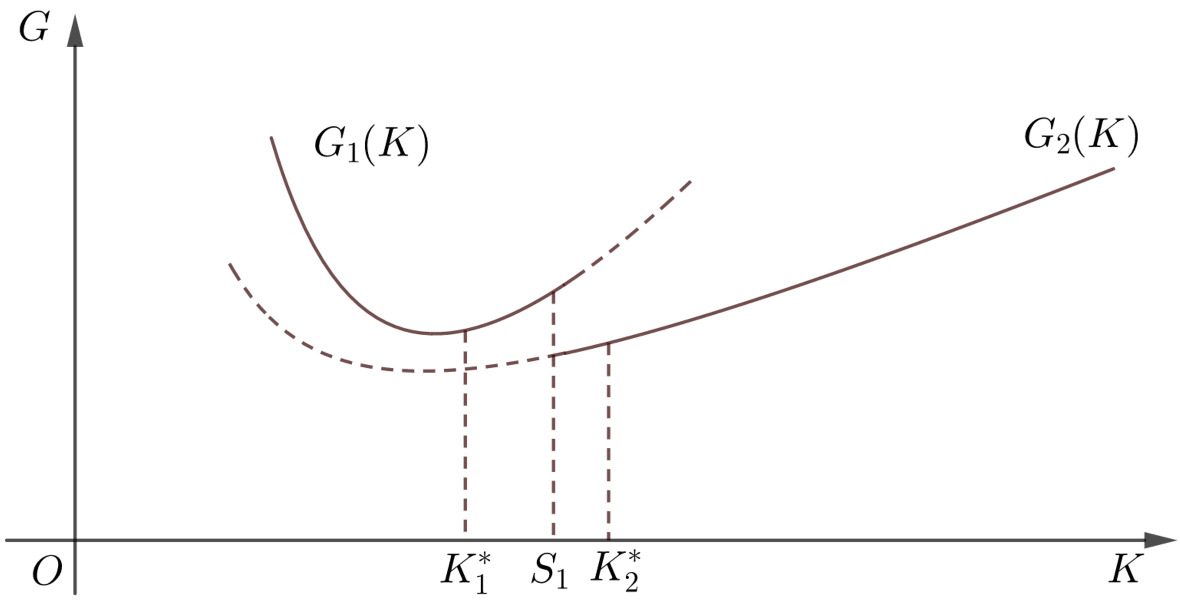

Figure 3: The algorithm to find the K^* optimal value in case of two prices

This case is described in Figure 3.

If $K_2^* < S_1$, then $G_2(S_1) = \dfrac{LH}{S_1} + \dfrac{IP_2S_1}{2} + P_2H$ is the minimum of total cost function with $K \geq S_1$.

Thereafter, we get $K_1^* = \sqrt{\dfrac{2LH}{IP_1}}$ and $G_1\left(K_1^*\right) = \sqrt{2LHIP} + P_1H$

We can divide into three categories as follows: (see Figure 4-6)

- Case 1: If $G_2(S_1) < G_1\left(K_1^*\right)$ then $K^* = S_1$
- Case 2: If $G_2(S_1) > G_1\left(K_1^*\right)$ then $K^* = K_1^*$

- Case 3: If $G_2(S_1) = G_1\left(K_1^*\right)$ then $K^* = K_1^* = S_1$

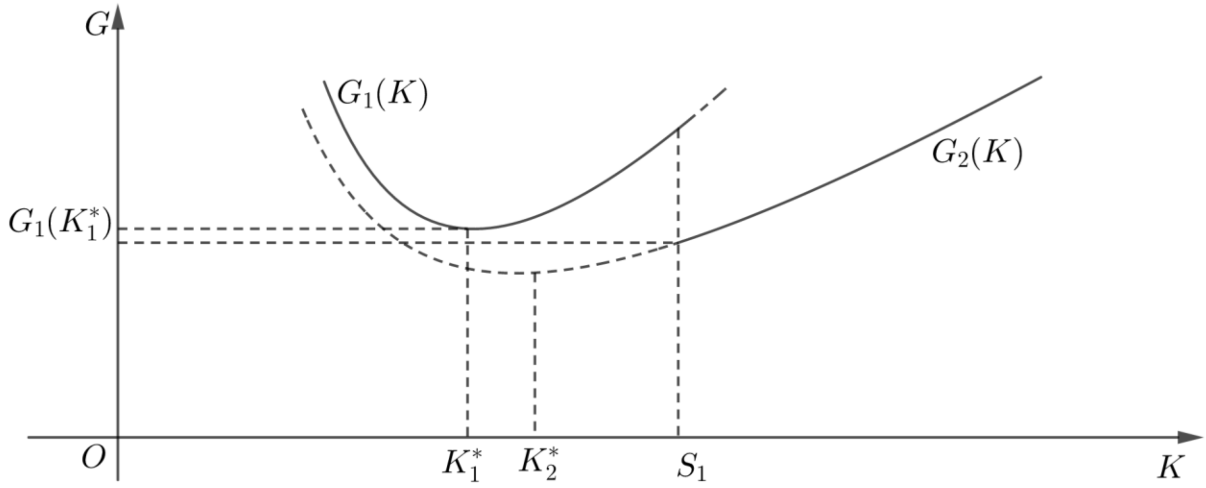

Figure 4 : Case 1

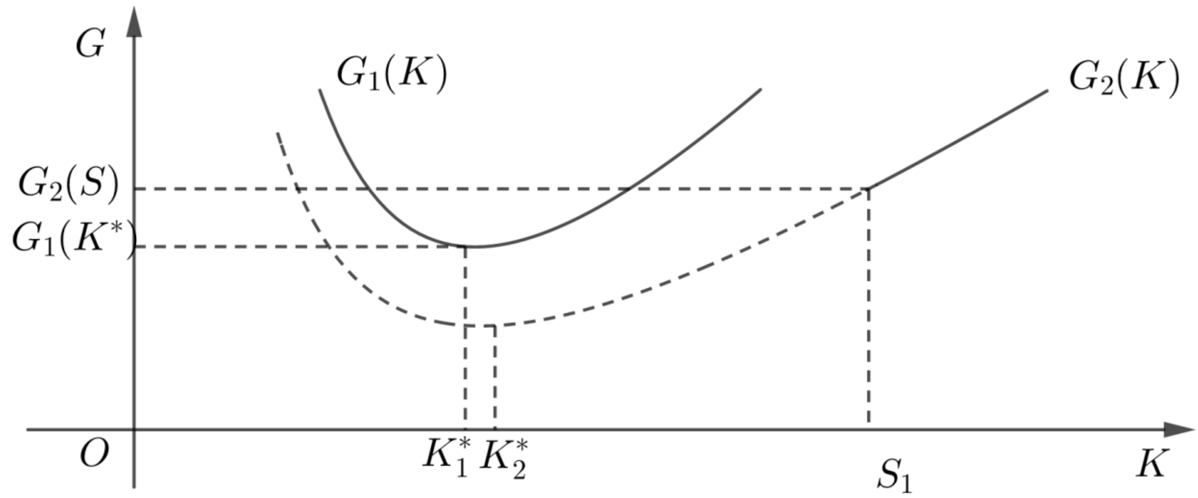

Figure 5 : Case 2

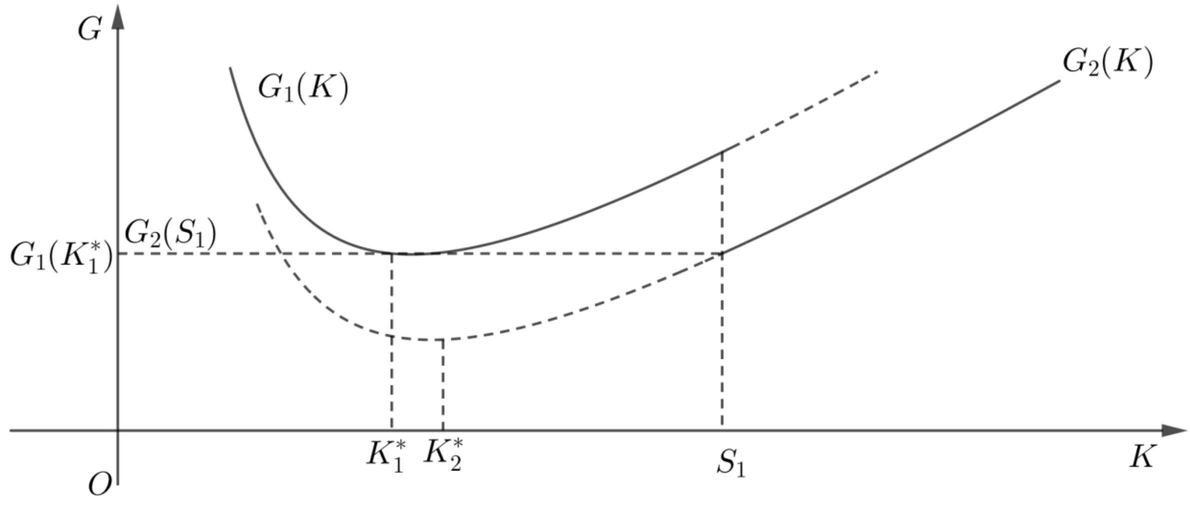

Figure 6 : Case 3

In general case, we also consider the $G_n(K)$ function to find the minimum of $G(K)$ function in the $[S_{n-1};+\infty)$ interval. If this value is minimum, we get $K^* K_n^*$ is the value where $G(K)$ is minimum (with $q<+\infty$); otherwise, we take $G_n(S_{n-1})$ is the local minimum value.

In $[S_{n-2};S_{n-1})$ interval, we find $G(K)$ is the minimum value of G_{n-1} function; and compare to the minimum value in previous interval, etc. Thereafter, we get the first value so that $K_i^* \in [S_{i-1};S_i)$ is K^*.

To help readers see applications of this model, we go over two examples in the following subsection.

4.3 Examples

4.3.1 Example 3

Assuming that the demand for only one kind of good in the T period is 500,000 units. The production density $N=200$ and the sale density $d=100$ units. The ordering costs is \$500 each time, the price is \$50/product, the coefficient of carrying costs is 0.01, lead time is two months. Please determine the optimal order quantity K^* and the total cost minimum $F\left(K^*\right)$, thereby calculating $G\left(K^*\right)$.

Solution

We have the optimal order quantity calculated by

$$K^* = \sqrt{\frac{2LH}{IP\left(1-\dfrac{d}{N}\right)}} = \sqrt{\frac{2\times 500\times 500,000}{0.01\times 50\left(1-\dfrac{100}{200}\right)}} = 44721.36 \text{ (tons)}$$

and the total cost minimum $F\left(K^*\right)$ is given by:

$$F\left(K^*\right)=\frac{H}{K^*}L+\frac{K^*\left(1-\frac{d}{N}\right)}{2}IP$$
$$=\frac{500,000}{44721.36}500+\frac{44721.36\left(1-\frac{100}{200}\right)}{2}0.01\times 50\,.$$
$$=11180.34 \text{ (dollars)}$$

Thus, the total cost is

$$G\left(K^*\right)=11180.34+500,000\times 50=25,011,180 \text{ (dollars)}$$

It can be seen that, the optimal order times $n^*=\frac{H}{K^*}=\frac{500,000}{44721.36}\,;\ 11$ times.

and the optimal review period is given by $t^*=\frac{1}{n^*}=\frac{1}{11}$ year.

Thereafter, we get the optimal reorder point:

$$B^*=H\left(T_0-t^*\left[\frac{T_0}{t^*}\right]\right)=500,000\left(\frac{1}{6}-\frac{1}{11}\right)=37878.79 \text{ (tons)}$$

4.3.2 Example 4

Supposing that in a company that sells one kind of electric bulb, the total consuming products are 20,000 barrels/year. The ordering cost is \$20 each time; the coefficient of carrying cost is 20%, and sales are regular. The lead time is negligible. If they order 2,000 barrels or more, the barrel price will be \$120; otherwise, the price is \$125/barrel. Determine the purchased quantities in each time so that the total cost is minimum, and find the review period and corresponding reorder points.

Solution

According to the assumption, it can be seen that $H=20,000$ barrels; $L=\$20$; $I=0.2; P_1=\$125; S_1=2000$ cases; $P_2=\$120$. Then one has

$$K_2^*=\sqrt{\frac{2LH}{IP_2}}=\sqrt{\frac{2\times 20\times 20000}{0,2\times 120}}\,.$$
$$=182,5742\,;\ 183 \text{ barrels}$$

Since $K_2^*<2000=S_1$,

$$\begin{aligned} F_2(S_1) &= \frac{LH}{S_1} + \frac{IP_2S_1}{2} + P_2H \\ &= \frac{20 \times 20000}{2000} + \frac{0,1 \times 120 \times 2000}{2} + 120 \times 20000 . \\ &= 2412200 \end{aligned}$$

On the other hand, one has:

$$K_1^* = \sqrt{\frac{2LH}{IP_1}} = \sqrt{\frac{2 \times 20 \times 20000}{0,2 \times 125}} = 178.8854 \; ; \; 179 ,$$

then

$$\begin{aligned} F_1\left(K_1^*\right) &= \frac{IP_1K_1^*}{2} + \frac{LH}{K_1^*} + P_1H \\ &= \frac{0,2 \times 125 \times 179}{2} + \frac{20 \times 20000}{179} + 125 \times 20000 . \\ &= 2504472 \end{aligned}$$

Hence, $F_2(S_1) = 2412200 < 2504472 = F_1\left(K_1^*\right)$.

Thereafter, we obtain, the optimal order quantities are: $K^* = S_1 = 2000$.

The purchase times are $\frac{20000}{2000} = 10$ in one year; review period is $\frac{365}{10} = 36.5$ days.

5 Discussions and Conclusions

In this paper, we provide the general concept and principle of reserve management with applications in computational sciences. We first present the origin and the theory of the Wilson model. The Wilson model plays a very important role in many areas, including computational sciences, economics, construction, business, and nance. Thereafter, we give two examples to illustrate the applications of the Wilson model so that academics and practitioners can learn how to use the Wilson model. We note that all the issues developed in our paper are very important issues, but, so far, we have not seen any book or paper discussing the examples we illustrated. Thus, the issues developed in our paper are now and have some good contributions to the literature.

In addition, we provide four examples to help readers understand and apply the model. The results of the examples in our article are trustworthy and useful for managers to use the Wilson model to get the optimal order quantity and reserve management costs. If managers do not use this approach, their order quantity and reserve management costs will not be optimal, and the managers could pay a higher price. Thus, this paper is a useful reference for managers, academics, and practitioners in getting the optimal order quantity and reducing the reserve management cost.

About the extension of this issue, it will be known that when one gets the optimal order quantity and reserve management cost, sometimes, one could be interested in the problems of finding the shortest path in the distribution of the company's goods to another company. Combining these two issues becomes a very engaging and meaningful topic in practice. Until now, there are scholars who have not yet researched and investigated it. About the approach to address the problem of finding the shortest path, readers may refer to Gallo and Pallottino (1988), Pallottino and Scutella (1998), Feillet et al. (2004), Truong et al. (2019), etc. These are interesting research directions in the hereafter.

Furthermore, the Wilson model presented in this article can be used to address several practical problems. This model will work actively if the data set and figures have missing values or errors in measurement. Readers may refer to Little (1992), Lee et al. (2021) and Pho (2022a, b) for the methods to solve the problems that have missing values.

Moreover, the Wilson model plays an extremely crucial role in current economics. Wilson model is a widespread model utilized to obtain the optimal order quantity and reduce reserve man-agement costs. The applications of this model are abundant, not only in economics but also in many other sectors such as finance, construction, education, etc. Nevertheless, not many scholars have applied this model in the current period. We can utilize this model to optimal order quantity and reserve management cost in numerous other disciplines such as engineering, business, technology, banking, etc. Readers may refer to Edeki, et al. (2021), Guimarães, et al. (2021), Hon, et al. (2021), McAleer (2021), Moslehpour, et al. (2021), Nhan, et al. (2021), Jaiswal, et al. (2022), Oanh, et al. (2022), TajMazinani, et al. (2022), Truong, et al. (2021), Islam, et al. (2021), Wong (2020, 2021, 2022), and Woo, et al. (2020) for more examples in different areas to apply the Wilson model.

For the limitations of the Wilson model, it can be seen that the optimal reorder point B^* is provided as in the formula (5). In some of the cases, the result of the floor of $\frac{T_0}{t^*}$ is $\frac{T_0}{t^*}$. In this situation, we cannot get the result of B^*. This is the biggest limitation when applying this model. It would be very significant if we find a solution to overcome this problem in the future. This is also the direction of our interest in this article.

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